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Advertising in Asymmetric Competing Supply Chains

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Abstract

Advertising is a crucial tool for demand creation and market expansion. When a manufacturer uses a retailer as a channel for reaching end customers, the advertising strategy takes on an additional dimension: which party will perform the advertising to end customers. Cost sharing ("cooperative advertising") arrangements proliferate the option by decoupling the execution of the advertising from its funding. We examine the efficacy of cost sharing in a model of two competing manufacturer-retailer supply chains who sell partially substitutable products that may differ in market size. Some counterintuitive findings suggest that the firms performing the advertising would rather bear the costs entirely, if this protects their unit profit margin. We also evaluate the implications of advertising strategy for overall supply chain efficiency and consumer welfare.

Keyword: manufacturer advertising; retailer advertising; cost sharing; supply chain competition; game theory

1 Introduction

Having a great product to sell is not enough. At some point in the life of almost every business, advertising becomes a crucial tool for demand creation and market expansion. By one estimate, 2010's advertising activity totaled more than \$300 billion in the United States and \$500 billion worldwide.[1] While both parties in the supply chain or channel[2] can simultaneously advertise the product, a common practice is for one or the other to take nearly exclusive responsibility for the advertising. For example, major retailers Walmart and Target frequently advertise certain products so that many of their thousands of global suppliers do not feel the need to. In contrast, Mengniu, an Asian dairy manufacturer, handles all advertising activities while expressly prohibiting its retailers from doing any (Ni, 2007). In franchising systems, franchisors such as McDonald's Corporation often perform all advertising on behalf of their franchisees.

However, for one party to perform the advertising does not necessitate that this party must bear

[1]Source: http://en.wikipedia.org/wiki/Advertising.

[2]Throughout this paper we will use the terms channel and supply chain interchangeably, taking into account any preexisting customs in the research and practitioner communities.

all the costs. Cost sharing has often been implemented in the form of cooperative advertising (e.g., Berger, 1972; He et al., 2009; Huang and Li, 2001; Jorgensen et al., 2000; Xie and Neyret, 2009). In 2002 manufacturers gave approximately $60 to $65 billion in promotional assistance to their retail partners (Arnold, 2003). Franchisees are frequently required to share advertising costs with their franchisors.

Advertising, including manufacturer advertising, retailer advertising, and cooperative advertising, has been documented very well in the extant literature (see Bagwell, 2005; Chen et al., 2009; Iyer et al., 2005; Little, 1979). To the best of our knowledge, none of it has comprehensively examined what advertising strategies might arise in competing supply chains with asymmetric market sizes, and how cost sharing might influence the outcome. This paper intends to answer these questions, with explicit consideration of competition at both the manufacturer and the retailer levels.

We will present a model of two competing supply chains, where in each supply chain a manufacturer sells its product exclusively through a downstream retailer.[3] This is representative of distribution conditions for some products in categories such as gasoline, soft-drink concentrates, beer, automobiles, clothing, fast food, fork-lift trucks, and heavy farm equipment (Doraiswamy et al., 1979; McGuire and Staelin, 1983). Similar models have been widely adopted in the extant literature (e.g., Ha and Tong, 2008; McGuire and Staelin, 1983; Wu et al., 2007). Our point of departure is in incorporating advertising with the potential for cost-sharing, and allowing asymmetry in market size. To focus on the disparity in advertising cost-efficiency between manufacturer advertising and retailer advertising, we assume that at most a single party in each supply chain, either the manufacturer or the retailer, will advertise. We compare scenarios with and without cost sharing for the advertising, for games structured as follows: (Stage 1) the designated potential advertisers decide to advertise or not; (Stage 2) the manufacturers simultaneously determine their own wholesale prices and advertising levels (if the game considers manufacturer advertising); and (Stage 3) the retailers simultaneously set their own retail prices and advertising levels (if the game considers retailer advertising). For each game we characterize the sub-game perfect equilibrium.

We first investigate manufacturer advertising and retailer advertising without cost sharing. Our

[3] We have analyzed additional structures, including a monopoly common retailer and a duopoly common retailer channel, and found results consistent with those presented here.

analysis demonstrates that in manufacturing advertising a dominant equilibrium strategy is for both manufacturers to advertise; however, they can encounter a Prisoner's Dilemma. That is, while a manufacturer can earn more by advertising regardless of whether its rival also advertises, the advertising can intensify the competition to a point where eventually both manufacturers are made worse off. This occurs when product substitutability is sufficiently high. When the manufacturers advertise, they tend to increase the wholesale prices to cover some of the advertising costs, which in turn elevates retail prices and exacerbates double marginalization. Under retailer advertising, an asymmetric equilibrium (in which only one retailer advertises) emerges because the smaller (less powerful) retailer becomes averse to competition when product substitutability is high. When the retailers advertise, the manufacturers reduce the wholesale prices, which enables the retailers to enhance their advertising levels and lower retail prices, consequently bolstering competition between the supply chains. When product substitutability is sufficiently low, the benefits of reduced double marginalization in retailer advertising significantly outweigh the strengths of manufacturer advertising. However, as product substitutability grows, the supply chain competition will reach such a level that the advertising levels need to be kept in check. Manufacturer advertising does that better than does retailer advertising.

We next study the impact of sharing the cost of the advertising. In manufacturer advertising with cost sharing, we find that the manufacturers generally prefer cost sharing. However, they encounter a Prisoner's Dilemma when the cost sharing rate is substantially high. This is intuitive because a higher cost sharing rate induces the manufacturers to engage in an advertising war that backfires. Retailer advertising has a similar dynamic, with the retailers becoming more sensitive to cost sharing and refraining from advertising when the cost sharing rate is too high. Adding cost sharing does not change the general preferences of manufacturers and retailers about who should do the advertising. However, because cost sharing intensifies product competition, retailer advertising becomes less attractive to all parties when product substitutability is sufficiently high.

At a prima facie level, cost sharing would seem to benefit the parties that advertise since they obtain "free" money from their supply chain partners. This is true for manufacturer advertising as long as the cost sharing rate is low. Surprisingly the retailers in our model do not welcome cost sharing when they are the ones to advertise, realizing that in this case "what one hand giveth, the other hand taketh away.” With the manufacturers increasing wholesale prices to compensate for the advertising subsidies they pay out, the retailers end up worse off even with their advertising-stimulated revenue gains. This surprising discovery may help explain industry reports that while many manufacturers make the funds available, “much of the cooperative advertising funds money goes unspent, as relatively few retailers and wholesalers pursue cooperative agreements.”[4] In practice, retailers who advertise may prefer additional side payments from the manufacturers or insist on wholesale price reduction rather than explicit cost sharing.

Besides examining the outcomes for the individual firms in the competing supply chains, we are also able to comment on overall supply chain performance and outcomes for the end consumer. For each supply chain, if advertising is performed, doing so with cost sharing is superior when and only when the cost sharing rate is sufficiently low. Regarding consumer welfare, more intense competition generally leads to lower retail prices and larger demand; therefore, advertising with cost sharing is better for consumers than that without. If cost sharing is to be performed, consumers fare better when the manufacturers handle the advertising instead of the retailers when the cost sharing rate is sufficiently high, because increased cost sharing for retailer advertising pushes up wholesale prices and in turn the retail prices.

Our work is related to the large volume of literature on advertising in the past several decades (see Bagwell (2005), Little (1979), and the references therein), which we will not exhaustively review due to space limitations. It is worth noting that few works have examined the market expansion effect of advertising as modeled in our work. For example, recent studies on competitive advertising involving two retailers or channels typically assume a fixed unit mass of consumers (e.g., along a Hotelling line, as in Chen et al., 2009; Iyer et al., 2005; Shaffer and Zettelmeyer, 2004, 2009; von der Fehr and Stevik, 1998; Wu et al., 2009), thus the expansion effect on the (aggregate) market is assumed away. Specifically, in these models a firm can increase its own demand if it is the only one advertising but aggregate demand remains constant if both competitors advertise. A body of research studies advertising from empirical and other perspectives different from ours (Erickson, 2003; Tellis, 2004), which thus far has not focused on how the efficacy of advertising’s market expansion ability varies with product substitutability, channel asymmetry,

[4]Source: http://www.inc.com/encyclopedia/cooperative-advertising.html.

and the extent of any cost sharing. The literature on channel structures is vast (see Cai, 2010; Cattani et al., 2004; Chen et al., 2007; Gilbert and Bhaskaran-Nair, 2009; Ingene and Parry, 2004; Mukhopadhyay et al., 2008; Ryan et al., 2012; Tsay and Agrawal, 2004; Wang et al., 2011), but most entries focus on matters other than the advertising structures with and without cost sharing.

A research stream on cooperative advertising does exist, but most entries have focused on a vertical channel with a single manufacturer and a single retailer (bilateral monopoly) (e.g., Berger, 1972; He et al., 2009; Huang and Li, 2001; Jorgensen et al., 2000; Xie and Neyret, 2009). Among the exceptions, Bergen and John (1997) considered a Hotelling model with a manufacturer selling through two retailers and found cooperative advertising to be an efficient coordination mechanism. Karray and Zaccour (2007) discussed a duopoly common retailer channel and suggested that results from bilateral monopoly models do not apply to competitive scenarios. Yan et al. (2006) compared cooperative advertising between Bertrand and Stackelberg competitions in a dual exclusive channel and demonstrated that the advertising can increase the players' profits in both game settings. Doraiswamy et al. (1979) studied the equilibrium in a symmetric dual exclusive channel with pure advertising effort (no cost-sharing) under the condition that the retailers will always advertise if the manufacturers do not. Our work diverges from these papers by providing a more comprehensive equilibrium analysis (including asymmetric equilibrium and multiple equilibria in manufacturer, retailer, and hybrid advertising structures with asymmetric channels) and explicitly studies the impact of cost sharing on players' preferences towards advertising strategy.

The remainder of this paper is organized as follows. We describe the model in Section 2. We study manufacturer advertising and retailer advertising in Section 3. The discussion on advertising with cost sharing resides in Section 4. We analyze supply chain efficiency and consumer welfare in Section 5 and conclude in Section 6. Appendix A (Online Supplements) explores additional properties of advertising effort levels, and extends the analysis to structures that we term "hybrid advertising" (in which one supply chain uses manufacturer advertising while the other uses retailer advertising) and "all efforts" (in which both the manufacturer and retailer advertise in each supply chain). All proofs are relegated to Appendix B (Online Supplements).

2 The Model

We consider a dual exclusive channel model, also referred to as dual exclusive supply chains (Ha and Tong, 2008; McGuire and Staelin, 1983), defined as two manufacturer-retailer dyads whose products compete in the end-customer market. We diverge from the extant literature by explicitly incorporating advertising decision-making and allowing asymmetry between the demand functions addressed by the two dyads.

In our notation the index i $(i = 1, 2)$ identifies the channel or supply chain or product. D_i represents the demand for the product produced and sold by supply chain i. Retail prices are p_i, and wholesale prices are w_i. A_i is supply chain i's initial base demand/market, meaning the amount that would be consumed when $p_i = 0$, no advertising is performed, and the supply chains do not compete. e_{mi} is the advertising intensity of Manufacturer i, whereas e_{ri} denotes the advertising level by Retailer i. With the impact of advertising, the new base demand becomes

$$\alpha_i = A_i + \mathbf{1}_{mi} e_{mi} + \mathbf{1}_{ri} e_{ri}, \tag{1}$$

where $\mathbf{1}_{mi} = 0$ or 1 is the indicator of whether Manufacturer i advertises in supply chain i. Similarly, $\mathbf{1}_{ri} = 0$ or 1 is the indicator of whether Retailer i advertises in supply chain i. Our formulation of the decision problems of the channel parties will enforce the logical necessity that a player that cannot couple a choice to not advertise (setting its own indicator variable to 0) with a positive advertising intensity. A player that chooses to advertise is free to follow through with virtually zero advertising intensity, though. The assumption that a firm can strategically commit in this way to advertising or non-advertising is in line with Banerjee and Bandyopadhyay (2003), Doraiswamy et al. (1979), Dukes (2009), and Wang et al. (2011). Some well-known retailers, such as Costco, and manufacturers, such as Ferrari, follow non-advertising strategies (CBCRadio, 2012). Bonnevier and Boodh (2011) observe that the non-advertising approach has been utilized by famous brands such as Maison Martin Margiela (a French fashion brand) and Ladurée (a French food company), and has recently increased in frequency among clothing brands, restaurants, and industries distributing goods of less durable character.

Because one of our main goals in this paper is to compare the efficacy of cost sharing in manufacturer advertising and retailer advertising, we will restrict attention to structures in which each supply chain contains at most one advertiser. In mathematical shorthand, this requires $\mathbf{1}_{mi} + \mathbf{1}_{ri} \leq 1$.

These respective functions represent the cost of advertising effort:

$$C(e_{mi}) = \lambda_{mi} e_{mi}^2 \text{ and } C(e_{ri}) = \lambda_{ri} e_{ri}^2.$$

The quadratic form conveys diminishing returns, which follows naturally from a presumption that rational managers will always target the "lowest-hanging fruit," so that subsequent improvements are progressively more difficult. This is consistent with Chen et al. (2009), Desai (1997), Doraiswamy et al. (1979), Tsay and Agrawal (2000), and the references therein. To enable fair comparison among the various advertising structures and for parsimony, we assume $\lambda_{mi} = \lambda_{ri} = 1$. Our sensitivity analysis shows that this does not compromise our findings.

Demand for product i takes the following form, which has precedent in works such as Ingene and Parry (2004):

$$D_i = \frac{\alpha_i - \theta\alpha_{3-i} - p_i + \theta p_{3-i}}{1-\theta^2}, i = 1, 2. \tag{2}$$

In this construction θ $(0 \leq \theta < 1)$ captures product substitutability, while the impact of advertising is embedded in the α_i values as governed by Eq. (1).[5]

To communicate the potential asymmetry between the markets faced by the two supply chains,

[5] The specific form of this demand function comes from consideration of the utility/surplus function of a representative consumer, as developed in Spence (1976), Dixit (1979), Shubik and Levitan (1980), Singh and Vives (1984), and Ingene and Parry (2007). This customer's utility is

$$U \equiv \sum_{i=1,2} (\alpha_i D_i - D_i^2/2) - \theta D_1 D_2 - \sum_{i=1,2} p_i D_i, \tag{3}$$

Since its introduction, this utility function has been widely utilized in the economics, marketing, and other related literature (see Cai et al., 2012; Choi and Coughlan, 2006; Ingene and Parry, 2004, 2007; Qiu, 1997; Singh and Vives, 1984). It exhibits the classical economic properties that the utility of owning a product decreases as the consumption of the substitute product increases, and the representative consumer's marginal utility for a product diminishes as the consumption of the product increases. It also implies that the value of using multiple substitutable products is less than the sum of the separate values of using each product on its own (Samuelson, 1974). When $\theta = 0$, the products are purely monopolistic; as θ goes to 1, the products converge to purely substitutable.

Maximization of Eq. (3) yields the demand in Eq. (2).

we define

$$\Omega \equiv \frac{A_1}{A_2}.$$

We also refer to Ω as *base demand ratio*. If $\Omega > 1$, supply chain 1's initial base demand is larger than supply chain 2's. This parameter will play a prime role in framing the findings of this research.

For parsimony, production costs and supply chain operational costs are normalized to zero.[6] The parameter η_i articulates how the cost of any advertising in supply chain i will be allocated, where $\eta_i = 0$ if the advertising party bears the cost entirely, while $0 < \eta_i \le 1$ indicates cost-sharing (cooperative advertising).

Manufacturer i 's and Retailer i's profits are then, respectively,

$$\Pi_{mi} = D_i w_i - \mathbf{1}_{mi}(1-\eta_i)e_{mi}^2 - \mathbf{1}_{ri}\eta_i e_{ri}^2, \quad (4)$$

$$\Pi_{ri} = D_i(p_i - w_i) - \mathbf{1}_{mi}\eta_i e_{mi}^2 - \mathbf{1}_{ri}(1-\eta_i)e_{ri}^2. \quad (5)$$

As noted earlier, the indicator variables designate the party that will perform any advertising for the channel. The variable combinations are summarized in the following table.

Table 1: Parameters specifying how advertising is performed and funded in supply chain i.

	No Cost Sharing	Cost Sharing
Manufacturer i advertises	$\mathbf{1}_{mi} = 1; \mathbf{1}_{ri} = 0; \eta_i = 0$	$\mathbf{1}_{mi} = 1; \mathbf{1}_{ri} = 0; 0 < \eta_i \le 1$
Retailer i advertises	$\mathbf{1}_{mi} = 0; \mathbf{1}_{ri} = 1; \eta_i = 0$	$\mathbf{1}_{mi} = 0; \mathbf{1}_{ri} = 1; 0 < \eta_i \le 1$

Manufacturer advertising and retailer advertising each proceed as a three-stage game. In Stage 1, the designated potential advertisers commit to advertising or not. In Stage 2, the manufacturers simultaneously determine their own wholesale prices and advertising levels (if the game considers manufacturer advertising). In Stage 3, the retailers simultaneously set their own retail prices and advertising levels (if the game considers retailer advertising).

[6] We have also analyzed cases with asymmetric non-zero operational costs and found that all our qualitative results hold. These can be obtained with the simple adjustment $\Omega \equiv \frac{A_1 - c_1}{A_2 - c_2}$, where c_i denotes the unit operational cost in supply chain i.

The following sections will examine manufacturer advertising and retailer advertising, and study the impact of cost sharing. In each subgame each party will seek to independently maximize its profit as defined above. We will obtain and analyze the sub-game perfect equilibrium outcomes.

3 Advertising without Cost Sharing

To separate the effect of how the advertising is performed from the effect of how it is funded, we first study manufacturing advertising and retailer advertising with no cost sharing.

3.1 Manufacturer Advertising

Manufacturing advertising can manifest in four different ways: both manufacturers advertise (MM), only Manufacturer 1 advertises (MN), only Manufacturer 2 advertises (NM), or neither manufacturer advertises (NN). We identify each subgame with a two-character string in which the first character describes who advertises in the first supply chain (M for the manufacturer, N for none), and likewise for the second character and the second supply chain. Table 1 indicates how these games map to parameter settings. Specifically, $\mathbf{1}_{ri} = 0$ and $\mathbf{1}_{mi} = 1$ in Eqs. (4) and (5) for MM, MN, and NM whenever Manufacturer i would advertise for supply chain i; otherwise $\mathbf{1}_{mi} = 0$. All our discussions presume the common feasible domains of the specific cases, as detailed in the Appendix.

In all four subgames, in the first stage the manufacturers simultaneously determine their respective optimal wholesale prices and advertising level(s). $\mathbf{1}_{ri} = 0$ and $\mathbf{1}_{mi} = 1$ in Eqs. (4) and (5) for MM, MN, and NM whenever Manufacturer i would advertise for supply chain i; otherwise $\mathbf{1}_{mi} = 0$. The retailers then simultaneously determine their respective retail prices. This specifies the manufacturers' profits, which implies an equilibrium for the stage of the game in which each manufacturer decides whether or not to advertise.

Because advertising increases the base demand for both products, allowing the option to advertise would seem to potentially increase the players' profits. This following lemma confirms this.

Lemma 1 *Under manufacturer advertising, a manufacturer benefits from its own advertising but is hurt by the rival manufacturer's. That is, for Manufacturer* 1*, MN outperforms NN and MM outperforms NM, while the opposite is true for Manufacturer* 2*.*

Lemma 1 is straightforward. It echoes the conventional wisdom that a manufacturer is rewarded for its own advertising but negatively affected by its competitor's. While a manufacturer's advertising generates more demand for its own product, it also encroaches on the other manufacturer's existing markets. This intensifies their channel/product competition. The rival manufacturer then has no choice but to step up its own advertising effort. Therefore, advertising is a dominant equilibrium strategy for both manufacturers as stated below.

Theorem 1 *Under manufacturer advertising, MM is the unique equilibrium strategy. However, the manufacturers can encounter a Prisoner's Dilemma if product substitutability is sufficiently high (e.g.,* $0.823 \leq \theta < 0.940$ *when* $\Omega = 1$*).*

Theorem 1 suggests that both manufacturers benefit from advertising when product substitutability is low. However, advertising could make both manufacturers worse off in MM than in NN when product substitutability is sufficiently intense. Figure 1 graphically illustrates Theorem 1. The explicit forms of all boundary values, such as the $\hat{\Omega}$ terms shown in the various figures and analytical results, are uniformly very complicated so we relegate these to Appendix B.

When product substitutability is lower, each manufacturer behaves more like a monopolist. Here advertising significantly increases each supply chain's own demand without encroaching on the other's too much. Furthermore, double marginalization is reduced by the intensified supply chain competition stimulated by the advertising. The manufacturers thus find advertising to be mutually beneficial, as illustrated in the Pareto Zone of Figure 1.

However, as product substitutability grows, advertising intensifies the horizontal competition between supply chains. The retailers must cut retail prices, pressuring both manufacturers to reduce the wholesale prices and thereby their profit margins. Beyond a certain level of substitutability the

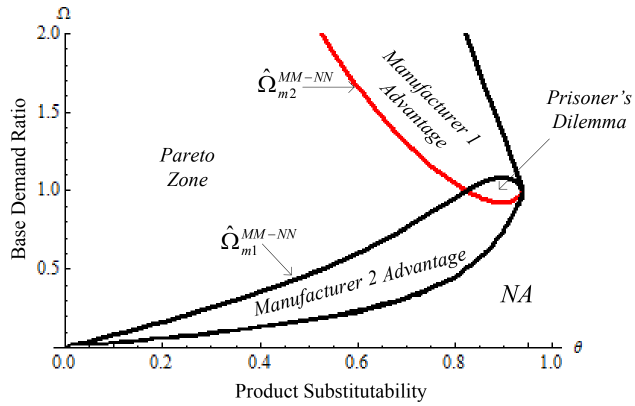

Figure 1: Manufacturers' profit comparison between MM and NN under manufacturer advertising. (Throughout this paper NA identifies the area corresponding to infeasible parameter combinations.)

manufacturers face a Prisoner's Dilemma. Both prefer that neither advertises, but if either party does not then the other has positive incentive to advertise. The practical implication for manufacturers is that they should sufficiently differentiate their products. This is even more important for a manufacturer with a smaller base market. When supply chain competition is sufficiently intense (the products are highly substitutable), this party loses more demand due to its rival's advertising that it can gain from its own advertising. This is depicted in Figure 1.

Whether the manufacturers can follow through on the initial commitment to non-advertising when unilateral deviation may provide benefit (albeit not in a sustainable way if in the context of a Prisoner's Dilemma) has been discussed in Dukes (2009) and Wang et al. (2011). Dukes (2009) argued, "The discussion...points to a potential benefit to firms if they could somehow commit themselves to not advertise. One way that firms might try to reduce competitive advertising is to use a common marketing agency to control the level of advertising. Another way is to induce regulated limits on advertising as has been done for professional services such as lawyers and doctors. Another possibility, which occurs in markets where advertising strategy in one period may depend on what happened in earlier periods (which are modeled as "repeated games"), is to undertake disciplinary advertising levels whenever rivals "cheat" by doing more advertising than was agreed (possibly implicitly) upon."

Do these findings apply only when the manufacturer in each supply chain does the advertising? We investigate by next considering retailer advertising.

3.2 Retailer Advertising

As with manufacturer advertising, retailer advertising has four possible outcomes: both retailers advertise (RR), only Retailer 1 advertises (RN), only Retailer 2 advertises (NR), or neither retailer advertises (NN). In all four cases $\mathbf{1}_{mi} = 0$ for $i = 1, 2$, while $\mathbf{1}_{ri} = 1$ whenever Retailer i advertises in supply chain i. In each subgame, the manufacturers determine the wholesale prices, and then the retailers simultaneously determine their respective retail prices and advertising levels.

We now compare the retailers' profits when they advertise and when they do not.

Lemma 2 *Under retailer advertising, there exist boundary values (denoted as $\hat{\Omega}$ with various superscript and subscript combinations) such that*

1. *Retailer 1 benefits from its own advertising when its rival does not advertise (going from NN to RN) if and only if $\Omega > \hat{\Omega}_{r1}^{RN-NN}(\theta)$, and when its rival advertises (going from NR to RR) if and only if $\Omega > \hat{\Omega}_{r1}^{RR-NR}(\theta)$, where*
   (a) $\hat{\Omega}_{r1}^{RN-NN}(\theta) < \hat{\Omega}_{r1}^{RR-NR}(\theta) < 1$*; and*
   (b) $\hat{\Omega}_{r1}^{RN-NN}(\theta)$ *and* $\hat{\Omega}_{r1}^{RR-NR}(\theta)$ *increase with* θ*.*
2. *Retailer 1 is hurt by its rival retailer's advertising when it does not advertise (going from NN to NR) if and only if $\Omega < \hat{\Omega}_{r1}^{NR-NN}(\theta)$, and when it advertises (going from RN to RR) if and only if $\Omega < \hat{\Omega}_{r1}^{RR-RN}(\theta)$, where*
   (a) $1 < \hat{\Omega}_{r1}^{RR-RN}(\theta) < \hat{\Omega}_{r1}^{NR-NN}(\theta)$*; and*
   (b) $\hat{\Omega}_{r1}^{RR-RN}(\theta)$ *and* $\hat{\Omega}_{r1}^{NR-NN}(\theta)$ *decrease with* θ*.*

The corresponding results for Retailer 2 can be stated by changing every instance of "1" in the variable indices to "2" and exchanging "NR" with "RN."

Consistent with Lemma 1 in the analysis of manufacturer advertising, Lemma 2 shows that a retailer can still earn extra profits from its own advertising but is hurt as its rival advertises. This is particularly true when product substitutability is low. However, a retailer might ultimately suffer from its own advertising but benefit from its rival's. This result differs from that under manufacturer advertising and runs counter to conventional wisdom. Unlike manufacturer advertising, retailer advertising lacks an intervening vertical cushion to soften the horizontal supply chain competition (in the sense of McGuire and Staelin (1983)), which consequently leads to higher advertising levels and drives the retail prices lower than those under manufacturer advertising. As product substitutability grows, the competition under retailer advertising becomes so intense, more than that under manufacturer advertising, that it outweighs the benefit of the accompanying reduction in double marginalization.

Consider the impact of channel asymmetry. When a retailer is in the supply chain with the smaller base market, it faces the prospect of earning insufficient incremental profit from its own advertising, which cannot compensate for the advertising costs incurred. This becomes more apparent when both retailers advertise, as compared to the case where only a single retailer advertises (i.e., when $\hat{\Omega}_{r1}^{RN-NN}(\theta) < \hat{\Omega}_{r1}^{RR-NR}(\theta)$). On the other hand, when a supply chain possesses the larger base market, its retailer is more resistant to the rival retailer's advertising and can benefit from the reduction in double marginalization due to its rival's advertising.

Evaluating the possibility of unilateral deviation from RR, RN, NR, and NN, we find that an asymmetric advertising strategy can be an equilibrium.

Theorem 2 *Under retailer advertising, RR is an equilibrium if and only if* $\hat{\Omega}_{r1}^{RR-NR}(\theta) < \Omega < \hat{\Omega}_{r2}^{RR-RN}(\theta)$*; RN is an equilibrium if and only if* $\hat{\Omega}_{r2}^{RR-RN}(\theta) < \Omega < \bar{\Omega}^{RR}(\theta)$*; NR is an equilibrium if and only if* $\underline{\Omega}^{RR}(\theta) < \Omega < \hat{\Omega}_{r1}^{RR-NR}(\theta)$*.*

Theorem 2 indicates that the existence of a specific equilibrium depends on the extent of product substitutability and the base demand disparity between supply chains 1 and 2, as illustrated in Figure 2.

If the retailers' respective base demands are comparable (as defined by the intermediate range $\hat{\Omega}_{r1}^{RR-NR}(\theta) < \Omega < \hat{\Omega}_{r2}^{RR-RN}(\theta)$), both retailers would benefit from advertising. However, if one supply chain has significantly larger base market than the other and product substitutability is high,

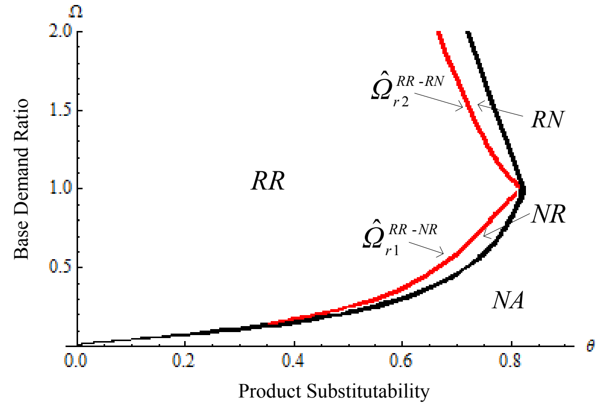

Figure 2: Equilibrium result in retailer advertising.

then the other retailer would rather stay out of the advertising game. Intensifying the competition is simply counterproductive when the competition level is already high. If, say, supply chain 1's base market is larger than supply chain 2's, then RN is an equilibrium as long as $\hat{\Omega}_{r2}^{RR-RN}(\theta) < \Omega < \bar{\Omega}^{RR}(\theta)$. Retailer 2 prefers to not pick a fight from its weak position, allowing Retailer 1 to capture more of the market through its unilateral advertising.

Unlike in manufacturer advertising, where a Prisoner's Dilemma may arise, subgames RN and NR emerge as the unique equilibria in their corresponding feasible areas. This finding is consistent with the observation of Banerjee and Bandyopadhyay (2003) that "private label brands that never advertise in categories, such as beer continue to thrive in markets in which large entrenched national brands command a high share of the consumers' mind."

4 Advertising with Cost Sharing

When advertising with cost sharing, also known in practice as *cooperative advertising*, one player performs the advertising while its supply chain partner bears some portion of the cost. Recent studies on cooperative advertising, mainly in a single manufacturer-retailer channel (see Berger, 1972; Huang and Li, 2001; Jorgensen et al., 2000; Xie and Neyret, 2009), suggest that both the manufacturer and the retailer can benefit from cost sharing. This naturally calls for investigation of whether the result extends to competitive settings, which our model is well-prepared to address.

As in the existing literature, we define cost sharing within a supply chain as the advertising party's collecting from the other party a proportion, η_i, of the advertising cost. The appropriate profit expressions come from applying the salient parameters from Table 1 to Eqs. (4) and (5). For example, in CSRR (RR with cost-sharing), the profits of Manufacturer i and Retailer i are, for i=1,2,

$$\begin{aligned} \Pi_{CSRR-mi} &= D_i w_i - \eta_i e_{ri}^2, \\ \Pi_{CSRR-ri} &= D_i(p_i - w_i) - (1-\eta_i)e_{ri}^2. \end{aligned} \tag{6}$$

In CSMM (MM with cost-sharing), the profits of Manufacturer i and Retailer i are, for i=1,2,

$$\begin{aligned} \Pi_{CSMM-mi} &= D_i w_i - (1-\eta_i)e_{mi}^2, \\ \Pi_{CSMM-ri} &= D_i(p_i - w_i) - \eta_i e_{mi}^2. \end{aligned} \tag{7}$$

For completeness we also analyze CSMN, CSNM, CSRN, CSNR, and CSNN (i.e., NN), where at most one player in the system advertises. The corresponding demand outcomes, D_i, follow Eq. (2). For analytic tractability, we focus on the symmetric setting (i.e., $A_1 = A_2 = 1$ so that $\Omega = 1$), which will be sufficient to deliver our managerial findings. We will analyze the asymmetric case (i.e., $\Omega \neq 1$) numerically. To establish that the efficacy of cost sharing comes from the advertising structure rather than the specific cost sharing rates or channel asymmetry, we unify the cost sharing rates by letting $\eta_i = \eta$. In reality η would result endogenously from the balance of power between the manufacturer and the retailer. That falls beyond the scope of our model, so we will report how each party's profits vary with η.

Below we will separately examine manufacturer advertising with cost sharing and retailer advertising with cost sharing. This will highlight the impact of the advertising structure. Then we will compare the structures with cost sharing to the ones without, to demonstrate the impact of cost sharing.

4.1 Manufacturer Advertising and Retailer Advertising, Both with Cost Sharing

In manufacturer advertising with cost sharing, the manufacturers determine their wholesale prices and advertising level(s) simultaneously in the first stage. In the second stage the retailers set the retail prices. We need consider Manufacturer 1 only, as the results for Manufacturer 2 follow by symmetry. The appropriate profit expressions come from setting $\mathbf{1}_{ri} = 0$ and $\mathbf{1}_{mi} = 1$ in Eqs. (4) and (5) for CSMM, CSMN, and CSNM whenever Manufacturer i would advertise for supply chain i; otherwise $\mathbf{1}_{mi} = 0$. The following lemma reports the equilibrium analysis for the cases with and without manufacturer advertising.

Lemma 3 *Under manufacturer advertising with cost sharing given $\Omega = 1$, cooperative advertising is a dominant equilibrium strategy for both the manufacturers. However, the manufacturers encounter a Prisoner's Dilemma if the rate of sharing is sufficiently high (i.e., $\eta > \hat{\eta}_{mi}^{CSMM-NN}(\theta)$).*

Lemma 3 is an extension of Theorem 1 to the system with cost sharing. It suggests that advertising continues to be a dominant strategy for the manufacturers regardless of the cost sharing level. This is mutually beneficial for the manufacturers when product substitutability is sufficiently low. However, as product substitutability (θ) grows, the demand-stimulating impact of advertising diminishes. This is because additional advertising costs drive up the wholesale price, and in turn the retail prices, which worsens the double marginalization. At some point, advertising does not generate enough benefit to offset the disadvantages of the intensified competition. This is when the Prisoner's Dilemma emerges, which is similar to Theorem 1. The conditions for this can now be stated in terms of the extent of cost sharing: for "sufficiently high" η the Prisoner's Dilemma occurs because a higher cost sharing rate stimulates heavier advertising from the manufacturers, which further worsens the double marginalization and reduces what the manufacturers can gain from advertising. The threshold for the Prisoner's Dilemma, $\hat{\eta}_{mi}^{CSMM-NN}(\theta)$, is a decreasing function of θ, which can be established numerically. This corroborates the mechanism described earlier: the lower the $\hat{\eta}_{mi}^{CSMM-NN}(\theta)$, the larger the set of circumstances for which the Prisoner's Dilemma in manufacturer advertising arises; and the higher the product substitutability, the lower the $\hat{\eta}_{mi}^{CSMM-NN}(\theta)$.

In retailer advertising with cost sharing (with profit functions generated by $\mathbf{1}_{mi} = 0$ for $i = 1, 2$ while $\mathbf{1}_{ri} = 1$), the manufacturers simultaneously determine their respective wholesale prices in the first stage. In the second stage, the retailers simultaneously determine their respective retail prices and amounts of advertising (if any). The following lemma documents the outcomes with and without retailer advertising.

Lemma 4 *Under retailer advertising with cost sharing given $\Omega = 1$, CSRR is an equilibrium if and only if $\eta < \hat{\eta}_{r1}^{CSRR-CSNR}(\theta)$, while NN could be an equilibrium if and only if $\eta > \hat{\eta}_{r1}^{CSRN-NN}(\theta)$. The retailers can encounter a Prisoner's Dilemma when both advertise if the cost sharing rate is sufficiently high (e.g., $\hat{\eta}_{r1}^{CSRR-NN}(\theta) < \eta < \eta_{r1}^{CSRN-NN}(\theta)$) .*

Lemma 4 shows that both advertising and no advertising can be equilibria for the retailers. So cost sharing is a key determinant of whether the retailers will choose to advertise. Recall that for retailer advertising without cost sharing, Theorem 2 reported that advertising is the unique equilibrium for the retailers under the symmetric demand setting ($\Omega = 1$). In light of that finding, Lemma 4 demonstrates that a high cost sharing rate surprisingly might be disadvantageous to the retailers. Shifting advertising costs to the manufacturers encourages the retailers to increase their advertising levels and intensifies the supply chain competition, which in turn erodes the retailers' profits. This becomes more pronounced as the product substitutability level increases. Cost sharing also induces the manufacturers to increase the wholesale prices, which further reduces the retailers' profits. Counterintuitively, if the cost sharing rate is low, the retailers can benefit from advertising; otherwise, the cost sharing will not motivate them to advertise. We further find a small range ($\hat{\eta}_{r1}^{CSRR-NN}(\theta) < \eta < \hat{\eta}_{r1}^{CSRN-NN}(\theta)$) such that CSRR is the unique dominant equilibrium strategy for the retailers. However, NN remains more profitable than CSRR for the retailers, which is again a Prisoner's Dilemma.

When $\Omega \neq 1$ numerically we find the subgame CSMM continues to dominate CSMN, CSNM, and NN throughout the feasible domain. For retailer advertising with cost sharing, the subgame CSRN can be the unique equilibrium if supply chain 1's initial base demand is larger than supply chain 2's ($\Omega > 1$) and the product substitutability is sufficiently large. CSNR can be the unique equilibrium if Chain 2's initial base demand is larger than Chain 1's ($\Omega < 1$) and the product substitutability is sufficiently large. This observation is similar to that without cost sharing, although the equilibrium boundary line between the CSRR and CSRN/CSNR regions shifts leftward because the cost sharing intensifies the horizontal competition.

4.2 The Value of Cost Sharing

The extant literature on cooperative advertising has generally found cost sharing to be an effective channel coordination mechanism in a single channel setting (see Berger, 1972; Huang and Li, 2001; Jorgensen et al., 2000; Xie and Neyret, 2009). This is not surprising since cost sharing gives the manufacturer an additional instrument for influencing the retailer to advertise more and thereby increase the channel's demand. We demonstrate that this property might not survive the addition of supply chain competition, inasmuch as higher advertising levels could undesirably intensify the competition in certain scenarios, e.g., retailer advertising with cost sharing. As discussed previously, the retailers are more sensitive to their own advertising, due to the absence of the competitive buffer that the intermediaries provide in manufacturer advertising. This leads to our main research question: *When would cost sharing be mutually beneficial for the manufacturers and the retailers?*

The answer comes from comparing the players' profits in manufacturer and retailer advertising with and without cost sharing. In the symmetric case (i.e., $\Omega = 1$), CSRR is the only equilibrium in retailer advertising with cost sharing and RR is the unique equilibrium in retailer advertising without cost sharing. The same is true of CSMM and MM in manufacturer advertising with and without cost sharing, respectively. So we compare CSRR to RR and CSMM to MM, as expressed in the following theorem.

Theorem 3 *In the symmetric setting (i.e., $\Omega = 1$), there exist $\hat{\eta}_{r1}^{CSMM-MM}(\theta) < \hat{\eta}_{m1}^{CSMM-MM}(\theta) < \hat{\eta}_{m1}^{CSRR-RR}(\theta)$, such that*

1. *Under retailer advertising, the manufacturers prefer cost sharing (CSRR) to no cost sharing (RR) if and only if $\eta < \hat{\eta}_{m1}^{CSRR-RR}(\theta)$, while the retailers always prefer no cost sharing (RR) to cost sharing (CSRR).*

2. *Under manufacturer advertising, the manufacturers prefer cost sharing (CSMM) to no cost sharing (MM) if and only if $\eta < \hat{\eta}_{m1}^{CSMM-MM}(\theta)$, while the retailers prefer cost sharing (CSMM) to no cost sharing (MM) if and only if $\eta < \hat{\eta}_{r1}^{CSMM-MM}(\theta)$.*

Theorem 3 catalogs a divergence between manufacturers and retailers in preferences towards cost sharing. When the retailers are responsible for advertising, the manufacturers see some merit in sharing some of the advertising costs to encourage sufficient advertising. This holds as long as the cost sharing rate is not too high and product substitutability is sufficiently low, because the manufacturers can recover their advertising subsidy by increasing the wholesale prices. These advantages do not persist when the cost sharing rate is high, as wholesale prices, and in turn retail prices, then need to be raised to a level so high that demand decreases. This is more pronounced when product substitutability is high.

A more surprising result is that retailers who advertise would rather do so without cost sharing. This is because the manufacturers will simply increase the wholesale prices to recover some of the advertising subsidy. Higher retail prices follow, which counteracts the demand-stimulating impact of the advertising. This theoretical result is consistent with the industry report cited earlier, which stated that much of manufacturers' cooperative advertising funds "goes unspent, as relatively few retailers and wholesalers pursue cooperative agreements."

In fact, our model enables a crisp statement of the deeper point. The advertising level is not the end goal of the manufacturer, only an intermediate step on the way to increased sales (and presumably increased profit). Cooperative advertising fees are tied only to this intermediate activity, whereas the wholesale price impacts the retailer on every unit sold. That is, they are both ways to share costs, but are structurally different. In this light, the wholesale price can be seen as the mechanism that is more directly tied to the desired end outcome. Indeed, the CEO of a consumer electronics manufacturer whose products are sold through more than 36,000 retail storefronts in North America has noted that his firm uses no cooperative advertising at all. He has found that his retail partners will promote the product aggressively when the manufacturer assures them of an attractive margin on each unit sold (Finnegan, 2011).

In CSMM, both the manufacturers and the retailers can benefit from cost sharing, as long as the cost sharing rate and product substitutability are relatively small. Because $\hat{\eta}_{r1}^{CSMM-MM}(\theta) < \hat{\eta}_{m1}^{CSRR-RR}(\theta) < \hat{\eta}_{m1}^{CSRR-RR}(\theta)$, manufacturers are more likely than retailers to advocate cost sharing in both CSRR and CSMM for any cost sharing rate.[7] On the other hand, if the cost sharing

[7]Manufacturers prefer a higher sharing rate in CSMM. Since RR dominates CSRR for the retailers, the preferred

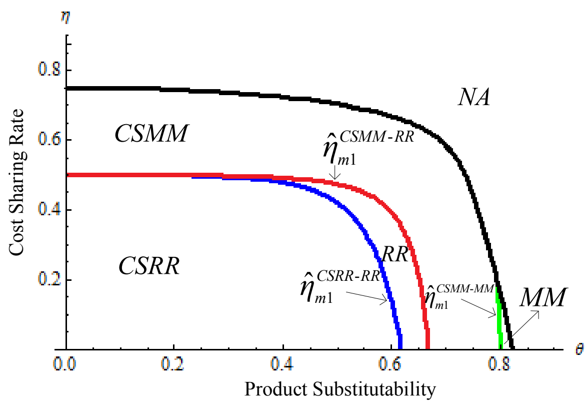

Figure 3: Manufacturer 1's preferences regarding RR, MM, CSRR and CSMM given $\Omega = 1$.

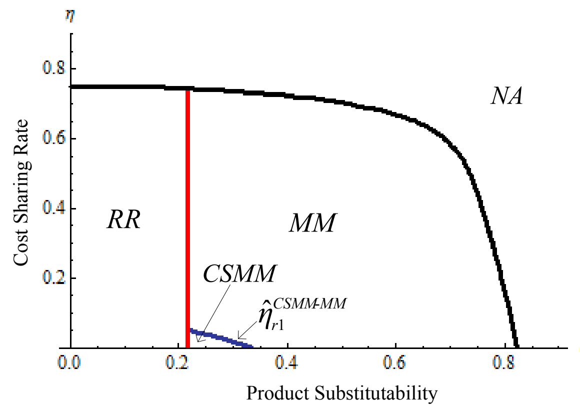

Figure 4: Retailer 1's preferences regarding RR, MM, CSRR and CSMM given $\Omega = 1$.

rate is too high, cost sharing becomes undesirable for all players, because the advertisers are motivated to intensify the advertising and therefore the degree of competition, while the accompanying increased wholesale prices put downward pressure on demand and profits for all players. The best cost sharing rate is somewhere in between, so as to strike a balance among these forces.

Figures 3 and 4 summarize the manufacturers' and the retailers' preferences regarding MM, RR, CSMM, and CSRR. Figure 3 shows that CSRR performs best for the manufacturers when the cost sharing rate and product substitutability are low. As product substitutability grows, RR gains favor. However, the dominance quickly shifts to CSMM with further increases in the cost sharing rate and product substitutability. MM dominates if product substitutability becomes very high, as the benefit of less intense competition at the manufacturer level prevails. Figure 4 presents the perspective of the retailers, who prefer RR when product substitutability is low, since RR is effective in expanding markets when both supply chains are relatively monopolistic. MM generally dominates when product substitutability is high, although CSMM could outperform MM in a limited set of conditions when the cost sharing rate is very low and product substitutability has intermediate magnitude.

We now investigate the asymmetric case where $\Omega \neq 1$. For both manufacturer advertising and retailer advertising, we again compare the two three-stage games, one with cost sharing and the other without. In manufacturer advertising, the qualitative insight of Theorem 3 continues to hold

cost sharing rate for the retailers is zero. The negotiation of the cost sharing rate is not the focus on this paper.

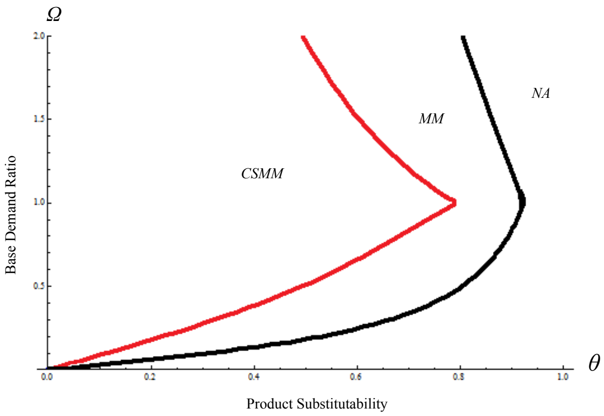

Figure 5: Manufacturers' equilibrium preference in manufacturer advertising with and without cost sharing given $\eta = 0.25$.

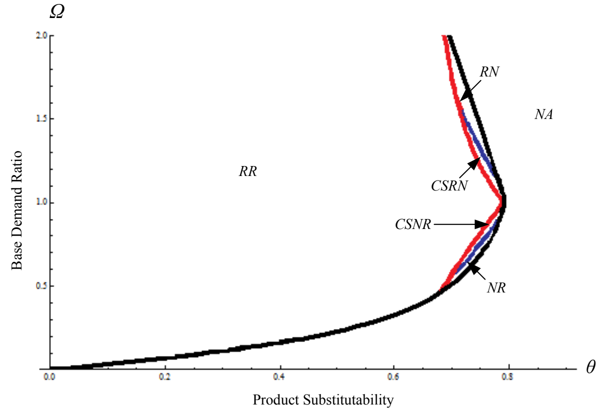

Figure 6: Retailers' equilibrium preference in manufacturer advertising with and without cost sharing given $\eta = 0.25$.

when $\Omega \neq 1$. As Figure 5 illustrates, CSMM outperforms MM when the product substitutability is low, and MM dominates otherwise. This reflects the trade-off between the market expansion effect and the competition effect. Cost sharing enhances the advertising level and hence boosts a supply chain's initial base demand; however, it also intensifies the horizontal competition. The market expansion effect is preferred to the competition effect when product substitutability is low. The advantage of MM grows as the channel asymmetry increases, because the smaller manufacturer has to scale back the advertising effort more significantly with cost sharing than without.

For retailer advertising, as depicted by Figure 6, RR dominates other structures in the majority of the feasible domain, especially when product substitutability is not high. The rationale is similar to that of Theorem 3, in which the retailers prefer no cost sharing so as to decrease supply chain competition. Nevertheless, in a small region, CSRN and CSNR can emerge as the unique choices for the retailers. This is because the horizontal competition is lowered significantly by the absence of advertising from the smaller retailer. The larger retailer benefits from the market expansion effect. However, the advantage erodes as the supply chains become more asymmetric. Consequently, RN and NR outperform CSRN and CSNR for the retailers when product substitutability is sufficiently high and the supply chain asymmetry is significant.

Table 2: Rank ordering of supply chain efficiency when $\Omega = 1$.

θ	[0,0.63)	[0.63,0.68)	[0.68,0.69)	[0.69,0.71)	[0.71,0.73)	[0.73,0.80)
RR	1	2	3	4	4	5
MM	2	1	1	1	1	1
RN	3	3	2	2	3	3
MN	4	4	4	3	2	2
NN	5	5	5	5	5	4

5 Extensions

This section incorporates additional metrics of performance, specifically total supply chain profit instead of individual firm profit, as well as consumer welfare.

5.1 Supply Chain Efficiency

We define *supply chain efficiency* as the sum of all players' profits. To investigate supply chain efficiency for all previously studied advertising structures, we start with the cases without cost sharing under the symmetric demand setting ($\Omega = 1$). By symmetry, MN has the same as NM, and likewise RN and NR are equally efficient. Table 2 shows the rank ordering of the 6 structures as product substitutability varies.

Table 2 demonstrates that RR performs the best when product substitutability is low ($\theta < 0.63$). Table 2 also shows that NN is not necessarily the worst and, as product substitutability becomes sufficiently high, RR becomes the worst because of the intense supply chain competition. This property is reminiscent of the Prisoner's Dilemma, although RR is not a dominant strategy like MM in Theorem 1. The trends in rank ordering confirm that retailer advertising worsens supply chain efficiency, more so than does manufacturer advertising, if supply chain competition becomes too intense.

We then extend to the case of asymmetric channels and compare all the subgames in manufacturer and retailer advertising. Figure 7 displays the subgame that gives the highest supply chain efficiency for each feasible combination of Ω and θ. RR dominates most of the time, but gives way to MM when product substitutability becomes sufficiently high. At the extremes RR takes the

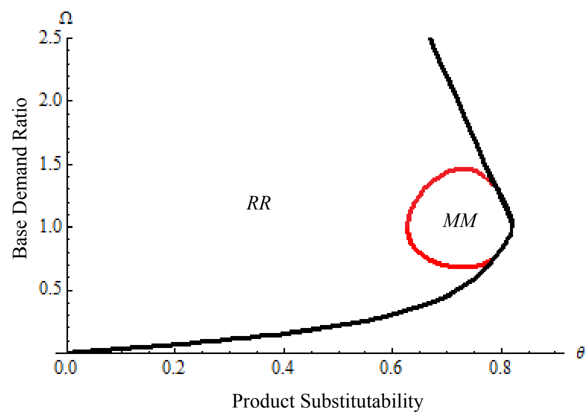

Figure 7: Comparison of supply chain efficiency among all subgames of manufacturer and retailer advertising without cost sharing.

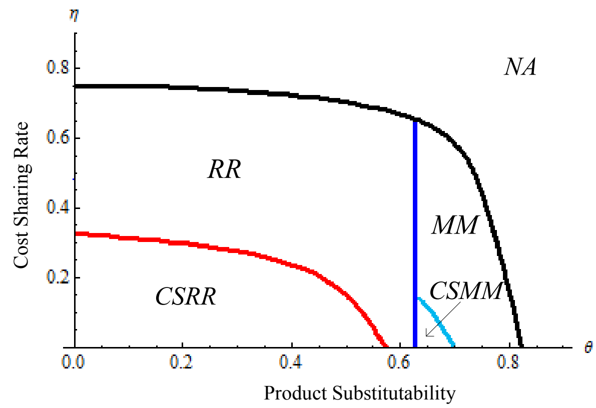

Figure 8: Comparison of supply chain efficiency all subgames of manufacturer and retailer advertising with and without cost sharing, given $\Omega = 1$.

lead because the intensifying competition increases total profit in the supply chain with larger base demand more than it takes away from the supply chain with the smaller base demand.

Next we consider the impact of cost sharing, for which we compare all subgames in manufacturer retailer and retailer advertising with and without cost sharing for the symmetric case ($\Omega = 1$). Figure 8 identifies the subgame with the highest supply chain efficiency for each combination of η and θ.

Figure 8 shows that CSRR could actually yield the highest supply chain efficiency when product substitutability and the cost sharing rate are low, although we saw earlier that the retailers have individual incentive to oppose it. This suggests that to attain the highest supply chain efficiency might require additional side payments from the manufacturers to the retailers. When the cost sharing rate is high and product substitutability remains relatively low, RR takes over the lead, which is reasonable given that RR is efficient when product substitutability is low and retailers strongly prefer RR to CSRR when the cost sharing rate is high. As product substitutability grows, CSMM and MM dominate. MM yields the highest supply chain efficiency when product substitutability is sufficiently high and the cost sharing rate is high. CSRR is strongest when the cost sharing rate and product substitutability are low. This comes at the expense of the retailers. As mentioned earlier, in such a situation side payments from the manufacturers to the retailers could enable a Pareto improvement vis-a-vis retailer advertising with cost sharing.

5.2 Consumer Welfare

Consumer welfare, denoted as U and subscripted with the advertising structure being used, is based on the utility of the representative consumer in Eq. (3). The structures without cost sharing can be rank ordered as follows for any (θ, Ω) in the common feasible domain.

Theorem 4 *The advertising structures without cost sharing give rise to consumer welfare outcomes with the following relative orderings.*

1. $U_{RR} > U_{MM}$;
2. $U_{RN} > U_{MN} > U_{NN}$; $U_{NR} > U_{NM} > U_{NN}$;
3. *If* $\Omega \geq (<) 1$, *then* $U_{RN} \geq (<) U_{NR}$, *and* $U_{MN} \geq (<) U_{NM}$.

Consumers obtain more utility from retailer advertising because this induces greater competition and consequently lower retail prices (and greater consumption than with manufacturer advertising). For asymmetric channels the retail price in the supply chain with larger base market is lower than with manufacturer advertising.

Comparing CSRR, CSMM, RR, and MM for the symmetric case ($\Omega = 1$) demonstrates the impact of cost sharing. Figure 9 illustrates that CSMM and CSRR dominate RR and MM. This is because cost sharing motivates increased advertising, resulting in increased consumption. Consumers benefit from CSRR because of the intensified competition. However if the cost sharing rate is high, CSMM becomes superior because the retail prices increase significantly in CSRR. While not illustrated in this chart since it only shows $\Omega = 1$, as the channel asymmetry grows, the region of CSMM dominance gradually shrinks to the left.

While this view of consumer welfare is the standard approach for researchers using this class of demand model, we acknowledge that the nature of advertising is such that the advertising itself

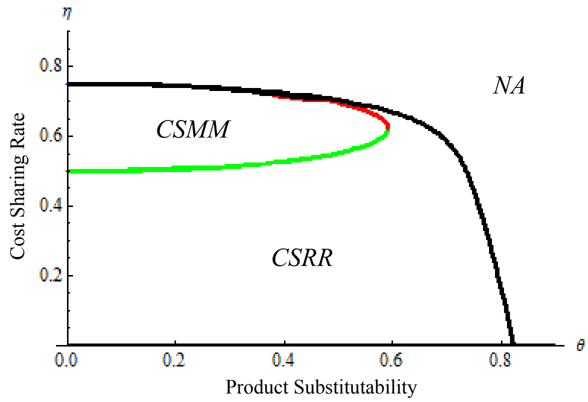

Figure 9: Comparison of consumer welfare among all subgames of manufacturer and retailer advertising with and without cost sharing, given $\Omega = 1$.

could benefit consumers in some ways that are separate from price and total consumption. For instance, increased advertising could improve the shopping process or consumption experience by providing valuable information. We leave consideration of such intangibles to future research.

6 Conclusion and Discussion

This paper evaluates the efficacy of manufacturer advertising and retailer advertising with and without cost sharing in a dual exclusive channel model with asymmetric competing supply chains. Our results offer managerial insights to better understand a variety of advertising strategies in practice. First, it is a dominant strategy for both manufacturers to advertise at a positive level in manufacturer advertising, although a Prisoner's Dilemma may occur. In retailer advertising, asymmetric advertising structures can arise as equilibria. Our analysis demonstrates that commitment to not advertising in competitive supply chains is credible. Second, whereas cost sharing can help both manufacturers and retailers, surprisingly it might hurt the retailers when they are the ones doing the advertising. This helps explain why cooperative advertising arrangements are not universally welcomed in practice by the parties performing the advertising, corroborating the empirical evidence we have presented. To achieve retailer buy-in requires that the manufacturers (or upstream firms) do not substantially increase the wholesale prices in conjunction with the advertising cost subsidy. In the end, wholesale price reduction may be the more effective way to stimulate retailer advertising effort. Retailers should also attempt to avoid engaging in an advertising war, especially under cost sharing. Our extended analysis suggests that supply chain efficiency is higher with retailer advertising if product substitutability is low, but otherwise is higher with manufacturer advertising. We have shown that advertising with cost sharing provides the highest consumer welfare by intensifying the competition between supply chains.

Our work provides a general framework for understanding how channel structure interacts with decisions around advertising and other market expansion efforts of a similar ilk, which opens numerous avenues for future research. First, this paper has focused on dual exclusive channels or supply chains, and other channel structures merit attention. Our preliminary analysis of other channel structures, including a monopoly common retailer and a duopoly common retailer channel, has yielded results consistent with this paper. Second, the cost sharing rate in our model is exogenous. Practically and theoretically, the rate can be negotiated within a Nash bargaining framework. Third, this paper inherits the Stackelberg game setting from McGuire and Staelin (1983), Coughlan (1985), and many others. A different decision structure might alter some of our findings (see Choi, 1991; Xie and Neyret, 2009). Finally, to prevent an already complicated formulation from becoming intractable we have omitted certain potentially interesting features, such as asymmetric information, asymmetric operational costs, demand uncertainty, and externalities from advertising (i.e., spillover effects and the resulting free riding). We believe the qualitative findings of this paper to be robust to such extensions, and eagerly await the future research that can offer definitive resolution.

Acknowledgements

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Online Supplement for "Advertising in Asymmetric Competing Supply Chains"

Appendix A further studies properties for advertising effort levels, hybrid advertising structures, and all efforts. Appendix B includes all proofs for the main findings of the paper.

Appendix A

A.1 Properties for Advertising Effort Levels

This subsection explores additional properties for advertising effort levels. Corollary 1 addresses the case of manufacturer advertising and retailer advertising without cost sharing. Corollary 2 addresses the case of manufacturer advertising and retailer advertising with cost sharing. We consider how the advertising effort level responds to the change of channel substitutability θ and base demand ratio Ω.

Corollary 1 *Under advertising without cost sharing, we obtain the following properties.*

1. *Chain* 1*'s advertising effort level* e *increases with* Ω *(e.g.,* $\frac{\partial e^*_{MM-mi}}{\partial \Omega} > 0$ *and* $\frac{\partial e^*_{RR-ri}}{\partial \Omega} > 0$*), whereas Chain* 2*'s advertising effort level* e *decreases with* Ω *(e.g.,* $\frac{\partial e^*_{MM-mi}}{\partial \Omega} < 0$ *and* $\frac{\partial e^*_{RR-ri}}{\partial \Omega} < 0$*).*

2. *The advertising effort level* e *does not always increase with* θ*. More specifically,* $\frac{\partial e^*_{MM-mi}}{\partial \theta} > 0$ *iff* $\Omega > \Omega^{e-\theta}_{MM}$ *and* $\frac{\partial e^*_{RR-ri}}{\partial \theta} > 0$ *iff* $\Omega > \Omega^{e-\theta}_{RR}$*, where*

$$\Omega^{e-\theta}_{MM} = \frac{784 - 384\theta^2 - 2056\theta^4 + 2992\theta^6 - 1711\theta^8 + 460\theta^{10} - 48\theta^{12}}{4\theta\left(1092 - 3388\theta^2 + 4281\theta^4 - 2824\theta^6 + 1034\theta^8 - 200\theta^{10} + 16\theta^{12}\right)},$$
$$\Omega^{e-\theta}_{RR} = \frac{729 - 567\theta^2 - 2808\theta^4 + 5448\theta^6 - 4064\theta^8 + 1424\theta^{10} - 192\theta^{12}}{2\theta\left(2187 - 8640\theta^2 + 13812\theta^4 - 11472\theta^6 + 5280\theta^8 - 1280\theta^{10} + 128\theta^{12}\right)}.$$

Corollary 1 shows that a player's advertising effort increases with its own base demand but decreases with its rival's. A player's advertising effort increases with channel substitutability level (θ) if and only if the player has an advantage in market size; otherwise, increasing the advertising effort will intensify the competition level between the supply chains.

Corollary 2 *Under advertising with cost sharing given $\Omega = 1$, we obtain the following properties.*

1. *For CSMM, the advertising effort level increases with θ iff $\theta > \theta_{CSMM}$;*
2. *For CSRR, the advertising effort level increases with θ iff $\eta > \eta_{CSRR}$, where θ_{CSMM} and θ_{CSRR} are unique in the feasible domain, where*

$$\begin{aligned}\theta_{CSMM} &= \{\theta| -4+20\theta+4\theta^2-16\theta^3-\theta^4+4\theta^5=0\},\\ \eta_{CSRR} &= \frac{-6+31\theta+7\theta^2-28\theta^3-2\theta^4+8\theta^5+\sqrt{\theta^2+2\theta^3-8\theta^4}}{2\left(-4+20\theta+4\theta^2-16\theta^3-\theta^4+4\theta^5\right)}.\end{aligned}$$

Corollary 2 shows that in CSMM, the advertising effort level increases if and only if the channel substitutability is sufficiently high, while in CSRR, the advertising effort level increases if and only if the cost sharing rate is very high.

A.2 Hybrid Advertising Structures

For completeness, we now turn our attention to *hybrid advertising structures*, in which the sole advertising provider in each supply chain need not be the same kind of firm as in the other supply chain. In other words, both the manufacturer and the retailer in each supply chain can freely decide whether or not to advertise. We label the two additional structures as follows: In MR, Manufacturer 1 advertises in supply chain 1 and Retailer 2 advertises in supply chain 2; In RM, Retailer 1 and Manufacturer 2 are the ones to advertise in their respective supply chains. The requisite profit functions follow from Eqs. (4) and (5) by setting $\mathbf{1}_{m1} = 1$ and $\mathbf{1}_{r2} = 1$ for MR, or $\mathbf{1}_{m2} = 1$ and $\mathbf{1}_{r1} = 1$ for RM, with all remaining indicators in each case set to zero.

Hybrid structures are more difficult to analyze than manufacturer/retailer advertising because of the interdependence of the decisions of the manufacturer and the retailer within each supply chain. For instance, with pure manufacturer advertising, Manufacturer 1 simply need only choose which of NM and MM provides itself with higher profit. However, in a hybrid structure, Manufacture 1 could consider abandoning advertising in anticipation that Retailer 1 would advertise. But this would require that Retailer 1 must profit more in RM than in MM; otherwise, the players would be at odds about which side should advertise. Said differently, Manufacturer 1 and Retailer 1 are a coalition in the sense that they have to coordinate on who advertises in order to obtain a mutual benefit. So we must compare the performance of different effort structures from a coalition's perspective.

To describe the stability of the advertising structure, we introduce the concept of *strong channel equilibrium*, in which no coalition of players within the same channel/supply chain can profitably deviate from the current state.[8] So, for MM to not be a strong channel equilibrium would mean that at least a manufacturer-retailer dyad would be better off by simultaneously defecting to either RM or MR.

Lemma 6 in the Appendix documents the comparison of MR and RM and the earlier advertising structures from the perspective of Manufacturer 1 and Retailer 1 as a coalition. Those findings lead to the following equilibrium results.

Theorem 5 *For hybrid advertising structures:*

1. *MM is a strong channel equilibrium if* $\hat{\Omega}_{r2}^{MR-MM}(\theta) < \Omega < \hat{\Omega}_{r1}^{RM-MM}(\theta)$ *in* $\theta \in [0.424, 0.823]$*;*
2. *RR is a strong channel equilibrium if* $\hat{\Omega}_{m1}^{MR-RR}(\theta) < \Omega < \hat{\Omega}_{m2}^{RM-RR}(\theta)$ *in* $\theta \in [0, 0.775]$*;*
3. *MR is a strong channel equilibrium if* $\Omega < \min\{\hat{\Omega}_{m2}^{MR-MM}(\theta), \max\{\hat{\Omega}_{r1}^{MR-RR}(\theta), \hat{\Omega}_{m1}^{MR-RR}(\theta)\}\}$*;*
4. *RM is a strong channel equilibrium if* $\Omega > \max\{\hat{\Omega}_{m1}^{RM-MM}(\theta), \min\{\hat{\Omega}_{r2}^{RM-MM}(\theta), \hat{\Omega}_{m2}^{RM-MM}(\theta)\}\}$.

Figure 10 graphically illustrates Theorem 5.

[8] Strong channel equilibrium is a special case of strong equilibrium that limits the coalition to the players within the same supply chain. For a definition of strong equilibrium, please see Aumann (1959) and Bernheim et al. (1987).

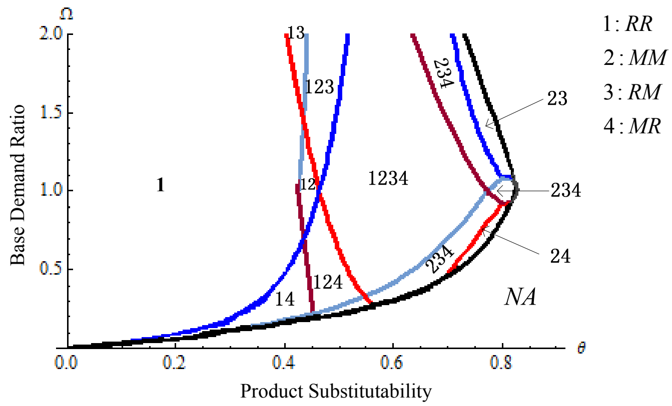

Figure 10: Equilibrium for hybrid advertising structures. 1 refers to RR, 2 to MM, 3 to RM, and 4 to MR. These numerical labels are a more compact way to present the equilibria for each region.

RR is the sole strong channel equilibrium if product substitutability is low because retailer advertising is more efficient in expanding the market, as well as reducing double marginalization. This is sufficient to offset any losses caused by intensified competition, when the supply chains are relatively monopolistic. As product substitutability grows, the advantages of RR erode but are sufficient to retain its equilibrium status unless product substitutability becomes too high (i.e., $\theta > 0.775$). MM exhibits stability as long as product substitutability is sufficiently high (i.e., $\theta > 0.424$). The strong channel equilibrium areas of MR and RM are asymmetric due to their advertising structure asymmetry. When a supply chain has the larger base demand, the supply chain is more likely to favor retailer advertising while the other supply chain sticks with the manufacturer's. Either MR or RM becomes unstable when product substitutability is sufficiently high and the supply chain with smaller base demand uses retailer advertising, because low retail prices and high effort costs force both players in the supply chain with smaller demand to switch to a more balanced advertising structure (i.e., MM). This confirms that manufacturer advertising is more stable when supply chain competition is intense, although the Prisoner's Dilemma persists.

A.3 All Efforts

The main body of this paper presents the analysis of advertising that is performed solely by either the manufacturer or the retailer in each supply chain. We now consider the scenario in which manufacturers and retailers advertise simultaneously, which we call *all efforts* (AE).

Let e_{mi} denote the advertising by Manufacturer $i, i = 1, 2$, and e_{ri} denote the advertising by Retailer $i, i = 1, 2$. We adapt Eq. (1)'s representation of base demand in channel i to become

$$\alpha_i = A_i + e_{mi} + e_{ri}.$$

This additive form, used for reasons of tractability, does not capture any diminishing returns when manufacturers and retailers both advertise to the same target market (Venkatesh and Kamakura, 2003), or any synergies for that matter.

The players' profit functions are given by, for i=1,2,

$$\begin{aligned} \Pi_{mi} &= D_i w_i - k_{mi} e_{mi}^2, \\ \Pi_{ri} &= D_i (p_i - w_i) - k_{ri} e_{ri}^2, \end{aligned}$$

where k_{mi} and k_{ri} are cost coefficients for the efforts of manufacturers and retailers, respectively. In the game, the manufacturers simultaneously determine wholesale prices w_i and effort levels e_{mi} in the first stage, and in the second the retailers simultaneously determine retail prices p_i and advertising levels e_{ri}. To simplify the following analysis we set $A_1 = A_2 = 1$ and $k_{m2} = k_{r1} = k_{r2} = 1$, while focusing on changes in the value of k_{m1}.

Lemma 5 *Given $A_1 = A_2 = 1$ and $k_{12} = k_{21} = k_{22} = 1$, Manufacturer 1's advertising effort decreases with its cost coefficient (k_{m1}).*

The proof of Lemma 5 also indicates that as the cost coefficient goes to infinity, the corresponding advertising level converges to zero. In our numerical analysis this property emerges for all players without the restrictions $A_1 = A_2 = 1$ and $k_{12} = k_{21} = k_{22} = 1$. Further, all else equal, the player with the lower cost coefficient will exert the higher advertising effort.

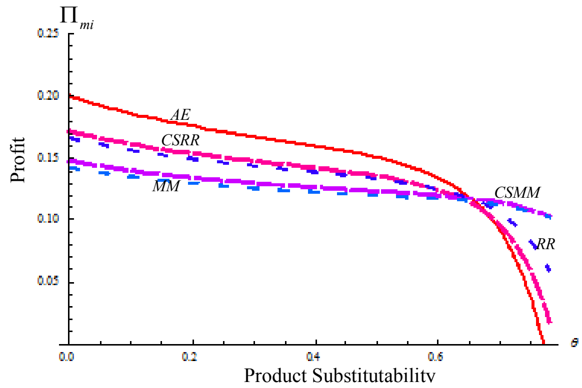

Figure 11: Manufacturers' profit comparison among AE, CSRR, CSMM, MM, and RR, given $\Omega = 1$ and $\eta = 0.2$

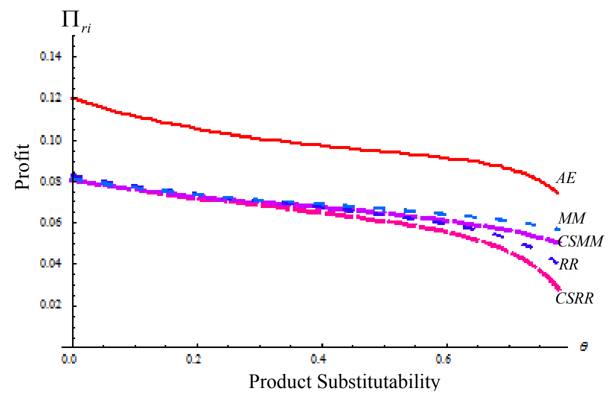

Figure 12: Retailers' profit comparison among AE, CSRR, CSMM, MM, and RR, given $\Omega = 1$ and $\eta = 0.2$

We now numerically compare AE to the previously analyzed advertising games. Note that the following representative example will convey the major qualitative insights even if the parameter values are changed. When channel substitutability is relatively low, AE outperforms MM, RR, CSMM, and CSRR for the manufacturers (Figure 11) whereas it dominates all other structures for the retailers (Figure 12). We also find that AE could be more preferable to retailers than manufacturers, because AE imposes more effort costs upon the manufacturers than upon the retailers. AE could perform worse than other advertising structures for the manufacturers if channel substitutability is substantially high. This is because AE results in more combined efforts than any other game, which significantly intensifies horizontal channel competition and incites a pricing war between the channels.

We include the AE analysis for the sake of completeness, although its complexity limits the availability of generalizable insights. In any case, this paper's main model is better suited to address our central research questions, whose industry motivations are presented in detail in Section 1. By focusing on manufacturer-only or retailer-only advertising, while allowing cost sharing, we can more sharply illuminate the impact of where control of the advertising decision is located in the supply chain, and the interplay between that control and the source of the advertising's funding in a competitive setting.

Appendix B

In our notation the index i $(i = 1, 2)$ identifies the channel or supply chain. Unless indicated otherwise, all equations below hold for $i = 1, 2$.

Proof of Lemma 1: To compare MM, MN, NM, and NN, we solve each subgame by reverse induction. More specifically, we first compute the retailers' best-response retail prices, then substitute them into the manufacturers' profit functions, and finally solve the manufacturers' first-order conditions for wholesale prices and advertising levels. Each subgame has a unique equilibrium. Comparing the manufacturers' profits across all subgames yields the subgame perfect equilibrium for the whole game.

In MM, given w_i and e_i, Retailer i's profits are concave with respect to p_i because $\frac{\partial^2 \Pi_{MM-ri}}{\partial p_i^2} = -\frac{2}{1-\theta^2} < 0$˙The best response retail price function can be obtained by solving from the first-order condition.

$$p_i(w_i, e_i) = \frac{(2-\theta^2)(A_i + e_i) - \theta(A_{3-i} + e_{3-i}) + 2w_i + \theta w_{3-i}}{4-\theta^2}, i = 1, 2.$$

Then, substituting $p_i(w_i, e_i)$ into the manufacturers' profit functions, we get

$$\Pi_{MM-mi}(w_i, e_i) = \frac{(2-\theta^2)\, q_i w_i + w_i \left((2-\theta^2)\,(A_i - w_i) - \theta\,(A_{3-i} + q_{3-i} - w_{3-i})\right) - (1-\theta^2)(4-\theta^2) q_i^2}{(1-\theta)(4-\theta)}.$$

The corresponding Hessian matrix is negative definite because

$$\frac{\partial^2 \Pi_{MM-mi}(w_i, e_i)}{\partial w_i^2} = -\frac{2\,(2-\theta^2)}{(1-\theta^2)\,(4-\theta^2)} < 0$$

and

$$\frac{\partial^2 \Pi_{MM-mi}(w_i, e_i)}{\partial w_i^2} \frac{\partial^2 \Pi_{MM-mi}(w_i, e_i)}{\partial e_i^2} - \frac{\partial^2 \Pi_{MM-mi}(w_i, e_i)}{\partial w_i \partial e_i} - \frac{\partial^2 \Pi_{MM-mi}(w_i, e_i)}{\partial e_i \partial w_i}$$
$$= \frac{28 - 52\theta^2 + 27\theta^4 - 4\theta^6}{(4 - 5\theta^2 + \theta^4)^2} > 0.$$

So, we can obtain the optimal w^*_{MM-i} and e^*_{MM-mi}. Replacing them into $p_i(w_i, e_i)$produces the optimal retail prices p^*_{MM-i}.

In summary, the unique equilibrium for MM is:

$$w^*_{MM-i} = \frac{2\,(4 - 5\theta^2 + \theta^4)\left((14 - 17\theta^2 + 4\theta^4)\, A_i - 2\theta\,(2-\theta^2)\, A_{3-i}\right)}{196 - 492\theta^2 + 417\theta^4 - 140\theta^6 + 16\theta^8},$$

$$
\begin{aligned}
p^*_{MM-i} &= \frac{4\left(3-4\theta^2+\theta^4\right)\left(\left(14-17\theta^2+4\theta^4\right)A_i-2\theta\left(2-\theta^2\right)A_{3-i}\right)}{196-492\theta^2+417\theta^4-140\theta^6+16\theta^8},\\
e^*_{MM-mi} &= \frac{\left(2-\theta^2\right)\left(\left(14-17\theta^2+4\theta^4\right)A_i-2\theta\left(2-\theta^2\right)A_{3-i}\right)}{196-492\theta^2+417\theta^4-140\theta^6+16\theta^8},\\
D^*_{MM-i} &= \frac{2\left(2-\theta^2\right)\left(\left(14-17\theta^2+4\theta^4\right)A_i-2\theta\left(2-\theta^2\right)A_{3-i}\right)}{196-492\theta^2+417\theta^4-140\theta^6+16\theta^8},\\
\Pi^*_{MM-mi} &= \frac{(2-\theta^2)(14-19\theta^2+4\theta^4)((14-17\theta^2+4\theta^4)A_i-2\theta(2-\theta^2)A_{3-i})^2}{(196-492\theta^2+417\theta^4-140\theta^6+16\theta^8)^2},\\
\Pi^*_{MM-ri} &= \frac{4(2-\theta^2)^2(1-\theta^2)((14-17\theta^2+4\theta^4)A_i-2\theta(2-\theta^2)A_{3-i})^2}{(196-492\theta^2+417\theta^4-140\theta^6+16\theta^8)^2}.
\end{aligned}
$$

For prices and demands to remain nonnegative requires

$$\left(14-17\theta^2+4\theta^4\right)A_i-2\theta\left(2-\theta^2\right)A_{3-i}\geq 0.$$

This is equivalent to $\frac{2\theta\left(2-\theta^2\right)}{14-17\theta^2+4\theta^4}\leq\Omega\leq\frac{14-17\theta^2+4\theta^4}{2\theta(2-\theta^2)}$, where the maximum feasible domain for θ is given by $[0, 0.940]$ because the upper bound of θ is obtained when the above two constraint boundary lines cross at $A_1 = A_2$.

For subgame NN, given the wholesale prices w_i, Retailer i's profit is concave with respect to p_i because $\frac{\partial^2\Pi_{NN-ri}}{\partial p_i^2}=-\frac{2}{1-\theta^2}<0$. The response function of the retail prices can be obtained by solving the following first-order conditions.

$$p_i(w_i)=\frac{\left(2-\theta^2\right)A_i-\theta A_{3-i}+2w_i+\theta w_{3-i}}{4-\theta^2}, i=1,2.$$

Substituting $p_i(w_i)$ into the manufacturers' profit function yields

$$\Pi_{NN-mi}(w_i)=\frac{w_i\left(\left(2-\theta^2\right)A_i-\theta A_{3-i}-2w_i+\theta^2 w_i+\theta w_{3-i}\right)}{4-5\theta^2+\theta^4}.$$

Manufacturer i's profit, $\Pi_{NN-mi}(w_i)$, is concave in w_i because $\frac{\partial^2\Pi_{NN-mi}}{\partial w_i^2}=-\frac{2\left(2-\theta^2\right)}{4-5\theta^2+\theta^4}<0$. So, we can obtain the unique and optimal wholesale prices w^*_{NN-i}. Substituting these into $p_i(w_i)$ delivers p^*_{NN-i}.

The unique subgame perfect equilibrium for NN is:

$$
\begin{aligned}
w^*_{NN-i} &= \frac{\left(8-9\theta^2+2\theta^4\right)A_i-\theta\left(2-\theta^2\right)A_{3-i}}{16-17\theta^2+4\theta^4},\\
p^*_{NN-i} &= \frac{2\left(3-\theta^2\right)\left(\left(8-9\theta^2+2\theta^4\right)A_i-\theta\left(2-\theta^2\right)A_{3-i}\right)}{64-84\theta^2+33\theta^4-4\theta^6},
\end{aligned}
$$

$$D_{NN-i}^{*} = \frac{(2-\theta^2)\left((8-9\theta^2+2\theta^4)A_i-\theta(2-\theta^2)A_{3-i}\right)}{64-148\theta^2+117\theta^4-37\theta^6+4\theta^8},$$
$$\Pi_{NN-mi}^{*} = \frac{(2-\theta^2)\left((8-9\theta^2+2\theta^4)A_i-\theta(2-\theta^2)A_{3-i}\right)^2}{(4-5\theta^2+\theta^4)(16-17\theta^2+4\theta^4)^2},$$
$$\Pi_{NN-ri}^{*} = \frac{(2-\theta^2)^2\left((8-9\theta^2+2\theta^4)A_i-\theta(2-\theta^2)A_{3-i}\right)^2}{(1-\theta^2)(64-84\theta^2+33\theta^4-4\theta^6)^2}.$$

For prices and demands to remain nonnegative requires

$$\left(8-9\theta^2+2\theta^4\right)A_i-\theta\left(2-\theta^2\right)A_{3-i}\geq 0.$$

This is equivalent to $\frac{\theta\left(2-\theta^2\right)}{8-9\theta^2+2\theta^4}\leq\Omega\leq\frac{8-9\theta^2+2\theta^4}{\theta(2-\theta^2)}$. The maximum feasible domain for θ is given by $\theta\in[0,1]$ as the upper bound of θ is obtained when the two constraint boundary lines cross.

For subgame MN, given w_i and e_1, Retailer i's profits are concave in p_i, because $\frac{\partial^2\Pi_{MN-ri}}{\partial p_i^2} = -\frac{1}{1-\theta^2}<0$. The best response retail prices derived from the first order conditions are

$$p_1(w_i,e_1) = \frac{(2-\theta^2)A_1-\theta A_2+2e_1-\theta^2e_1+2w_1+\theta w_2}{4-\theta^2};$$
$$p_2(w_i,e_1) = \frac{(2-\theta^2)A_2-\theta A_1-\theta w_1+\theta w_1+2w_2}{4-\theta^2}.$$

Substituting $p_i(w_i,e_1)$ into the manufacturers' profit functions yields

$$\Pi_{MN-m1}(w_i,e_1) = \frac{-(4-5\theta^2+\theta^4)e_1^2+(2-\theta^2)e_1w_1+w_1\left((2-\theta^2)A_1-\theta A_2-2w_1+\theta^2w_1+\theta w_2\right)}{4-5\theta^2+\theta^4};$$
$$\Pi_{MN-m2}(w_i,e_1) = \frac{w_2\left(-\theta A_1+(2-\theta^2)A_2-\theta e_1+\theta w_1-2w_2+\theta^2w_2\right)}{4-5\theta^2+\theta^4}.$$

The $\Pi_{MN-m1}(w_i,e_1)$ are concave on (w_1,e_1) because $\frac{\partial^2\Pi_{MN-m1}(w_i,e_1)}{\partial w_1^2}=-\frac{2(2-\theta^2)}{4-5\theta^2+\theta^4}<0$ and the second-order Hessian Matrix has determinant $\frac{\partial^2\Pi_{MN-m1}(w_i,e_1)}{\partial w_1^2}\frac{\partial^2\Pi_{MN-m1}(w_i,e_1)}{\partial e_1^2}-\frac{\partial^2\Pi_{MN-m1}(w_i,e_1)}{\partial w_1\partial e_1}\frac{\partial^2\Pi_{MN-m1}(w_i,e_1)}{\partial e_1\partial w_1}= \frac{28-52\theta^2+27\theta^4-4\theta^6}{(4-5\theta^2+\theta^4)^2}$, which is strictly positive in the feasible domain of $\theta\in[0,0.94]$. Meanwhile, $\Pi_{MN-m2}(w_i,e_1)$ is concave on w_2 because $\frac{\partial^2\Pi_{MN-m2}(w_2)}{\partial w_1^2}=-\frac{2(2-\theta^2)}{4-5\theta^2+\theta^4}<0$. So, we can obtain the unique equilibrium wholesale prices and advertising level

$$w_{MN-1}^{*} = \frac{2(4-5\theta^2+\theta^4)\left((8-9\theta^2+2\theta^4)A_1+\theta(-2+\theta^2)A_2\right)}{112-270\theta^2+221\theta^4-72\theta^6+8\theta^8};$$
$$w_{MN-2}^{*} = \frac{(4-5\theta^2+\theta^4)\left(2\theta(-2+\theta^2)A_1+(14-17\theta^2+4\theta^4)A_2\right)}{112-270\theta^2+221\theta^4-72\theta^6+8\theta^8};$$
$$e_{MN-1}^{*} = \frac{(2-\theta^2)\left((8-9\theta^2+2\theta^4)A_1-\theta(2-\theta^2)A_2\right)}{112-270\theta^2+221\theta^4-72\theta^6+8\theta^8},$$

and substituting these into $p_i(w_i, e_1)$ yields the following equilibrium retail prices

$$p^*_{MN-1} = \frac{4\left(3-4\theta^2+\theta^4\right)\left(\left(8-9\theta^2+2\theta^4\right)A_1+\theta\left(-2+\theta^2\right)A_2\right)}{112-270\theta^2+221\theta^4-72\theta^6+8\theta^8};$$
$$p^*_{MN-2} = \frac{2\left(3-4\theta^2+\theta^4\right)\left(2\theta\left(-2+\theta^2\right)A_1+\left(14-17\theta^2+4\theta^4\right)A_2\right)}{112-270\theta^2+221\theta^4-72\theta^6+8\theta^8}.$$

A similar process obtains the following demands and profits for Manufacturer 1 in MN and NM, [9]

$$D^*_{MN-1} = \frac{2\left(2-\theta^2\right)\left(\left(8-9\theta^2+2\theta^4\right)A_1-\theta\left(2-\theta^2\right)A_2\right)}{112-270\theta^2+221\theta^4-72\theta^6+8\theta^8},$$
$$D^*_{MN-2} = \frac{\left(2-\theta^2\right)\left(\left(14-17\theta^2+4\theta^4\right)A_2-2\theta\left(2-\theta^2\right)A_1\right)}{112-270\theta^2+221\theta^4-72\theta^6+8\theta^8},$$
$$\Pi^*_{MN-m1} = \frac{\left(2-\theta^2\right)\left(14-19\theta^2+4\theta^4\right)\left(\left(8-9\theta^2+2\theta^4\right)A_1-\theta\left(2-\theta^2\right)A_2\right)^2}{\left(112-270\theta^2+221\theta^4-72\theta^6+8\theta^8\right)^2},$$
$$D^*_{NM-1} = \frac{\left(2-\theta^2\right)\left(\left(14-17\theta^2+4\theta^4\right)A_1-2\theta\left(2-\theta^2\right)A_2\right)}{112-270\theta^2+221\theta^4-72\theta^6+8\theta^8},$$
$$D^*_{NM-2} = \frac{2\left(2-\theta^2\right)\left(\left(8-9\theta^2+2\theta^4\right)A_2-\theta\left(2-\theta^2\right)A_1\right)}{112-270\theta^2+221\theta^4-72\theta^6+8\theta^8},$$
$$\Pi^*_{NM-m1} = \frac{\left(4-\theta^2\right)\left(2-3\theta^2+\theta^4\right)\left(\left(14-17\theta^2+4\theta^4\right)A_1-2\theta\left(2-\theta^2\right)A_2\right)^2}{\left(112-270\theta^2+221\theta^4-72\theta^6+8\theta^8\right)^2}.$$

In the following, without loss of generality, we compare Manufacturer 1's profits across the various cases. To compare MN and NN, we use $\Delta\Pi^{MN-NN}_{m1}$ to denote Manufacturer 1's profit in MN minus its profit in NN. The earlier profit expressions yield

$$\Delta\Pi^{MN-NN}_{m1} = \frac{\left(2-\theta^2\right)^2\left(896-3232\theta^2+4570\theta^4-3222\theta^6+1191\theta^8-220\theta^{10}+16\theta^{12}\right)\left(\left(8-9\theta^2+2\theta^4\right)A_1-\theta\left(2-\theta^2\right)A_2\right)^2}{\left(4-5\theta^2+\theta^4\right)\left(16-17\theta^2+4\theta^4\right)^2\left(112-270\theta^2+221\theta^4-72\theta^6+8\theta^8\right)^2}.$$

The common lower and upper bounds of the constrained areas are defined by

$$\underline{\Omega}^{MN-NN}(\theta) = \frac{\theta\left(2-\theta^2\right)}{8-9\theta^2+2\theta^4} \quad \text{and} \quad \bar{\Omega}^{MN-NN}(\theta) = \frac{\left(14-17\theta^2+4\theta^4\right)}{2\theta\left(2-\theta^2\right)},$$

where the domain for θ is $\theta \in [0, 0.967]$. Then $\Delta\Pi^{MN-NN}_{m1} > 0$ as long as $896-3232\theta^2+4570\theta^4-3222\theta^6+1191\theta^8-220\theta^{10}+16\theta^{12} > 0$, which is always true in its feasible domain.

A similar approach shows for the comparison of NM with NN that

$$\Delta\Pi^{NM-NN}_{m1} = -\frac{\left(2-\theta^2\right)\left(\left(8-9\theta^2+2\theta^4\right)A_1-\theta\left(2-\theta^2\right)A_2\right)^2}{\left(4-5\theta^2+\theta^4\right)\left(16-17\theta^2+4\theta^4\right)^2} - \frac{\left(4-\theta^2\right)\left(2-3\theta^2+\theta^4\right)\left(\left(14-17\theta^2+4\theta^4\right)A_1-2\theta\left(2-\theta^2\right)A_2\right)^2}{\left(112-270\theta^2+221\theta^4-72\theta^6+8\theta^8\right)^2}$$

[9]The values for Manufacturer 2 can be obtained by replacing every 1 with 2 and vice versa. Other results are omitted for brevity.

$$< \quad 0,$$

This is supported by the common lower and upper bounds

$$\underline{\Omega}^{NM-NN}(\theta) = \frac{2\theta\left(2-\theta^2\right)}{14-17\theta^2+4\theta^4} \quad \text{and} \quad \bar{\Omega}^{NM-NN}(\theta) = \frac{\left(8-9\theta^2+2\theta^4\right)}{\theta\left(2-\theta^2\right)},$$

where $\theta \in [0, 0.967]$. As before, the upper limit for θ is obtained when the two constraint lines, $\underline{\Omega}^{NM-NN}(\theta)$ and $\bar{\Omega}^{NM-NN}(\theta)$, cross.

For the comparison of MM with NM we have

$$\begin{aligned}\Delta\Pi_{m1}^{MM-NM} \quad = \quad & \frac{28-52\theta^2+27\theta^4-4\theta^6}{\left(196-492\theta^2+417\theta^4-140\theta^6+16\theta^8\right)^2} - \frac{\left(4-\theta^2\right)\left(2-3\theta^2+\theta^4\right)}{\left(112-270\theta^2+221\theta^4-72\theta^6+8\theta^8\right)^2}\Bigg) \\ & \times \left(\left(14-17\theta^2+4\theta^4\right)A_1 - 2\theta\left(2-\theta^2\right)A_2\right)^2.\end{aligned}$$

The expression is strictly positive since $\frac{28-52\theta^2+27\theta^4-4\theta^6}{\left(196-492\theta^2+417\theta^4-140\theta^6+16\theta^8\right)^2} > \frac{\left(4-\theta^2\right)\left(2-3\theta^2+\theta^4\right)}{\left(112-270\theta^2+221\theta^4-72\theta^6+8\theta^8\right)^2}$ for any $\theta \in [0, 0.940]$ as required by MM.

For MM and MN we have

$$\begin{aligned}& \Delta\Pi_{m1}^{MM-MN} \\ = \quad & \left(2-\theta^2\right)\left(14-19\theta^2+4\theta^4\right)\left(\frac{\left(\left(14-17\theta^2+4\theta^4\right)A_1-2\theta\left(2-\theta^2\right)A_2\right)^2}{\left(196-492\theta^2+417\theta^4-140\theta^6+16\theta^8\right)^2} - \frac{\left(\left(8-9\theta^2+2\theta^4\right)A_1-\theta\left(2-\theta^2\right)A_2\right)^2}{\left(112-270\theta^2+221\theta^4-72\theta^6+8\theta^8\right)^2}\right).\end{aligned}$$

This is strictly negative between $\frac{2\theta\left(2-\theta^2\right)}{14-17\theta^2+4\theta^4}$ and $\frac{\left(14-17\theta^2+4\theta^4\right)}{2\theta\left(2-\theta^2\right)}$.

This progression indicates that Manufacturer 1 always benefits from providing advertising effort regardless of what the other manufacturer does, but is harmed by the other manufacturer's choice to advertise. The same techniques provide the corresponding results for Manufacturer 2. □

Proof of Theorem 1: The first part of Theorem 1 results directly from Lemma 1. The Prisoner's Dilemma can be demonstrated by comparing Manufacturer 1's profits in MM and NN. It is easy to show that the common feasible area of MM and NN is confined by MM's feasible area. Therefore,

$$\underline{\Omega}^{MM-NN}(\theta) = \frac{2\theta\left(2-\theta^2\right)}{14-17\theta^2+4\theta^4} \quad \text{and} \quad \bar{\Omega}^{MM-NN}(\theta) = \frac{\left(14-17\theta^2+4\theta^4\right)}{2\theta\left(2-\theta^2\right)}.$$

A special case is given by $\theta \in [0, 0.940]$ when the above two constraint lines cross. [10]

[10]The feasible range for θ becomes smaller as the base demand ratio diverges, as illustrated in Figure 1.

Define $\Delta\Pi_{m1}^{MM-NN}$ as Manufacturer 1's profit in MM minus its profit in NN. We have

$$\begin{aligned}\Delta\Pi_{m1}^{MM-NN} &= \left(2-\theta^2\right)\Bigg(\frac{\left(14-19\theta^2+4\theta^4\right)\left(\left(14-17\theta^2+4\theta^4\right)A_1-2\theta\left(2-\theta^2\right)A_2\right)^2}{\left(196-492\theta^2+417\theta^4-140\theta^6+16\theta^8\right)^2} \\ &\quad -\frac{\left(\left(8-9\theta^2+2\theta^4\right)A_1-\theta\left(2-\theta^2\right)A_2\right)^2}{\left(4-5\theta^2+\theta^4\right)\left(16-17\theta^2+4\theta^4\right)^2}\Bigg) \qquad \text{(A-1)}\end{aligned}$$

Making the change of variable $\Omega = A_1/A_2$ and solving $\Delta\Pi_{m1}^{MM-NN} = 0$ yields two roots:

$$\begin{aligned}\hat{\Omega}_{m1-1}^{MM-NN}(\theta) &= \frac{K_1+K_2\sqrt{56-146\theta^2+125\theta^4-39\theta^6+4\theta^8}}{175616-1034880\theta^2+K_3}, \\ \hat{\Omega}_{m1-2}^{MM-NN}(\theta) &= \frac{K_1-K_2\sqrt{56-146\theta^2+125\theta^4-39\theta^6+4\theta^8}}{175616-1034880\theta^2+K_3},\end{aligned}$$

where

$$\begin{aligned}K_1 &= 940800\theta-4982880\theta^3+11381440\theta^5-14694560\theta^7+11805760\theta^9-6117970\theta^{11}+2045720\theta^{13}-426080\theta^{15}+50240\theta^{17}-2560\theta^{19}, \\ K_2 &= \theta\left(6272-25544\theta^2+42844\theta^4-38414\theta^6+19905\theta^8-5968\theta^{10}+960\theta^{12}-64\theta^{14}\right), \\ K_3 &= 2677288\theta^4-3997072\theta^6+3806878\theta^8-2413562\theta^{10}+1031035\theta^{12}-293184\theta^{14}+53184\theta^{16}-5568\theta^{18}+256\theta^{20}.\end{aligned}$$

Since $\hat{\Omega}_{m1-2}^{MM-NN}(\theta)$ is below the common lower bound in cases MM and NN, we define

$$\hat{\Omega}_{m1}^{MM-NN}(\theta) = \min\{\hat{\Omega}_{m1-1}^{MM-NN}(\theta), \bar{\Omega}^{MM-NN}(\theta)\},$$

which is the boundary line for Manufacturer 1's preferences between MM and NN (shown in Figure 1). Note that $\hat{\Omega}_{m1-1}^{MM-NN}(\theta) \le 1$ in $\theta \in [0, 0.940]$.

Similarly, we can define

$$\hat{\Omega}_{m2}^{MM-NN}(\theta) = \min\{\hat{\Omega}_{m2-1}^{MM-NN}(\theta), \underline{\Omega}^{MM-NN}(\theta)\}$$

for Manufacturer 2 (shown in Figure 1), where

$$\hat{\Omega}_{m2-1}^{MM-NN}(\theta) = \frac{K_1+K_2\sqrt{56-146\theta^2+125\theta^4-39\theta^6+4\theta^8}}{\theta^2(\theta^2-2)^2K_4},$$

and

$$K_4 = 9464-34516\theta^2+49530\theta^4-35595\theta^6+13476\theta^8-2560\theta^{10}+192\theta^{12}.$$

Here we also characterize the monotonicity of optimal retail prices and demand with respect to θ within the common feasible range of θ in the different subgames. We consider only subgames NN and MM, as the others are similar. In NN,

$$\frac{\partial p_{NN-i}^*}{\partial\theta} = -\frac{4\theta\left(224-336\theta^2+201\theta^4-56\theta^6+6\theta^8\right)A_i}{\left(64-84\theta^2+33\theta^4-4\theta^6\right)^2}$$

$$- \quad \frac{2\left(384-456\theta^2+146\theta^4+33\theta^6-27\theta^8+4\theta^{10}\right)A_{3-i}}{\left(64-84\theta^2+33\theta^4-4\theta^6\right)^2}.$$

This is strictly negative because $224-336\theta^2+201\theta^4-56\theta^6+6\theta^8>0$ and $384-456\theta^2+146\theta^4+33\theta^6-27\theta^8+4\theta^{10}>0$ for any $\theta\in[0,1)$. Also

$$\begin{aligned}\frac{\partial D^*_{NN-i}}{\partial\theta} &= \frac{2\theta\left(704-2080\theta^2+2510\theta^4-1588\theta^6+559\theta^8-104\theta^{10}+8\theta^{12}\right)A_i}{\left(64-148\theta^2+117\theta^4-37\theta^6+4\theta^8\right)^2}\\ &- \frac{\left(256-176\theta^2-492\theta^4+764\theta^6-439\theta^8+117\theta^{10}-12\theta^{12}\right)A_{3-i}}{\left(64-148\theta^2+117\theta^4-37\theta^6+4\theta^8\right)^2}.\end{aligned}$$

This indicates that D^*_{NN-i} increases with θ if $\Omega>\frac{256-176\theta^2-492\theta^4+764\theta^6-439\theta^8+117\theta^{10}-12\theta^{12}}{2\theta(704-2080\theta^2+2510\theta^4-1588\theta^6+559\theta^8-104\theta^{10}+8\theta^{12})}$ but decreases with θ otherwise. So, if the supply chains are sufficiently asymmetric, the supply chain with the larger base market obtains more demand as product substitutability grows.

For MM,

$$\begin{aligned}\frac{\partial p^*_{MM-i}}{\partial\theta} = - &\quad \frac{8\theta\left(308-1820\theta^2+3393\theta^4-2960\theta^6+1369\theta^8-328\theta^{10}+32\theta^{12}\right)A_i}{\left(196-492\theta^2+417\theta^4-140\theta^6+16\theta^8\right)^2}\\ &- \frac{8\left(1176-3516\theta^2+3786\theta^4-1441\theta^6-330\theta^8+469\theta^{10}-148\theta^{12}+16\theta^{14}\right)A_{3-i}}{\left(196-492\theta^2+417\theta^4-140\theta^6+16\theta^8\right)^2}.\end{aligned}$$

This is strictly negative for any θ in the feasible range, ensuring nonnegative prices and demands.

$$\begin{aligned}\frac{\partial D^*_{MM-i}}{\partial\theta} &= \frac{16\theta\left(1092-3388\theta^2+4281\theta^4-2824\theta^6+1034\theta^8-200\theta^{10}+16\theta^{12}\right)A_i}{\left(196-492\theta^2+417\theta^4-140\theta^6+16\theta^8\right)^2}\\ &- \frac{4\left(784-384\theta^2-2056\theta^4+2992\theta^6-1711\theta^8+460\theta^{10}-48\theta^{12}\right)A_{3-i}}{\left(196-492\theta^2+417\theta^4-140\theta^6+16\theta^8\right)^2}.\end{aligned}$$

D^*_{MM-i} increases with θ if $\Omega>\frac{784-384\theta^2-2056\theta^4+2992\theta^6-1711\theta^8+460\theta^{10}-48\theta^{12}}{4\theta(1092-3388\theta^2+4281\theta^4-2824\theta^6+1034\theta^8-200\theta^{10}+16\theta^{12})}$, but decreases with θ otherwise. We can show comparable properties for the other subgames in a similar fashion. □

Proof of Lemma 2: This Lemma's proof is similar to that of Lemma 1.

More specifically, we first compute the retailers' best-response retail prices and advertising levels, then substitute them into the manufacturers' profit functions, and finally solve the manufacturers' first-order condition for wholesale prices. Each subgame has a unique equilibrium. Comparing the retailers' profits across all subgames gives the subgame perfect equilibrium for the entire game.

Here we start with RR. Other subgames can be solved similarly. Given w_i, the retailers' profits are jointly concave in p_i and e_i because $\frac{\partial\Pi^2_{RR-ri}(w_i)}{\partial p_i^2}=-\frac{2}{1-\theta^2}<0$ and the determinant of its

Hessian matrix

$$\begin{aligned}
& \frac{\partial^2 \Pi_{RR-ri}(w_i)}{\partial p_i^2}\frac{\partial^2 \Pi_{RR-ri}(w_i)}{\partial e_i^2} - \frac{\partial^2 \Pi_{RR-ri}(w_i)}{\partial p_i \partial e_i}\frac{\partial^2 \Pi_{RR-ri}(w_i)}{\partial e_i \partial p_i} \\
= & \frac{3-4\theta^2}{(1-\theta^2)^2} \\
> & 0
\end{aligned}$$

as long as $\theta < \frac{\sqrt{3}}{2}$, which is true in the feasible domain.

According to the first-order conditions,

$$\begin{aligned}
p_i(w_i) &= \frac{2\left(3-5\theta^2+2\theta^4\right)A_i + 4\theta\left(-1+\theta^2\right)A_{3-i} + 3w_i - 6\theta^2 w_i + 4\theta w_{3-i} - 4\theta^3 w_{3-i}}{9-16\theta^2+4\theta^4}; \\
e_i(w_i) &= \frac{\left(3-2\theta^2\right)A_i - 2\theta A_{3-i} - 3w_i + 2\theta^2 w_i + 2\theta w_{3-i}}{9-16\theta^2+4\theta^4}.
\end{aligned}$$

Substituting $p_i(w_i)$ and $e_i(w_i)$ into the manufacturers' profit functions yields

$$\Pi_{RR-mi}(w_i) = \frac{2w_i\left(\left(3-2\theta^2\right)A_i - 2\theta A_{3-i} - 3w_i + 2\theta^2 w_i + 2\theta w_{3-i}\right)}{9-16\theta^2+4\theta^4}.$$

$\Pi_{RR-mi}(w_i)$ is concave in w_i because $\frac{\partial^2 \Pi_{RR-mi}(w_i)}{\partial w_i^2} = -\frac{4\left(3-2\theta^2\right)}{9-16\theta^2+4\theta^4} < 0$ as long as $\theta < \frac{\sqrt{9-\sqrt{33}}}{2}$, which is true in the feasible domain. Therefore, the equilibrium wholesale prices w^*_{RR-i} are unique. The equilibrium retail prices p^*_{RR-i} and advertising levels e^*_{RR-ri} also follow from the equilibrium wholesale prices.

In summary, the unique equilibrium for RR is:

$$\begin{aligned}
w^*_{RR-i} &= \frac{\left(9-14\theta^2+4\theta^4\right)A_i - \theta\left(3-2\theta^2\right)A_{3-i}}{18-26\theta^2+8\theta^4}, \\
p^*_{RR-i} &= \frac{\left(15-26\theta^2+8\theta^4\right)\left(\left(9-14\theta^2+4\theta^4\right)A_i - \theta\left(3-2\theta^2\right)A_{3-i}\right)}{162-522\theta^2+560\theta^4-232\theta^6+32\theta^8}, \\
e^*_{RR-ri} &= \frac{\left(3-2\theta^2\right)\left(\left(9-14\theta^2+4\theta^4\right)A_i - \theta\left(3-2\theta^2\right)A_{3-i}\right)}{162-522\theta^2+560\theta^4-232\theta^6+32\theta^8}, \\
D^*_{RR-i} &= \frac{\left(3-2\theta^2\right)\left(\left(9-14\theta^2+4\theta^4\right)A_i - \theta\left(3-2\theta^2\right)A_{3-i}\right)}{81-261\theta^2+280\theta^4-116\theta^6+16\theta^8}, \\
\Pi^*_{RR-ri} &= \frac{\left(3-2\theta^2\right)^2\left(3-4\theta^2\right)\left(\left(9-14\theta^2+4\theta^4\right)A_i - \theta\left(3-2\theta^2\right)A_{3-i}\right)^2}{4\left(81-261\theta^2+280\theta^4-116\theta^6+16\theta^8\right)^2}, \\
\Pi^*_{RR-mi} &= \frac{\left(3-2\theta^2\right)\left(\left(9-14\theta^2+4\theta^4\right)A_i - \theta\left(3-2\theta^2\right)A_{3-i}\right)^2}{2\left(9-16\theta^2+4\theta^4\right)\left(9-13\theta^2+4\theta^4\right)^2}.
\end{aligned}$$

For the prices and demands in RR to be nonnegative requires

$$\left(9-14\theta^2+4\theta^4\right)A_i - \theta\left(3-2\theta^2\right)A_{3-i} \geq 0.$$

This is equivalent to $\frac{\theta\left(3-2\theta^2\right)}{9-14\theta^2+4\theta^4} \leq \Omega \leq \frac{9-14\theta^2+4\theta^4}{\theta(3-2\theta^2)}$, which implies the largest feasible domain for θ is given by $\theta \in [0, 0.823]$ and the upper bound of θ is reached when the above two constraint boundaries cross. Define

$$\underline{\Omega}^{RR} \equiv \frac{\theta\left(3-2\theta^2\right)}{9-14\theta^2+4\theta^4} \quad \text{and} \quad \bar{\Omega}^{RR} \equiv \frac{9-14\theta^2+4\theta^4}{\theta\left(3-2\theta^2\right)}.$$

The above constraint is the strictest of all cases in this paper.

In RN, given w_i, Retailer 1's profit is concave on (p_1, e_1) because $\frac{\partial^2 \Pi_{RN-r1}}{\partial p_1^2} = -\frac{2}{1-\theta^2} < 0$ and the second Hessian Matrix has determinant $\frac{\partial^2 \Pi_{RN-r1}}{\partial p_1^2}\frac{\partial^2 \Pi_{RN-r1}}{\partial e_1^2} - \frac{\partial^2 \Pi_{RN-r1}}{\partial p_1 \partial e_1}\frac{\partial^2 \Pi_{RN-r1}}{\partial e_1 \partial p_1} = \frac{3-4\theta^2}{(1-\theta^2)^2} > 0$, as long as $\theta < \frac{\sqrt{3}}{2}$ which is true on the common domain. Retailer 2's profit is concave on p_2 because $\frac{\partial^2 \Pi_{RN-r2}}{\partial p_2^2} = -\frac{2}{1-\theta^2} < 0$. The first-order conditions then yield

$$\begin{aligned}
p_1(w_1, w_2) &= \frac{2\left(2-3\theta^2+\theta^4\right)A_1 + 2\theta\left(-1+\theta^2\right)A_2 + 2w_1 - 3\theta^2 w_1 + 2\theta w_2 - 2\theta^3 w_2}{6-9\theta^2+2\theta^4}; \\
p_2(w_1, w_2) &= \frac{2\theta\left(-1+\theta^2\right)A_1 + \left(3-5\theta^2+2\theta^4\right)A_2 + 2\theta w_1 - 2\theta^3 w_1 + 3w_2 - 4\theta^2 w_2}{6-9\theta^2+2\theta^4}; \\
e_1(w_1, w_2) &= \frac{\left(2-\theta^2\right)A_1 - \theta A_2 - 2w_1 + \theta^2 w_1 + \theta w_2}{6-9\theta^2+2\theta^4}.
\end{aligned}$$

Substituting $p_i(w_1, w_2)$ and $e_1(w_1, w_2)$ into the manufacturers' profit functions yields

$$\begin{aligned}
\Pi_{RN-m1}(w_1) &= \frac{2w_1\left(\left(2-\theta^2\right)A_1 - \theta A_2 - 2w_1 + \theta^2 w_1 + \theta w_2\right)}{6-9\theta^2+2\theta^4}; \\
\Pi_{RN-m2}(w_2) &= \frac{w_2\left(-2\theta A_1 + \left(3-2\theta^2\right)A_2 + 2\theta w_1 - 3w_2 + 2\theta^2 w_2\right)}{6-9\theta^2+2\theta^4}.
\end{aligned}$$

$\Pi_{RN-m1}(w_1)$ is concave in w_i because $\frac{\partial^2 \Pi_{RN-m1}}{\partial w_1^2} = -\frac{4\left(2-\theta^2\right)}{6-9\theta^2+2\theta^4} < 0$ as long as $\theta < \frac{\sqrt{9-\sqrt{33}}}{2}$, which holds in the feasible area. So, the unique equilibrium wholesale prices w^*_{RN-i} are as follows:

$$\begin{aligned}
w^*_{RN-1} &= \frac{4\left(3-4\theta^2+\theta^4\right)A_1 + \theta\left(-3+2\theta^2\right)A_2}{24-30\theta^2+8\theta^4}; \\
w^*_{RN-2} &= \frac{\theta\left(-2+\theta^2\right)A_1 + 2\left(3-4\theta^2+\theta^4\right)A_2}{12-15\theta^2+4\theta^4}.
\end{aligned}$$

The equilibrium wholesale prices lead to the equilibrium retail prices p^*_{RN-i} and advertising level e^*_{RN-1} that follow.

$$\begin{aligned}
p^*_{RN-1} &= \frac{\left(10-15\theta^2+4\theta^4\right)\left(4\left(3-4\theta^2+\theta^4\right)A_1 + \theta\left(-3+2\theta^2\right)A_2\right)}{2\left(72-198\theta^2+183\theta^4-66\theta^6+8\theta^8\right)}; \\
p^*_{RN-2} &= \frac{\left(9-14\theta^2+4\theta^4\right)\left(\theta\left(-2+\theta^2\right)A_1 + 2\left(3-4\theta^2+\theta^4\right)A_2\right)}{72-198\theta^2+183\theta^4-66\theta^6+8\theta^8};
\end{aligned}$$

$$e^*_{RN-1} = \frac{(2-\theta^2)\left(4\left(3-4\theta^2+\theta^4\right)A_1-\theta\left(3-2\theta^2\right)A_2\right)}{2\left(72-198\theta^2+183\theta^4-66\theta^6+8\theta^8\right)}.$$

The equilibrium profits and demands for the retailers are given by

$$\begin{aligned}
\Pi^*_{RN-r1} &= \frac{\left(2-\theta^2\right)^2\left(3-4\theta^2\right)\left(4\left(3-4\theta^2+\theta^4\right)A_1-\theta\left(3-2\theta^2\right)A_2\right)^2}{4\left(72-198\theta^2+183\theta^4-66\theta^6+8\theta^8\right)^2},\\
\Pi^*_{RN-r2} &= \frac{\left(3-2\theta^2\right)^2\left(1-\theta^2\right)\left(2\left(3-4\theta^2+\theta^4\right)A_2-\theta\left(2-\theta^2\right)A_1\right)^2}{\left(72-198\theta^2+183\theta^4-66\theta^6+8\theta^8\right)^2},\\
D^*_{RN-1} &= \frac{\left(2-\theta^2\right)\left(4\left(3-4\theta^2+\theta^4\right)A_1-\theta\left(3-2\theta^2\right)A_2\right)}{72-198\theta^2+183\theta^4-66\theta^6+8\theta^8},\\
D^*_{RN-2} &= \frac{\left(3-2\theta^2\right)\left(2\left(3-4\theta^2+\theta^4\right)A_2-\theta\left(2-\theta^2\right)A_1\right)}{72-198\theta^2+183\theta^4-66\theta^6+8\theta^8}.
\end{aligned}$$

For these prices and demands to be nonnegative requires

$$4(3-4\theta^2+\theta^4)A_1 \geq \theta(3-2\theta^2)A_2 \quad \text{and} \quad 2(3-4\theta^2+\theta^4)A_2 \geq \theta(2-\theta^2)A_1,$$

which is equivalent to $\frac{\theta(3-2\theta^2)}{4(3-4\theta^2+\theta^4)} \leq \Omega \leq \frac{2(3-4\theta^2+\theta^4)}{\theta(2-\theta^2)}$. The largest feasible domain for θ is $[0, 0.902]$, as the upper bound of θ is obtained when the above two constraint boundaries cross.

In NR, symmetrically, the equilibrium for the retailers is given by

$$\begin{aligned}
\Pi^*_{NR-r1} &= \frac{\left(3-2\theta^2\right)^2\left(1-\theta^2\right)\left(2\left(3-4\theta^2+\theta^4\right)A_1-\theta\left(2-\theta^2\right)A_2\right)^2}{\left(72-198\theta^2+183\theta^4-66\theta^6+8\theta^8\right)^2},\\
\Pi^*_{NR-r2} &= \frac{\left(2-\theta^2\right)^2\left(3-4\theta^2\right)\left(4\left(3-4\theta^2+\theta^4\right)A_2-\theta\left(3-2\theta^2\right)A_1\right)^2}{4\left(72-198\theta^2+183\theta^4-66\theta^6+8\theta^8\right)^2},\\
D^*_{NR-1} &= \frac{\left(3-2\theta^2\right)\left(2\left(3-4\theta^2+\theta^4\right)A_1-\theta\left(2-\theta^2\right)A_2\right)}{72-198\theta^2+183\theta^4-66\theta^6+8\theta^8},\\
D^*_{NR-2} &= \frac{\left(2-\theta^2\right)\left(4\left(3-4\theta^2+\theta^4\right)A_2-\theta\left(3-2\theta^2\right)A_1\right)}{72-198\theta^2+183\theta^4-66\theta^6+8\theta^8}.
\end{aligned}$$

For the prices and demands to be nonnegative requires

$$2(3-4\theta^2+\theta^4)A_1 \geq \theta(2-\theta^2)A_2 \quad \text{and} \quad 4(3-4\theta^2+\theta^4)A_2 \geq \theta(3-2\theta^2)A_1,$$

where the largest feasible domain of θ is given by $\theta \in [0, 0.902]$ as the upper bound of θ is obtained when the above two constraint boundaries cross.

In the following, without loss of generality, we compare Retailer 1's profits in the various cases. To compare RN, NR, and NN, we can get their boundary values

$$\hat{\Omega}^{RN-NN}_{r1}(\theta) = \min\{\hat{\Omega}^{RN-NN}_{r1-1}(\theta), \bar{\Omega}^{RN-NN}(\theta)\},$$

$$\hat{\Omega}_{r1}^{NR-NN}(\theta) \;=\; \min\{\hat{\Omega}_{r1-1}^{NR-NN}(\theta), \bar{\Omega}^{NR-NN}(\theta)\},$$

where

$$\bar{\Omega}^{RN-NN}(\theta) = \frac{2\,(3-4\theta^2+\theta^4)}{\theta\,(2-\theta^2)} \quad \text{and} \quad \bar{\Omega}^{NR-NN}(\theta) = \frac{\theta(3-2\theta^2)}{4(3-4\theta^2+4\theta^4)},$$

which also ensure the nonnegative prices and demands for RN (NR). Meanwhile,

$$\begin{aligned}
\hat{\Omega}_{r1-1}^{RN-NN}(\theta) &= \frac{M_1 + M_2\sqrt{3-7\theta^2+4\theta^4}}{221184 - 1603584\theta^2 + M_3}, \\
\hat{\Omega}_{r1-1}^{NR-NN}(\theta) &= \frac{96 - 256\theta^2 + 270\theta^4 - 143\theta^6 + 38\theta^8 - 4\theta^{10}}{72\theta - 170\theta^3 + 142\theta^5 - 49\theta^7 + 6\theta^9},
\end{aligned}$$

where

$$\begin{aligned}
M_1 &= 552960\theta - 3571200\theta^3 + 10073280\theta^5 - 16355200\theta^7 + 16937420\theta^9 - 11694700\theta^{11} + 5452760\theta^{13} - 1694980\theta^{15} + 336120\theta^{17} - 38400\theta^{19} + 1920\theta^{21}, \\
M_2 &= \theta^3\left(4608 - 187200\theta^2 + 307200\theta^4 - 264180\theta^6 + 128870\theta^8 - 35820\theta^{10} + 5280\theta^{12} - 320\theta^{14}\right), \\
M_3 &= 51217920\theta^4 - 95037440\theta^6 + 113735520\theta^8 - 92128800\theta^{10} + 51543660\theta^{12} - 19930080\theta^{14} + 5225680\theta^{16} - 886320\theta^{18} + 87680\theta^{20} - 3840\theta^{22}.
\end{aligned}$$

We have $\Pi^*_{RN-r1} > \Pi^*_{NN-r1}$ if and only if $\Omega > \hat{\Omega}_{r1}^{RN-NN}(\theta)$ and $\Pi^*_{RR-r1} > \Pi^*_{NR-r1}$ if and only if $\Omega > \hat{\Omega}_{r1}^{RR-NR}(\theta)$. Contour plots clearly demonstrate that $\hat{\Omega}_{r1}^{RN-NN}(\theta) < \hat{\Omega}_{r1}^{RR-NR}(\theta) < 1$, and that $\hat{\Omega}_{r1}^{RN-NN}(\theta)$ and $\hat{\Omega}_{r1}^{RR-NR}(\theta)$ increase with θ. [11] These contour plots, similar to Figure 1 and others in this paper, are unique because θ is in $[0, 1)$, η is in $[0, 1]$, and we need only consider Ω in $[0, 1]$ (for cases where the base demands are not symmetric). When we cover these feasible domains, the function crosses the zero only once. We can provide any of the dozens of contour plots used in this paper, but omit them here to focus the exposition.

To compare RN, NR, and RR, we compute their boundary values as follows.

$$\begin{aligned}
\hat{\Omega}_{r1}^{RR-RN}(\theta) &= \min\{\hat{\Omega}_{r1-1}^{RR-RN}(\theta), \bar{\Omega}^{RR-RN}(\theta)\}, \\
\hat{\Omega}_{r1}^{RR-NR}(\theta) &= \min\{\hat{\Omega}_{r1-1}^{RR-NR}(\theta), \bar{\Omega}^{RR-NR}(\theta)\},
\end{aligned}$$

where

$$\bar{\Omega}^{RR-RN}(\theta) = \bar{\Omega}^{RR-NR}(\theta) = \frac{9-14\theta^2+4\theta^4}{\theta\,(3-2\theta^2)},$$

because $\frac{\theta\left(3-2\theta^2\right)}{(9-14\theta^2+4\theta^4)} > \frac{\theta\left(2-\theta^2\right)}{2(3-4\theta^2+\theta^4)} > \frac{\theta\left(3-2\theta^2\right)}{4(3-4\theta^2+\theta^4)}$, and

$$\hat{\Omega}_{r1-1}^{RR-RN}(\theta) \;=\; \frac{162 - 513\theta^2 + 642\theta^4 - 404\theta^6 + 128\theta^8 - 16\theta^{10}}{162\theta - 447\theta^3 + 434\theta^5 - 172\theta^7 + 24\theta^9},$$

[11] A contour line (also isoline or isarithm) of a function of two variables is a curve of all combinations of the two variables along which the function has a constant value (specifically zero in every one of our applications of this technique). For example, $\hat{\Omega}_{r1}^{RN-NN}(\theta)$ is a contour line of $\Pi^*_{RN-r1} - \Pi^*_{NN-r1} = 0$.

$$\hat{\Omega}_{r1-1}^{RR-NR}(\theta) = \frac{N_1 + 2N_2\sqrt{\left(1-\theta^2\right)^3\left(3-4\theta^2\right)}}{314928 - 2974320\theta^2 + N_3},$$

where

$$\begin{aligned}
N_1 &= 1049760\theta - 8806320\theta^3 + 32979960\theta^5 - 72691560\theta^7 \\
&\quad + 104618490\theta^9 - 103060040\theta^{11} + 70781320\theta^{13} - 33827760\theta^{15} + 11005120\theta^{17} - 2317760\theta^{19} + 284160\theta^{21} - 15360\theta^{23}, \\
N_2 &= \theta^3\left(-5832 + 28998\theta^2 - 57663\theta^4 + 59238\theta^6 - 33996\theta^8 + 10968\theta^{10} - 1856\theta^{12} + 128\theta^{14}\right), \\
N_3 &= 126214200\theta^4 - 317426040\theta^6 + 525630510\theta^8 - 602274360\theta^{10} \\
&\quad + 488572160\theta^{12} - 282244160\theta^{14} + 115121280\theta^{16} - 32323840\theta^{18} + 5933440\theta^{20} - 640000\theta^{22} + 30720\theta^{24}.
\end{aligned}$$

So the common feasible area for θ is $\theta \in [0, 0.823]$ where the upper bound of θ is reached when the nonnegativity constraint lines cross and the domain will be narrower as Ω decreases. We have $\Pi^*_{NR-r1} < \Pi^*_{NN-r1}$ if and only if $\Omega < \hat{\Omega}_{r1}^{NR-NN}(\theta)$ and $\Pi^*_{RR-r1} < \Pi^*_{RN-r1}$ if and only if $\Omega < \hat{\Omega}_{r1}^{RR-RN}(\theta)$. Contour plots demonstrate that $1 < \hat{\Omega}_{r1}^{RR-RN}(\theta) < \hat{\Omega}_{r1}^{NR-NN}(\theta)$, and that $\hat{\Omega}_{r1}^{RR-RN}(\theta)$ and $\hat{\Omega}_{r1}^{NR-NN}(\theta)$ decrease with θ.

Similar methods yield the boundary values for Retailer 2 in NR, RN, and NN as follows.

$$\begin{aligned}
\hat{\Omega}_{r2}^{RN-NN}(\theta) &= \min\{\hat{\Omega}_{r2-1}^{RN-NN}(\theta), \underline{\Omega}^{RN-NN}(\theta)\}, \\
\hat{\Omega}_{r2}^{NR-NN}(\theta) &= \min\{\hat{\Omega}_{r2-1}^{NR-NN}(\theta), \underline{\Omega}^{NR-NN}(\theta)\},
\end{aligned}$$

where

$$\begin{aligned}
\underline{\Omega}^{RN-NN}(\theta) &= \frac{\theta\left(3-2\theta^2\right)}{4\left(3-4\theta^2+\theta^4\right)}, \\
\underline{\Omega}^{NR-NN}(\theta) &= \frac{\theta\left(2-\theta^2\right)}{2\left(3-4\theta^2+\theta^4\right)}, \\
\hat{\Omega}_{r2-1}^{RN-NN}(\theta) &= \frac{2304 - 10008\theta^2 + 17854\theta^4 - 16922\theta^6 + 9189\theta^8 - 2858\theta^{10} + 472\theta^{12} - 32\theta^{14}}{672\theta - 2416\theta^3 + 3462\theta^5 - 2531\theta^7 + 996\theta^9 - 200\theta^{11} + 16\theta^{13}}, \\
\hat{\Omega}_{r2-1}^{NR-NN}(\theta) &= \frac{2M_1 + M_2\sqrt{3-7\theta^2+4\theta^4}}{\theta^2 M_4},
\end{aligned}$$

where

$$M_4 = 27648 - 156672\theta^2 + 383856\theta^4 - 536296\theta^6 + 472531\theta^8 - 272667\theta^{10} + 102920\theta^{12} - 24428\theta^{14} + 3296\theta^{16} - 192\theta^{18}.$$

The boundaries for Retailer 2 in NR, RN, and RR are as follows.

$$\hat{\Omega}_{r2}^{RR-RN}(\theta) = \min\{\hat{\Omega}_{r2-1}^{RR-RN}(\theta), \underline{\Omega}^{RR-RN}(\theta)\},$$

$$\hat{\Omega}_{r2}^{RR-NR}(\theta) = \min\{\hat{\Omega}_{r2-1}^{RR-NR}(\theta), \underline{\Omega}^{RR-NR}(\theta)\},$$

where

$$\begin{aligned}
\underline{\Omega}^{RR-RN}(\theta) &= \underline{\Omega}^{RR-NR}(\theta) = \frac{\theta\left(3-2\theta^2\right)}{9-14\theta^2+4\theta^4}, \\
\hat{\Omega}_{r2-1}^{RR-RN}(\theta) &= \frac{N_1+2\sqrt{N_2}}{\theta^2 N_4}, \\
\hat{\Omega}_{r2-1}^{RR-NR}(\theta) &= \frac{3888-19494\theta^2+39849\theta^4-42950\theta^6+26332\theta^8-9192\theta^{10}+1696\theta^{12}-128\theta^{14}}{11134\theta-4779\theta^3+7980\theta^5-6760\theta^7+3064\theta^9-704\theta^{11}+64\theta^{13}},
\end{aligned}$$

where

$$N_4 = 34992-256608\theta^2+833976\theta^4-1584432\theta^6+1950599\theta^8-1625544\theta^{10}+926840\theta^{12}-355712\theta^{14}+87536\theta^{16}-12416\theta^{18}+768\theta^{20}.$$

We have $\Pi^*_{NR-r2} > \Pi^*_{NN-r2}$ if and only if $\Omega < \hat{\Omega}_{r2}^{NR-NN}(\theta)$ and $\Pi^*_{RR-r2} > \Pi^*_{RN-r2}$ if and only if $\Omega < \hat{\Omega}_{r2}^{RR-RN}(\theta)$. We can show that $\hat{\Omega}_{r2}^{NR-NN}(\theta) > \hat{\Omega}_{r2}^{RR-RN}(\theta) > 1$, and that $\hat{\Omega}_{r2}^{NR-NN}(\theta)$ and $\hat{\Omega}_{r2}^{RR-RN}(\theta)$ decrease with θ. Also, we have $\Pi^*_{RN-r2} < \Pi^*_{NN-r2}$ if and only if $\Omega > \hat{\Omega}_{r2}^{RN-NN}(\theta)$ and $\Pi^*_{RR-r2} < \Pi^*_{NR-r1}$ if and only if $\Omega > \hat{\Omega}_{r2}^{RR-NR}(\theta)$. We observe that $1 > \hat{\Omega}_{r2}^{RR-NR}(\theta) > \hat{\Omega}_{r2}^{RN-NN}(\theta)$, and that $\hat{\Omega}_{r2}^{RR-NR}(\theta)$ and $\hat{\Omega}_{r2}^{RN-NN}(\theta)$ increase with θ. □

Proof of Theorem 2: Consider RN. As argued in Lemma 2, Retailer 2 prefers RN to RR as long as $\Omega > \hat{\Omega}_{r2}^{RR-RN}(\theta)$. Meanwhile, Retailer 1 prefers RN to NN as long as $\Omega > \hat{\Omega}_{r1}^{RN-NN}(\theta)$, where $\hat{\Omega}_{r1}^{RN-NN}(\theta) < \hat{\Omega}_{r2}^{RR-RN}(\theta)$. Thus neither retailer would deviate from RN as long as $\hat{\Omega}_{r2}^{RR-RN}(\theta) < \Omega < \bar{\Omega}^{RR}(\theta)$. NR also has this property by symmetry. By a similar argument, Retailer 2 prefers RR to RN as long as $\Omega < \hat{\Omega}_{r2}^{RR-RN}(\theta)$ and Retailer 1 prefers RR to NR as long as $\Omega > \hat{\Omega}_{r1}^{RR-NR}(\theta)$. Thus RR is an equilibrium if and only if $\hat{\Omega}_{r1}^{RR-NR}(\theta) < \Omega < \hat{\Omega}_{r2}^{RR-RN}(\theta)$. Worth noting is that at least one player could perform better in NN than in RR. However, this occurs outside the common feasible domain for $\theta \in [0, 0.823]$ so falls beyond the scope of our discussion. □

Proof of Lemma 3: Manufacturer advertising with cost sharing presents four possible outcomes: CSMM, CSMN, CSNM, and CSNN. The profit functions of CSMM are documented in Eq. (7) and those of CSMN and CSNM can be inferred similarly given that only one manufacturer advertises. CSNN is equivalent to NN (since cost sharing has no impact when no parties advertise), which was analyzed earlier. For brevity, below we list only equilibrium solutions for the symmetric setting ($A_1 = A_2 = 1$).

The equilibrium actions and outcomes for Manufacturer i and Retailer i in various cases are as follows. In CSMM:

$$w_{CSMM-i} = \frac{2(1-\eta)\left(4-5\theta^2+\theta^4\right)}{14-2\eta(2-\theta)(1+\theta)\left(4-\theta-2\theta^2\right)+\theta\left(4-\theta\left(17+2\theta-4\theta^2\right)\right)};$$

$$p_{CSMM-i} = \frac{4(1-\eta)\left(3-4\theta^2+\theta^4\right)}{14-2\eta(2-\theta)(1+\theta)\left(4-\theta-2\theta^2\right)+\theta\left(4-\theta\left(17+2\theta-4\theta^2\right)\right)};$$

$$e_{CSMM-mi} = \frac{2-\theta^2}{14-2\eta(2-\theta)(1+\theta)\left(4-\theta-2\theta^2\right)+\theta\left(4-\theta\left(17+2\theta-4\theta^2\right)\right)};$$

$$D_{CSMM-i} = \frac{2(1-\eta)\left(2-\theta^2\right)}{14-2\eta(2-\theta)(1+\theta)\left(4-\theta-2\theta^2\right)+\theta\left(4-\theta\left(17+2\theta-4\theta^2\right)\right)};$$

$$\Pi_{CSMM-mi} = \frac{(1-\eta)\left(2-\theta^2\right)\left(14-19\theta^2+4\theta^4-4\eta\left(4-5\theta^2+\theta^4\right)\right)}{\left(14-2\eta(2-\theta)(1+\theta)\left(4-\theta-2\theta^2\right)+\theta\left(4-\theta\left(17+2\theta-4\theta^2\right)\right)\right)^2};$$

$$\Pi_{CSMM-ri} = \frac{\left(2-\theta^2\right)^2\left(4-9\eta+4\eta^2-4(1-\eta)^2\theta^2\right)}{\left(14-2\eta(2-\theta)(1+\theta)\left(4-\theta-2\theta^2\right)+\theta\left(4-\theta\left(17+2\theta-4\theta^2\right)\right)\right)^2}.$$

In CSMN:

$$w_{CSMN-i} = \frac{2(1-\eta)(2-\theta)(1+\theta)\left(2-\theta-\theta^2\right)^2(4+\theta(1-2\theta))}{16(7-8\eta)-2(135-148\eta)\theta^2+13(17-18\eta)\theta^4-2(36-37\eta)\theta^6+8(1-\eta)\theta^8};$$

$$p_{CSMN-i} = \frac{4(1-\eta)(1-\theta)^2(1+\theta)(2+\theta)\left(3-\theta^2\right)(4+\theta(1-2\theta))}{16(7-8\eta)-2(135-148\eta)\theta^2+13(17-18\eta)\theta^4-2(36-37\eta)\theta^6+8(1-\eta)\theta^8};$$

$$e_{CSMN-m1} = \frac{\left(2-\theta^2\right)\left(2-\theta-\theta^2\right)(4+\theta(1-2\theta))}{16(7-8\eta)-2(135-148\eta)\theta^2+13(17-18\eta)\theta^4-2(36-37\eta)\theta^6+8(1-\eta)\theta^8};$$

$$D_{CSMN-i} = \frac{2(1-\eta)(1-\theta)(2+\theta)\left(2-\theta^2\right)(4+\theta(1-2\theta))}{16(7-8\eta)-2(135-148\eta)\theta^2+13(17-18\eta)\theta^4-2(36-37\eta)\theta^6+8(1-\eta)\theta^8};$$

$$\Pi_{CSMN-mi} = \frac{(1-\eta)(1-\theta)^2(2+\theta)^2\left(2-\theta^2\right)(4+\theta(1-2\theta))^2\left(14-19\theta^2+4\theta^4-4\eta\left(4-5\theta^2+\theta^4\right)\right)}{\left(16(7-8\eta)-2(135-148\eta)\theta^2+13(17-18\eta)\theta^4-2(36-37\eta)\theta^6+8(1-\eta)\theta^8\right)^2};$$

$$\Pi_{CSMN-ri} = \frac{(1-\theta)^2(2+\theta)^2\left(2-\theta^2\right)^2\left(4-9\eta+4\eta^2-4(1-\eta)^2\theta^2\right)(4+\theta(1-2\theta))^2}{\left(16(7-8\eta)-2(135-148\eta)\theta^2+13(17-18\eta)\theta^4-2(36-37\eta)\theta^6+8(1-\eta)\theta^8\right)^2}.$$

In CSNM:

$$w_{CSNM-i} = \frac{\left(4-5\theta^2+\theta^4\right)(14-2\eta(1-\theta)(2+\theta)(4+\theta(1-2\theta))-\theta(4+\theta(17-2\theta(1+2\theta))))}{16(7-8\eta)-2(135-148\eta)\theta^2+13(17-18\eta)\theta^4-2(36-37\eta)\theta^6+8(1-\eta)\theta^8};$$

$$p_{CSNM-i} = \frac{2\left(3-4\theta^2+\theta^4\right)(14-2\eta(1-\theta)(2+\theta)(4+\theta(1-2\theta))-\theta(4+\theta(17-2\theta(1+2\theta))))}{16(7-8\eta)-2(135-148\eta)\theta^2+13(17-18\eta)\theta^4-2(36-37\eta)\theta^6+8(1-\eta)\theta^8};$$

$$e_{CSNM-m2} = \frac{\left(2-\theta^2\right)\left(2-\theta-\theta^2\right)(4+\theta(1-2\theta))}{16(7-8\eta)-2(135-148\eta)\theta^2+13(17-18\eta)\theta^4-2(36-37\eta)\theta^6+8(1-\eta)\theta^8};$$

$$D_{CSNM-i} = \frac{\left(2-\theta^2\right)(14-2\eta(1-\theta)(2+\theta)(4+\theta(1-2\theta))-\theta(4+\theta(17-2\theta(1+2\theta))))}{16(7-8\eta)-2(135-148\eta)\theta^2+13(17-18\eta)\theta^4-2(36-37\eta)\theta^6+8(1-\eta)\theta^8};$$

$$\Pi_{CSNM-mi} = \frac{\left(4-\theta^2\right)\left(2-3\theta^2+\theta^4\right)\left(14-2\eta(1-\theta)(2+\theta)(4+\theta(1-2\theta))-\theta(4+\theta(17-2\theta(1+2\theta)))\right)^2}{\left(16(7-8\eta)-2(135-148\eta)\theta^2+13(17-18\eta)\theta^4-2(36-37\eta)\theta^6+8(1-\eta)\theta^8\right)^2};$$

$$\Pi_{CSNM-ri} = \frac{\left(2-\theta^2\right)^2\left(1-\theta^2\right)\left(14-2\eta(1-\theta)(2+\theta)(4+\theta(1-2\theta))-\theta(4+\theta(17-2\theta(1+2\theta)))\right)^2}{\left(16(7-8\eta)-2(135-148\eta)\theta^2+13(17-18\eta)\theta^4-2(36-37\eta)\theta^6+8(1-\eta)\theta^8\right)^2}.$$

The equilibria for the rival manufacturer and retailer follow by symmetry. For example, for Manufacturer 2 in CSMN, $w_{CSMN-1} = w_{CSNM-2}$ and $p_{CSMN-1} = p_{CSNM-2}$. To ensure a meaningful comparison, we enforce the common feasible domain for all cases. That is, $\eta < \hat{\eta}_{mi}^{CSMM}(\theta) \equiv \frac{14-4\theta-17\theta^2+2\theta^3+4\theta^4}{2(8-2\theta-9\theta^2+\theta^3+2\theta^4)}$.

We first compare CSMN and CSNN (i.e., NN). Define $\Delta\Pi_{m1}^{CSMN-NN} \equiv \Pi_{CSMN-m1} - \Pi_{NN-m1}$ as Manufacturer 1's profit in CSMN minus that in NN. This is strictly positive if and only if $\eta < \hat{\eta}_{m1}^{CSMN-NN}(\theta) \equiv \frac{896-3232\theta^2+4570\theta^4-3222\theta^6+1191\theta^8-220\theta^{10}+16\theta^{12}}{1024-3584\theta^2+4940\theta^4-3407\theta^6+1235\theta^8-224\theta^{10}+16\theta^{12}}$, which exceeds $\eta_{m1}^{CSMM}(\theta)$ and thus lies outside the common feasible domain. Hence, $\Pi_{CSMN-m1} > \Pi_{NN-m1}$ throughout the common feasible domain. Next define $\Delta\Pi_{m1}^{CSMM-CSNM} \equiv \Pi_{CSMM-m1} - \Pi_{CSNM-m1}$. By contour plotting, we find $\Delta\Pi_{m1}^{CSMM-CSNM} > 0$ for any θ and η in the feasible domain. Therefore, Manufacturer 1 always benefits from its own advertising under cost sharing at any cost sharing rate. So does Manufacturer 2. Thus, CSMM is the unique equilibrium for manufacturer advertising with cost sharing given $\Omega = 1$.

We now compare CSMM with NN. Define $\Delta\Pi_{mi}^{CSMM-NN} \equiv \Pi_{CSMM-mi} - \Pi_{NN-mi}$. This is strictly positive if and only if $\eta < \hat{\eta}_{mi}^{CSMM-NN}(\theta) \equiv \frac{2\left(14-7\theta-27\theta^2+9\theta^3+14\theta^4-2\theta^5-2\theta^6\right)}{32-16\theta-58\theta^2+19\theta^3+29\theta^4-4\theta^5-4\theta^6}$, which is outside the common feasible domain when $\theta < 0.676$. Therefore, $\Pi_{CSMM-mi} < \Pi_{NN-mi}$ when $\eta > \hat{\eta}_{mi}^{CSMM-NN}(\theta)$; otherwise, $\Pi_{CSMM-mi} \geq \Pi_{NN-mi}$. Figure 13 illustrates this property, which implies that the manufacturers might encounter a Prisoner's Dilemma under manufacturer advertising with cost sharing.□

Proof of Lemma 4: Retailer advertising with cost sharing presents four possible outcomes: CSRR, CSRN, CSNR, and CSNN. The profit functions under CSRR are documented in Eq. (6) and those of CSRN and CSNR can be inferred similarly given that only one retailer advertises. Again, CSNN is equivalent to Case NN. For brevity, we present findings only for the symmetric setting of $A_1 = A_2 = 1$.

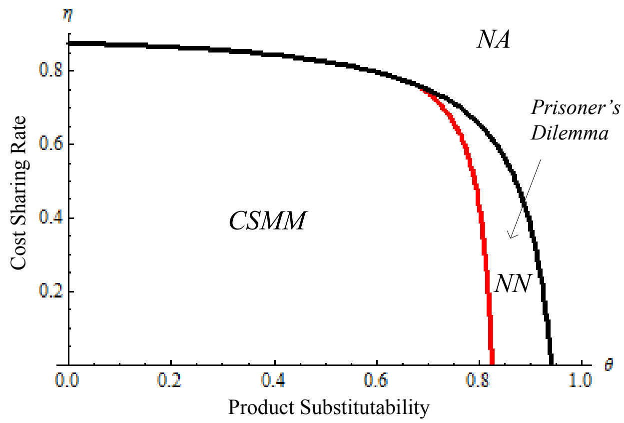

Figure 13: Manufacturer 1's preference between CSMM and NN, given $\Omega = 1$

The equilibrium actions and outcomes for Manufacturer i and Retailer i in various cases are as follows.

In CSRR:

$$\begin{aligned}
w_{CSRR-i} &= \frac{9-30\eta+36\eta^2-16\eta^3-2(1-\eta)(8-\eta(17-10\eta))\theta^2+4(1-\eta)^3\theta^4}{18-63\eta+76\eta^2-32\eta^3+2(1-\eta)^2(3-4\eta)\theta-2(1-\eta)(14-\eta(31-18\eta))\theta^2-4(1-\eta)^3\theta^3+8(1-\eta)^3\theta^4};\\
p_{CSRR-i} &= \frac{15-50\eta+58\eta^2-24\eta^3-2(1-\eta)(13-4\eta(7-4\eta))\theta^2+8(1-\eta)^3\theta^4}{18-63\eta+76\eta^2-32\eta^3+2(1-\eta)^2(3-4\eta)\theta-2(1-\eta)(14-\eta(31-18\eta))\theta^2-4(1-\eta)^3\theta^3+8(1-\eta)^3\theta^4};\\
e_{CSRR-ri} &= \frac{(1-\eta)\left(3-4\eta-2(1-\eta)\theta^2\right)}{18-63\eta+76\eta^2-32\eta^3+2(1-\eta)^2(3-4\eta)\theta-2(1-\eta)(14-\eta(31-18\eta))\theta^2-4(1-\eta)^3\theta^3+8(1-\eta)^3\theta^4};\\
D_{CSRR-i} &= \frac{2(1-\eta)^2\left(3-4\eta-2(1-\eta)\theta^2\right)}{18-63\eta+76\eta^2-32\eta^3+2(1-\eta)^2(3-4\eta)\theta-2(1-\eta)(14-\eta(31-18\eta))\theta^2-4(1-\eta)^3\theta^3+8(1-\eta)^3\theta^4};\\
\Pi_{CSRR-mi} &= \frac{(1-\eta)^2\left((3-4\eta)^2(6-\eta(13-8\eta))-4(1-\eta)(3-4\eta)(11-2\eta(12-7\eta))\theta^2+4(1-\eta)^2(22-7\eta(7-4\eta))\theta^4-16(1-\eta)^4\theta^6\right)}{\left(18-63\eta+76\eta^2-32\eta^3+2(1-\eta)^2(3-4\eta)\theta-2(1-\eta)(14-\eta(31-18\eta))\theta^2-4(1-\eta)^3\theta^3+8(1-\eta)^3\theta^4\right)^2};\\
\Pi_{CSRR-ri} &= \frac{(1-\eta)^3\left(3-4\eta-2(1-\eta)\theta^2\right)^2\left(3-4\eta-4(1-\eta)\theta^2\right)}{\left(18-63\eta+76\eta^2-32\eta^3+2(1-\eta)^2(3-4\eta)\theta-2(1-\eta)(14-\eta(31-18\eta))\theta^2-4(1-\eta)^3\theta^3+8(1-\eta)^3\theta^4\right)^2}.
\end{aligned}$$

In CSRN:

$$\begin{aligned}
w_{CSRN-1} &= \frac{\left(6-4\eta(3-2\eta)-(9-2(9-5\eta)\eta)\theta^2+2(1-\eta)^2\theta^4\right)CS_3}{CS_1};\\
p_{CSRN-1} &= \frac{\left(2(5-2\eta(5-3\eta))-(15-2(15-8\eta)\eta)\theta^2+4(1-\eta)^2\theta^4\right)CS_3}{CS_1};\\
e_{CSRN-r1} &= \frac{(1-\eta)\left(2-\theta^2\right)CS_3}{CS_1};\\
D_{CSRN-1} &= \frac{(1-\eta)^2\left(2-\theta^2\right)CS_3}{CS_2};\\
\Pi_{CSRN-m1} &= \frac{(1-\eta)^2\left(2-\theta^2\right)\left(2(6-\eta(13-8\eta))-(18-(37-20\eta)\eta)\theta^2+4(1-\eta)^2\theta^4\right)CS_3^2}{4CS_2^2};\\
\Pi_{CSRN-r1} &= \frac{(1-\eta)^3\left(2-\theta^2\right)^2\left(3-4\eta-4(1-\eta)\theta^2\right)CS_3^2}{4CS_2^2}.
\end{aligned}$$

In CSNR:

$$\begin{aligned}
w_{CSNR-1} &= \frac{\left(6-9\theta^2+2\theta^4-2\eta\left(4-5\theta^2+\theta^4\right)\right)CS_4}{CS_1};\\
p_{CSNR-1} &= \frac{\left(9-12\eta-2(7-8\eta)\theta^2+4(1-\eta)\theta^4\right)CS_4}{CS_1};\\
D_{CSNR-1} &= \frac{\left(3-4\eta-2(1-\eta)\theta^2\right)CS_4}{CS_1};\\
\Pi_{CSNR-m1} &= \frac{\left(3-4\eta-2(1-\eta)\theta^2\right)\left(6-9\theta^2+2\theta^4-2\eta\left(4-5\theta^2+\theta^4\right)\right)CS_4^2}{4CS_2^2};\\
\Pi_{CSNR-r1} &= \frac{\left(1-\theta^2\right)\left(3-4\eta-2(1-\eta)\theta^2\right)^2CS_4^2}{4CS_2^2}.
\end{aligned}$$

In the above,

$$\begin{aligned}
CS_1 &= 8(3-4\eta)(6-\eta(13-8\eta))-4(99-\eta(331-2(189-74\eta)\eta))\theta^2\\
&\quad +2(183-2\eta(293-3\eta(106-39\eta)))\theta^4-4(1-\eta)(33-\eta(69-37\eta))\theta^6+16(1-\eta)^3\theta^8;\\
CS_2 &= 4(3-4\eta)(6-\eta(13-8\eta))-2(99-\eta(331-2(189-74\eta)\eta))\theta^2\\
&\quad +(183-2\eta(293-3\eta(106-39\eta)))\theta^4-2(1-\eta)(33-\eta(69-37\eta))\theta^6+8(1-\eta)^3\theta^8;\\
CS_3 &= 12-2\eta(1-\theta)(2+\theta)(4+\theta(1-2\theta))-\theta\left(3+2\theta\left(8-\theta-2\theta^2\right)\right);\\
CS_4 &= 2(6-\eta(13-8\eta))-4(1-\eta)^2\theta-(16-3(11-6\eta)\eta)\theta^2+2(1-\eta)^2\theta^3+4(1-\eta)^2\theta^4.
\end{aligned}$$

The equilibrium actions and outcomes for the other manufacturer and retailer in each setting can be easily obtained by symmetry. For example, $w_{CSRN-1}=w_{CSNR-2}$ and $p_{CSRN-1}=p_{CSNR-2}$. The common feasible area for all forms of retailer advertising with cost sharing is $\eta<\frac{3-2\theta^2}{2(2-\theta^2)}\equiv\hat{\eta}_{ri}^{CSRR}(\theta)$.

We now compare CSRN and CSNN. Define $\Delta\Pi_{r1}^{CSRN-NN}\equiv\Pi_{CSRR-r1}-\Pi_{NN-r1}$ as Retailer 1's profits in CSRN minus the one in NN. We prove the existence of $\hat{\eta}_{r1}^{CSRN-NN}(\theta)$ by characterizing $\Delta\Pi_{r1}^{CSRN-NN}=0$ through a contour plot. The threshold curve is then uniquely represented by $\hat{\eta}_{r1}^{CSRN-NN}(\theta)$, because there are only two viable parameters. $\hat{\eta}_{r1}^{CSRN-NN}(\theta)$ is in the middle of the feasible domain and decreases with θ.

Note that $\hat{\eta}_{r1}^{CSRN-NN}$ is equivalent to $\hat{\eta}_{r2}^{CSNR-NN}$ by symmetry. Therefore, no retailer will unilaterally deviate from NN if and only if $\eta>\hat{\eta}_{r1}^{CSRN-NN}$.

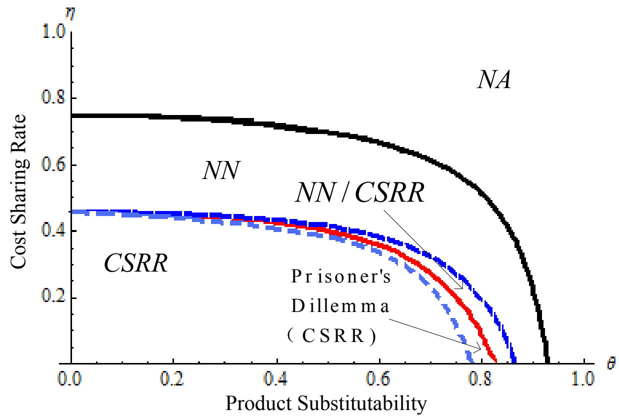

Figure 14: Equilibrium analysis in retailer advertising with cost sharing, given $\Omega = 1$.

Now compare CSRR and CSRN. Define $\Delta\Pi_{r2}^{CSRR-CSRN} \equiv \Pi_{CSRR-r2} - \Pi_{CSRN-r2}$. We obtain $\hat{\eta}_{r2}^{CSRR-CSRN}(\theta)$ from the contour plot of $\Delta\Pi_{r2}^{CSRR-CSRN} = 0$. $\hat{\eta}_{r2}^{CSRR-CSRN}(\theta)$ is in the middle of feasible domain and decreases with θ.

Note that $\hat{\eta}_{r1}^{CSRR-CSNR}$ is equivalent to $\hat{\eta}_{r2}^{CSRR-CSRN}$. Therefore, no retailer will unilaterally deviate from CSRR if and only if $\eta < \hat{\eta}_{r1}^{CSRR-CSNR}$. It is worth noting that $\eta_{r1}^{CSRN-NN} < \hat{\eta}_{r1}^{CSRR-CSNR}$. That is, a domain exists in which both CSRR and NN can be equilibria, as illustrated in Figure 14.

We now compare CSRR and NN. Define $\Delta\Pi_{r1}^{CSRR-NN} \equiv \Pi_{CSRR-r1} - \Pi_{NN-r1}$. By contour plotting we obtain a unique $\hat{\eta}^{CSRR-NN}(\theta)$ from $\Delta\Pi_{r1}^{CSRR-NN} = 0$. Since $\hat{\eta}^{CSRR-NN}(\theta) < \hat{\eta}_{r1}^{CSRN-NN} < \hat{\eta}_{r1}^{CSRR-CSNR}$, the retailers encounter a Prisoner's Dilemma when $\hat{\eta}_{r1}^{CSRR-NN}(\theta) < \eta < \hat{\eta}_{r1}^{CSRN-NN}(\theta)$, because both retailers are harmed by their advertising even though advertising is a dominant equilibrium strategy. Figure 14 summarizes all the above findings. □

Proof of Theorem 3: Because of symmetry the following proof needs only to consider Manufacturer 1 and Retailer 1. We first compare Manufacturer 1's profits between CSMM and MM and between CSRR and RR. Contour plotting shows that Manufacturer 1 prefers CSMM to MM as long as $\eta < \hat{\eta}_{m1}^{CSMM-MM}(\theta)$, where

$$\hat{\eta}_{m1}^{CSMM-MM}(\theta) = \frac{196 - 548\theta^2 - 16\theta^3 + 485\theta^4 + 8\theta^5 - 156\theta^6 + 16\theta^8}{8\,(28 - 75\theta^2 - 2\theta^3 + 64\theta^4 + \theta^5 - 20\theta^6 + 2\theta^8)}.$$

When $\hat{\eta}_{m1}^{CSMM-MM}(\theta) < \eta < \hat{\eta}_{mi}^{CSMM}(\theta)$, which is the common feasible area for all cases of manufacturer advertising with cost sharing, Manufacturer 1 prefers MM to CSMM. Similarly, Manu-

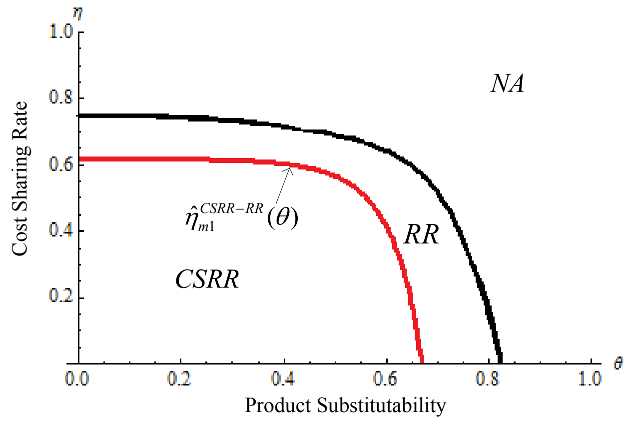

Figure 15: Manufacturer 1's preference between RR and CSRR given $\Omega = 1$.

facturer 1 prefers CSRR to RR as long as $\eta < \hat{\eta}_{m1}^{CSRR-RR}(\theta)$, where $\hat{\eta}_{m1}^{CSRR-RR}(\theta)$ is illustrated in Figure 15. If $\hat{\eta}_{m1}^{CSRR-RR}(\theta) < \eta < \hat{\eta}_{ri}^{CSRR}(\theta)$, which is the common feasible area for all cases of retailer advertising with cost sharing, Manufacturer 1 prefers RR to CSRR.

Now consider Retailer 1's profit differences between CSMM and MM and between CSRR and RR. Methods similar to those described earlier show that Retailer 1 prefers CSMM to MM as long as $\eta < \hat{\eta}_{r1}^{CSMM-MM}(\theta)$, where

$$\hat{\eta}_{r1}^{CSMM-MM}(\theta) = \frac{28 - 48\theta - 148\theta^2 + 64\theta^3 + 199\theta^4 - 20\theta^5 - 100\theta^6 + 16\theta^8}{4\left(60 + 16\theta - 160\theta^2 - 32\theta^3 + 151\theta^4 + 20\theta^5 - 59\theta^6 - 4\theta^7 + 8\theta^8\right)}.$$

If $\hat{\eta}_{r1}^{CSMM-MM}(\theta) < \eta < \hat{\eta}_{mi}^{CSMM}(\theta)$, Retailer 1 prefers MM to CSMM. We have $\hat{\eta}_{r1}^{CSMM-MM}(\theta) < \hat{\eta}_{m1}^{CSMM-MM}(\theta) < \hat{\eta}_{m1}^{CSRR-RR}(\theta)$. The contour plot on the θ,η plane shows that RR dominates CSRR for Retailer 1 throughout the entire feasible domain. □

Proof of Theorem 4: We explicitly present the proof for $U_{RR} > U_{MM}$ only. The others are similar in nature so we omit due to their length. They are available on request.

Consumer welfare (U, with superscripts and subscripts following the conventions used throughout this paper) is based on the utility of the representative consumer in Eq. (3). Some algebra yields

$$\begin{aligned} \triangle U^{RR-MM}(\theta) &\equiv U_{RR} - U_{MM} \\ &= \frac{\left(6 - 5\theta^2 + 2\theta^4\right)\left(n_1 \times \Omega^2 + n_2 \times 2\theta\Omega + n_3\right) A_2^2}{2\left(196 - 492\theta^2 + 417\theta^4 - 140\theta^6 + 16\theta^8\right)^2 \left(81 - 261\theta^2 + 280\theta^4 - 116\theta^6 + 16\theta^8\right)^2}, \end{aligned}$$

where

$$n_1 = 1238328 - 102024360\theta^2 + 372154380\theta^4 - 791899470\theta^6 + 1090522310\theta^8 - 1019350860\theta^{10} + 659563400\theta^{12} - 295403280\theta^{14}$$

$$
\begin{aligned}
& +89787720\theta^{16} - 17652480\theta^{18} + 2022400\theta^{20} - 102400\theta^{22}; \\
n_2 \quad = \quad & 86184 - 1271628\theta^{2} + 6466026\theta^{4} - 17175409\theta^{6} + 27728341\theta^{8} - 29181402\theta^{10} + 20675252\theta^{12} - 9940664\theta^{14} \\
& +3197248\theta^{16} - 658176\theta^{18} + 78336\theta^{20} - 4096\theta^{22}; \\
n_3 \quad = \quad & 1238328 - 10202436\theta^{2} + 37215438\theta^{4} - 79189947\theta^{6} + 109052231\theta^{8} - 101935086\theta^{10} + 65956340\theta^{12} - 29540328\theta^{14} \\
& +89787720\theta^{16} - 17652480\theta^{18} + 2022400\theta^{20} - 102400\theta^{22}.
\end{aligned}
$$

In the common feasible domain, $\triangle U^{RR-MM}(\theta,\Omega)$ is convex with respect to Ω, and increases with Ω for $\Omega > 0$. Furthermore, $\triangle U^{RR-MM}(\theta,\Omega) > \triangle U^{RR-MM}(\theta,0) = \frac{\left(6-5\theta^2+2\theta^4\right)n_1}{2(196-492\theta^2+417\theta^4-140\theta^6+16\theta^8)^2(81-261\theta^2+280\theta^4-11}$ which is positive in the common feasible domain. Hence $U_{RR} > U_{MM}$. □

Proof of Corollary 1: The following discussion is based on the common feasible domain under both manufacturer and retailer advertising; that is, $\Omega \in [\frac{\theta(3-2\theta^2)}{9-14\theta^2+4\theta^4}, \frac{9-14\theta^2+4\theta^4}{\theta(3-2\theta^2)}]$ and $\theta \in [0, 0.823]$. Without loss of generality, we let $A_{3-i} = 1$ and then $\Omega = A_i$. For supply chain 1's advertising level, we consider the relationship between e and Ω for MM:

$$\frac{\partial e^*_{MM-m1}}{\partial \Omega} = \frac{(2-\theta^2)(14-17\theta^2+4\theta^4)}{196-492\theta^2+417\theta^4-140\theta^6+16\theta^8} > 0.$$

For RR,

$$\frac{\partial e^*_{RR-r1}}{\partial \Omega} = \frac{(3-2\theta^2)(9-14\theta^2+4\theta^4)}{162-522\theta^2+560\theta^4-232\theta^6+32\theta^8},$$

which is positive in the feasible domain. So, supply chain 1's advertising effort level increases with Ω. For supply chain 2's advertising level, we consider the relationship between e and Ω for MM:

$$\frac{\partial e^*_{MM-m2}}{\partial \Omega} = -\frac{2\theta\left(2-\theta^2\right)^2}{196-492\theta^2+417\theta^4-140\theta^6+16\theta^8} < 0.$$

For RR,

$$\frac{\partial e^*_{RR-r2}}{\partial \Omega} = -\frac{\theta\left(3-2\theta^2\right)^2}{162-522\theta^2+560\theta^4-232\theta^6+32\theta^8} < 0,$$

Similar results arise in the other subgames, which are omitted here for brevity.

Now consider the relationship between e and θ. For MM,

$$\frac{\partial e^*_{MM-m1}}{\partial \theta} = \frac{2\left(\begin{array}{c} -784+384\theta^2+2056\theta^4-2992\theta^6+1711\theta^8-460\theta^{10}+48\theta^{12} \\ +4\theta\left(1092-3388\theta^2+4281\theta^4-2824\theta^6+1034\theta^8-200\theta^{10}+16\theta^{12}\right)A_1 \end{array}\right)}{\left(196-492\theta^2+417\theta^4-140\theta^6+16\theta^8\right)^2},$$

which is positive if and only if $A_1 > \frac{784-384\theta^2-2056\theta^4+2992\theta^6-1711\theta^8+460\theta^{10}-48\theta^{12}}{4\theta(1092-3388\theta^2+4281\theta^4-2824\theta^6+1034\theta^8-200\theta^{10}+16\theta^{12})} \doteq \Omega^{e-\theta}_{MM}$. For RR,

$$\frac{\partial e^*_{RR-r1}}{\partial\theta}=\frac{\begin{array}{c}-729+567\theta^2+2808\theta^4-5448\theta^6+4064\theta^8-14240\theta^{10}+192\theta^{12}\\+2\theta\left(2187-8640\theta^2+13812\theta^4-11472\theta^6+5280\theta^8-1280\theta^{10}+128\theta^{12}\right)A_1\end{array}}{2\left(81-261\theta^2+280\theta^4-116\theta^6+16\theta^8\right)^2},$$

which is positive if and only if $A_1>\frac{729-567\theta^2-2808\theta^4+5448\theta^6-4064\theta^8+14240\theta^{10}-192\theta^{12}}{2\theta(2187-8640\theta^2+13812\theta^4-11472\theta^6+5280\theta^8-1280\theta^{10}+128\theta^{12})}\doteq\Omega^{e-\theta}_{RR}$.

Similar results arise in the other subgames, which are omitted here for brevity. □

Proof of Corollary 2: The following discussion is based on the common feasible domain of CSMM and CSRR, that is $\eta<\frac{14-4\theta-17\theta^2+2\theta^3+4\theta^4}{2(8-2\theta-9\theta^2+\theta^3+2\theta^4)}=\hat{\eta}^{CSMM}_{mi}(\theta)$. For CSMM,

$$\frac{\partial e_{CSMM-mi}}{\partial\theta}=\frac{2(1-\eta)\left(-4+20\theta+4\theta^2-16\theta^3-\theta^4+4\theta^5\right)}{\left(14+4\theta-17\theta^2-2\theta^3+4\theta^4-2\eta\left(8+2\theta-9\theta^2-\theta^3+2\theta^4\right)\right)^2},$$

which is positive if and only if $-4+20\theta+4\theta^2-16\theta^3-\theta^4+4\theta^5>0$. We define $\theta_{CSMM}\doteq arg\{\theta|-4+20\theta+4\theta^2-16\theta^3-\theta^4+4\theta^5=0\}$, which is unique in the feasible domain. For CSRR,

$$\frac{\partial e_{CSRR-ri}}{\partial\theta}=\frac{2(1-\eta)^3\begin{pmatrix}-9+48\theta+12\theta^2-48\theta^3-4\theta^4+16\theta^5\\+4\eta^2\left(-4+20\theta+4\theta^2-16\theta^3-\theta^4+4\theta^5\right)\\-4\eta\left(-6+31\theta+7\theta^2-28\theta^3-2\theta^4+8\theta^5\right)\end{pmatrix}}{\begin{pmatrix}-18-6\theta+28\theta^2+4\theta^3-8\theta^4+4\eta^3\left(8+2\theta-9\theta^2-\theta^3+2\theta^4\right)\\-2\eta^2\left(38+11\theta-49\theta^2-6\theta^3+12\theta^4\right)+\eta\left(63+20\theta-90\theta^2-12\theta^3+24\theta^4\right)\end{pmatrix}^2},$$

which is positive as long as $\eta>\frac{-6+31\theta+7\theta^2-28\theta^3-2\theta^4+8\theta^5+\sqrt{\theta^2+2\theta^3-8\theta^4}}{2(-4+20\theta+4\theta^2-16\theta^3-\theta^4+4\theta^5)}\doteq\eta_{CSRR}$. □

Proof of Theorem 5:

We provide results for Manufacturer 1 and Retailer 1 here, and invoke symmetry for Manufacturer 2 and Retailer 2. The following lemma is needed to prove Theorem 5.

Lemma 6 *Consider Manufacturer* 1 *and Retailer* 1 *in a scenario of hybrid advertising. Boundary values exist such that*

1. *Both Manufacturer* 1 *and Retailer* 1 *simultaneously prefer RM to MM if and only if* $\Omega>$

$\hat{\Omega}_{r1}^{RM-MM}(\theta)$*, but prefer MM to RM if and only if* $\Omega < \hat{\Omega}_{m1}^{RM-MM}(\theta)$*, where* $\hat{\Omega}_{r1}^{RM-MM}(\theta) > \hat{\Omega}_{m1}^{RM-MM}(\theta)$.

2. *Both Manufacturer* 1 *and Retailer* 1 *simultaneously prefer MR to RR if and only if* $\Omega < \min\{\hat{\Omega}_{r1-2}^{MR-RR}(\theta),$
$\hat{\Omega}_{m1}^{MR-RR}(\theta)\}$*, but prefer RR to MR if and only if* $\Omega > \max\{\hat{\Omega}_{r1-2}^{MR-RR}(\theta), \hat{\Omega}_{m1}^{MR-RR}(\theta)\}$.

3. *Both Manufacturer* 1 *and Retailer* 1 *simultaneously prefer RM to NM if and only if* $\Omega > \hat{\Omega}_{r1}^{RM-NM}(\theta)$*, but prefer NM to RM if and only if* $\Omega < \hat{\Omega}_{m1}^{RM-NM}(\theta)$.

4. *Both Manufacturer* 1 *and Retailer* 1 *always prefer MR to NR.*

Proof of Lemma 6: We first follow the itemized sequence of results in Lemma 6 and then extend our proof to Manufacturer 2 and Retailer 2.

(1) *Compare MM to RM and MR.* We directly start with the unique equilibrium of RM that follows.

$$
\begin{aligned}
w^*_{RM-1} &= \frac{(6-9\theta^2+2\theta^4)((21-30\theta^2+8\theta^4)A_1-2\theta(3-2\theta^2)A_2)}{4(63-180\theta^2+172\theta^4-64\theta^6+8\theta^8)},\\
w^*_{RM-2} &= \frac{(6-9\theta^2+2\theta^4)(2(3-4\theta^2+\theta^4)A_2-\theta(2-\theta^2)A_1)}{63-180\theta^2+172\theta^4-64\theta^6+8\theta^8},\\
p^*_{RM-1} &= \frac{(10-15\theta^2+4\theta^4)\left((21-30\theta^2+8\theta^4)A_1-2\theta\left(3-2\theta^2\right)A_2\right)}{4\left(63-180\theta^2+172\theta^4-64\theta^6+8\theta^8\right)},\\
p^*_{RM-2} &= \frac{(9-14\theta^2+4\theta^4)\left(2\left(3-4\theta^2+\theta^4\right)A_2-\theta\left(2-\theta^2\right)A_1\right)}{63-180\theta^2+172\theta^4-64\theta^6+8\theta^8},\\
e^*_{RM-r1} &= \frac{(2-\theta^2)\left((21-30\theta^2+8\theta^4)A_1-2\theta\left(3-2\theta^2\right)A_2\right)}{4\left(63-180\theta^2+172\theta^4-64\theta^6+8\theta^8\right)},\\
e^*_{RM-m2} &= \frac{(3-2\theta^2)\left(2\left(3-4\theta^2+\theta^4\right)A_2-\theta\left(2-\theta^2\right)A_1\right)}{2\left(63-180\theta^2+172\theta^4-64\theta^6+8\theta^8\right)},\\
D^*_{RM-1} &= \frac{(2-\theta^2)\left((21-30\theta^2+8\theta^4)A_1-2\theta\left(3-2\theta^2\right)A_2\right)}{2\left(63-180\theta^2+172\theta^4-64\theta^6+8\theta^8\right)},\\
D^*_{RM-2} &= \frac{(3-2\theta^2)\left(2\left(3-4\theta^2+\theta^4\right)A_2-\theta\left(2-\theta^2\right)A_1\right)}{63-180\theta^2+172\theta^4-64\theta^6+8\theta^8},\\
\Pi^*_{RM-r1} &= \frac{(2-\theta^2)^2\left(3-4\theta^2\right)\left((21-30\theta^2+8\theta^4)A_1-2\theta\left(3-2\theta^2\right)A_2\right)^2}{16\left(63-180\theta^2+172\theta^4-64\theta^6+8\theta^8\right)^2},\\
\Pi^*_{RM-r2} &= \frac{(3-2\theta^2)^2\left(1-\theta^2\right)\left(2\left(3-4\theta^2+\theta^4\right)A_2-\theta\left(2-\theta^2\right)A_1\right)^2}{\left(63-180\theta^2+172\theta^4-64\theta^6+8\theta^8\right)^2},
\end{aligned}
$$

$$\Pi^*_{RM-m1} = \frac{(2-\theta^2)(6-9\theta^2+2\theta^4)\left((21-30\theta^2+8\theta^4)A_1-2\theta(3-2\theta^2)A_2\right)^2}{8\left(63-180\theta^2+172\theta^4-64\theta^6+8\theta^8\right)^2},$$
$$\Pi^*_{RM-m2} = \frac{(63-144\theta^2+92\theta^4-16\theta^6)\left(2(3-4\theta^2+\theta^4)A_2-\theta(2-\theta^2)A_1\right)^2}{4\left(63-180\theta^2+172\theta^4-64\theta^6+8\theta^8\right)^2}.$$

For prices and demands to be nonnegative requires

$$(21-30\theta^2+8\theta^4)A_1-2\theta(3-2\theta^2)A_2 \geq 0 \quad \text{and} \quad 2(3-4\theta^2+\theta^4)A_2-\theta(2-\theta^2)A_1 \geq 0.$$

Thus, the common lower and upper bounds for RM and MM are defined as follows:

$$\underline{\Omega}^{RM-MM}(\theta) = \frac{2\theta(3-2\theta^2)}{21-30\theta^2+8\theta^4} \quad \text{and} \quad \bar{\Omega}^{RM-MM}(\theta) = \frac{2(3-4\theta^2+\theta^4)}{\theta(2-\theta^2)}.$$

The feasible domain for θ is $\theta \in [0, 0.876]$, where the upper bound of θ arises when the above two constraint lines cross, which is narrower than the domain of MM but wider than that of RR. Following the same steps as in the proof of Lemma 1, the boundary values of $\hat{\Omega}^{RM-MM}_{r1}(\theta)$ and $\hat{\Omega}^{RM-MM}_{m1}(\theta)$ result from equating the profits of RM and those of MM:

$$\hat{\Omega}^{RM-MM}_{r1}(\theta) = \min\{\hat{\Omega}^{RM-MM}_{r1-1}(\theta), \bar{\Omega}^{RM-MM}(\theta)\},$$
$$\hat{\Omega}^{RM-MM}_{m1}(\theta) = \min\{\hat{\Omega}^{RM-MM}_{m1-1}(\theta), \bar{\Omega}^{RM-MM}(\theta)\},$$

where

$$\hat{\Omega}^{RM-MM}_{r1-1}(\theta) = \frac{2(k_1+16k_2\sqrt{3-7\theta^2+4\theta^4})}{1037232-12940704\theta^2+k_3},$$
$$\hat{\Omega}^{RM-MM}_{m1-1}(\theta) = \frac{2(k_4+4\sqrt{2}k_2\sqrt{84-240\theta^2+223\theta^4-74\theta^6+8\theta^8})}{14521248-125505072\theta^2+k_5},$$

$$\begin{aligned}
k_1 &= 1481176\theta-13970088\theta^3+58294432\theta^5-141471600\theta^7+221268845\theta^9-233757712\theta^{11} \\
&\quad +169938520\theta^{13}-84878840\theta^{15}+28501760\theta^{17}-6123520\theta^{19}+757760\theta^{21}-40960\theta^{23}, \\
k_2 &= \theta^3\left(12348-662760\theta^2+1485430\theta^4-1810480\theta^6+1309880\theta^8-575840\theta^{10}+150480\theta^{12}-21440\theta^{14}+1280\theta^{16}\right), \\
k_3 &= 653420880\theta^4-1833419280\theta^6+3239983790\theta^8-3835461920\theta^{10}+3137639640\theta^{12}-1795305120\theta^{14} \\
&\quad +715888640\theta^{16}-194739200\theta^{18}+34426880\theta^{20}-3563520\theta^{22}+163840\theta^{24}, \\
k_4 &= 20744640\theta-157560480\theta^3+535812480\theta^5-1079136000\theta^7+1434589940\theta^9-1327460950\theta^{11} \\
&\quad +877542420\theta^{13}-417840280\theta^{15}+142223920\theta^{17}-33732160\theta^{19}+5287680\theta^{21}-491520\theta^{23}+20480\theta^{25}, \\
k_5 &= 4888679040\theta^4-11340070560\theta^6+17443955180\theta^8-18760361370\theta^{10}+14496201740\theta^{12}-8144737400\theta^{14}
\end{aligned}$$

$$+3328237752\theta^{16}-977815504\theta^{18}+201036680\theta^{20}-27438808\theta^{22}+2232320\theta^{24}-8192\theta^{26}.$$

$\Pi^*_{RM-r1} > \Pi^*_{MM-r1}$ if and only if $\Omega > \hat{\Omega}^{RM-MM}_{r1}(\theta)$ and $\Pi^*_{RM-m1} > \Pi^*_{MM-m1}$ if and only if $\Omega > \hat{\Omega}^{RM-MM}_{m1}(\theta)$. The contour plots clearly show that $\hat{\Omega}^{RM-MM}_{r1}(\theta) > \hat{\Omega}^{RM-MM}_{m1}(\theta)$ and that $\hat{\Omega}^{RM-MM}_{r1}(\theta)$ and $\hat{\Omega}^{RM-MM}_{m1}(\theta)$ increase with θ.

The equilibrium for MR is:

$$\begin{aligned}
D^*_{MR-1} &= \frac{(3-2\theta^2)\left(2\left(3-4\theta^2+\theta^4\right)A_1-\theta\left(2-\theta^2\right)A_2\right)}{63-180\theta^2+172\theta^4-64\theta^6+8\theta^8},\\
D^*_{MR-2} &= \frac{(2-\theta^2)\left(\left(21-30\theta^2+8\theta^4\right)A_2-2\theta\left(3-2\theta^2\right)A_1\right)}{2\left(63-180\theta^2+172\theta^4-64\theta^6+8\theta^8\right)},\\
\Pi^*_{MR-r1} &= \frac{(3-2\theta^2)^2\left(1-\theta^2\right)\left(2\left(3-4\theta^2+\theta^4\right)A_1-\theta\left(2-\theta^2\right)A_2\right)^2}{\left(63-180\theta^2+172\theta^4-64\theta^6+8\theta^8\right)^2},\\
\Pi^*_{MR-r2} &= \frac{(2-\theta^2)^2\left(3-4\theta^2\right)\left(\left(21-30\theta^2+8\theta^4\right)A_2-2\theta\left(3-2\theta^2\right)A_1\right)^2}{16\left(63-180\theta^2+172\theta^4-64\theta^6+8\theta^8\right)^2},\\
\Pi^*_{MR-m1} &= \frac{\left(63-144\theta^2+92\theta^4-16\theta^6\right)\left(2\left(3-4\theta^2+\theta^4\right)A_1-\theta\left(2-\theta^2\right)A_2\right)^2}{4\left(63-180\theta^2+172\theta^4-64\theta^6+8\theta^8\right)^2},\\
\Pi^*_{MR-m2} &= \frac{(2-\theta^2)\left(6-9\theta^2+2\theta^4\right)\left(\left(21-30\theta^2+8\theta^4\right)A_2-2\theta\left(3-2\theta^2\right)A_1\right)^2}{8\left(63-180\theta^2+172\theta^4-64\theta^6+8\theta^8\right)^2}.
\end{aligned}$$

For prices and demands to be nonnegative requires

$$2\left(3-4\theta^2+\theta^4\right)A_1-\theta\left(2-\theta^2\right)A_2\geq 0 \quad \text{and} \quad \left(21-30\theta^2+8\theta^4\right)A_2-2\theta\left(3-2\theta^2\right)A_1\geq 0.$$

The common lower and upper bounds for RM and MM are:

$$\underline{\Omega}^{MR-MM}(\theta)=\frac{\theta\left(2-\theta^2\right)}{2\left(3-4\theta^2+\theta^4\right)} \quad \text{and} \quad \bar{\Omega}^{MR-MM}(\theta)=\frac{21-30\theta^2+8\theta^4}{2\theta\left(3-2\theta^2\right)}.$$

As with RM, the common feasible domain for θ is $\theta\in[0,0.876]$. The boundary lines of $\hat{\Omega}^{MR-MM}_{r2}(\theta)$ and $\hat{\Omega}^{MR-MM}_{m2}(\theta)$ can be obtained by equating the profits of MR and those of MM. Contour plots show that $\hat{\Omega}^{MR-MM}_{r2}(\theta) < \hat{\Omega}^{MR-MM}_{m2}(\theta)$ and that $\hat{\Omega}^{MR-MM}_{r2}(\theta)$ and $\hat{\Omega}^{MR-MM}_{m2}(\theta)$ decrease in θ.

(2) *Compare RR to MR and RM*. The common lower and upper bounds for MR and RR are:

$$\underline{\Omega}^{MR-RR}(\theta)=\frac{\theta\left(3-2\theta^2\right)}{9-14\theta^2+4\theta^4} \quad \text{and} \quad \bar{\Omega}^{MR-RR}(\theta)=\frac{9-14\theta^2+4\theta^4}{\theta\left(3-2\theta^2\right)}.$$

The largest feasible domain for θ is $\theta\in[0,0.823]$, which is the same as that of RR. Boundary values of $\hat{\Omega}^{MR-RR}_{r1}(\theta)$ and $\hat{\Omega}^{MR-RR}_{m1}(\theta)$ result from equating the profits under MR and RR.

$$\hat{\Omega}^{MR-RR}_{r1}(\theta) \;=\; \min\{\hat{\Omega}^{MR-RR}_{r1-1}(\theta),\bar{\Omega}^{MR-RR}(\theta)\},$$

$$
\hat{\Omega}_{m1}^{MR-RR}(\theta) \quad = \quad \min\{\hat{\Omega}_{m1-1}^{MR-RR}(\theta), \bar{\Omega}^{MR-RR}(\theta)\},
$$

where

$$
\begin{aligned}
\hat{\Omega}_{r1-1}^{MR-RR}(\theta) &= \frac{l_1 + 2l_2\sqrt{\left(1-\theta^2\right)^3\left(3-4\theta^2\right)}}{19683 - 244944\theta^2 + l_3}, \\
\hat{\Omega}_{m1-1}^{MR-RR}(\theta) &= \frac{l_4 + \sqrt{2}l_5\sqrt{189 - 642\theta^2 + 700\theta^4 - 264\theta^6 + 32\theta^8}}{91854 - 740664\theta^2 + l_6},
\end{aligned}
$$

and where

$$
\begin{aligned}
l_1 &= 65561\theta - 583200\theta^3 + 2507760\theta^5 - 6676560\theta^7 \\
&\quad + 11843520\theta^9 - 14362160\theta^{11} + 11980720\theta^{13} - 6822400\theta^{15} + 2589440\theta^{17} - 623360\theta^{19} + 85760\theta^{21} - 5120\theta^{23}, \\
l_2 &= \theta^3\left(5103 - 25920\theta^2 + 52632\theta^4 - 55152\theta^6 + 32248\theta^8 - 10592\theta^{10} + 1824\theta^{12} - 128\theta^{14}\right), \\
l_3 &= 13306680\theta^4 - 41484960\theta^6 + 82566720\theta^8 - 110586720\theta^{10} \\
&\quad + 102338720\theta^{12} - 66050400\theta^{14} + 29577600\theta^{16} - 8989440\theta^{18} + 1766400\theta^{20} - 202240\theta^{22} + 10240\theta^{24}, \\
l_4 &= 306180\theta - 2094660\theta^3 + 6285600\theta^5 - 10910880\theta^7 + 12159840\theta^9 - 9112700\theta^{11} + 4657080\theta^{13} - 1602000\theta^{15} + 354400\theta^{17} - 45440\theta^{19} + 2560\theta^{21}, \\
l_5 &= \theta^3\left(567 - 2439\theta^2 + 4140\theta^4 - 3532\theta^6 + 1592\theta^8 - 360\theta^{10} + 32\theta^{12}\right), \\
l_6 &= 2643516\theta^4 - 5495256\theta^6 + 7370820\theta^8 - 6682176\theta^{10} + 4171948\theta^{12} - 1793432\theta^{14} + 520784\theta^{16} - 97504\theta^{18} + 10624\theta^{20} - 512\theta^{22}.
\end{aligned}
$$

$\Pi^*_{MR-r1} > \Pi^*_{RR-r1}$ if and only if $\Omega < \hat{\Omega}_{r1}^{MR-RR}(\theta)$ and $\Pi^*_{MR-m1} > \Pi^*_{RR-m1}$ if and only if $\Omega < \hat{\Omega}_{m1}^{MR-RR}(\theta)$. The contour plots show that $\hat{\Omega}_{r1}^{MR-RR}(\theta) > \hat{\Omega}_{m1}^{MR-RR}(\theta)$ when $\theta \in [0, 0.802]$, where $\hat{\Omega}_{r1}^{MR-RR}(\theta)$ is defined as $\hat{\Omega}_{r1-1}^{MR-RR}(\theta)$, whereas $\hat{\Omega}_{r1}^{MR-RR}(\theta) < \hat{\Omega}_{m1}^{MR-RR}(\theta)$ for $\theta \in [0.802, 0.823]$, where $\hat{\Omega}_{r1}^{MR-RR}(\theta)$ is equivalent to $\hat{\Omega}_{r1-2}^{MR-RR}(\theta)$; $\hat{\Omega}_{r1-1}^{MR-RR}(\theta)$ increases with θ within $[0, 0.630]$, $\hat{\Omega}_{r1-2}^{MR-RR}(\theta)$ decreases with θ within $[0.630, 0.823]$, and $\hat{\Omega}_{m1}^{MR-RR}(\theta)$ increases with θ in the common feasible area.

Boundary lines of $\hat{\Omega}_{r2}^{RM-RR}(\theta)$ and $\hat{\Omega}_{m2}^{RM-RR}(\theta)$ come from equating the profits of RM and RR, where $\hat{\Omega}_{r2}^{RM-RR}(\theta) < \hat{\Omega}_{m2}^{RM-RR}(\theta)$ when $\theta < 0.802$ but the direction of the inequality reverses when $\theta \in [0.802, 0.823]$; $\hat{\Omega}_{r2}^{RM-RR}(\theta)$ decreases with θ within $[0, 0.630]$, where $\hat{\Omega}_{r2}^{RM-RR}(\theta)$ is equivalent to $\hat{\Omega}_{r2-1}^{RM-RR}(\theta)$, and increases with θ within $[0.630, 0.823]$ where $\hat{\Omega}_{r2}^{RM-RR}(\theta)$ is defined as $\hat{\Omega}_{r2-2}^{RM-RR}(\theta)$, and $\hat{\Omega}_{m2}^{RM-RR}(\theta)$ decreases with θ.

(3) *Compare profits between NM (MN) and RM (MR).* The common lower and upper bounds for NM and RM are:

$$
\underline{\Omega}^{RM-NM}(\theta) = \frac{2\theta\left(3-2\theta^2\right)}{21 - 30\theta^2 + 8\theta^4} \quad \text{and} \quad \bar{\Omega}^{RM-NM}(\theta) = \frac{2(3 - 4\theta^2 + \theta^4)}{\theta(2-\theta^2)}.
$$

The largest feasible domain for θ is $\theta \in [0, 0.876]$. The boundary values of $\hat{\Omega}_{r1}^{RM-NM}(\theta)$ and $\hat{\Omega}_{m1}^{RM-NM}(\theta)$ come from equating the profits in RM and NM.

$$
\hat{\Omega}_{r1}^{RM-NM}(\theta) \quad = \quad \min\{\hat{\Omega}_{r1-1}^{RM-NM}(\theta), \bar{\Omega}^{RM-NM}(\theta)\},
$$

$$\hat{\Omega}_{m1}^{RM-NM}(\theta) \quad = \quad \min\{\hat{\Omega}_{m1-1}^{RM-NM}(\theta), \bar{\Omega}^{RM-NM}(\theta)\},$$

where

$$\begin{aligned}
\hat{\Omega}_{r1-1}^{RM-NM}(\theta) &= \frac{2(m_1 + 8m_2\sqrt{3 - 7\theta^2 + 4\theta^4})}{4148928 - 357598808\theta^2 + m_3}, \\
\hat{\Omega}_{m1-1}^{RM-NM}(\theta) &= \frac{2(m_4 + 4\sqrt{2}m_2\sqrt{24 - 66\theta^2 + 59\theta^4 - 19\theta^6 + 2\theta})}{8297856 - 708281128\theta^2 + m_5},
\end{aligned}$$

and where

$$\begin{aligned}
m_1 &= 5927040\theta - 45722888\theta^3 + 156707788\theta^5 - 314763000\theta^7 \\
&\quad + 411421650\theta^9 - 367223760\theta^{11} + 228263000\theta^{13} - 98748800\theta^{15} + 29118080\theta^{17} - 5570560\theta^{19} + 622080\theta^{21} - 30720\theta^{23}, \\
m_2 &= \theta^3 \left(7056 - 371700\theta^2 + 817870\theta^4 - 979240\theta^6 + 696520\theta^8 - 301280\theta^{10} + 77520\theta^{12} - 10880\theta^{14} + 640\theta^{16}\right), \\
m_3 &= 1378583640\theta^4 - 3137925000\theta^6 + 4689323390\theta^8 - 4840237520\theta^{10} \\
&\quad + 3534734040\theta^{12} - 1839190240\theta^{14} + 676712320\theta^{16} - 171806720\theta^{18} + 28597760\theta^{20} - 2805760\theta^{22} + 122880\theta^{24}, \\
m_4 &= 11854080\theta - 90457920\theta^3 + 308946960\theta^5 - 624137400\theta^7 \\
&\quad + 830360620\theta^9 - 766304790\theta^{11} + 502985060\theta^{13} - 236594360\theta^{15} + 79150960\theta^{17} - 18365760\theta^{19} + 2805760\theta^{21} - 253440\theta^{23} + 10240\theta^{25}, \\
m_5 &= 2723580720\theta^4 - 6236695080\theta^6 + 9473885140\theta^8 - 10068689370\theta^{10} \\
&\quad + 7695641980\theta^{12} - 4281329880\theta^{14} + 1733992720\theta^{16} - 505291520\theta^{18} + 103082240\theta^{20} - 13959680\theta^{22} + 1126400\theta^{24} - 40960\theta^{26}.
\end{aligned}$$

$\Pi^*_{RM-m1} > \Pi^*_{NM-m1}$ if and only if $\Omega > \hat{\Omega}_{m1}^{RM-NM}(\theta)$ and $\Pi^*_{RM-r1} > \Pi^*_{NM-r1}$ if and only if $\Omega > \hat{\Omega}_{r1}^{RM-NM}(\theta)$. The contour plots show that $\hat{\Omega}_{r1}^{RM-NM}(\theta) > \hat{\Omega}_{m1}^{RM-NM}(\theta)$ and that $\hat{\Omega}_{r1}^{RM-NM}(\theta)$ and $\hat{\Omega}_{m1}^{RM-NM}(\theta)$ increase with θ.

The boundary lines of $\hat{\Omega}_{r2}^{MR-MN}(\theta)$ and $\hat{\Omega}_{m2}^{MR-MN}(\theta)$ are obtained by equating the profits in RM and RR, where $\hat{\Omega}_{r2}^{MR-MN}(\theta) < \hat{\Omega}_{m2}^{MR-MN}(\theta)$. $\hat{\Omega}_{r2}^{MR-MN}(\theta)$ and $\hat{\Omega}_{m2}^{MR-MN}(\theta)$ decrease with θ.

(4) *Compare profits between MR (RM) and NR (RN).* The common lower and upper bounds for MR and NR are as follows:

$$\underline{\Omega}^{MR-NR}(\theta) = \frac{\theta\left(2 - \theta^2\right)}{2(3 - 4\theta^2 + \theta^4)} \quad \text{and} \quad \bar{\Omega}^{RM-NM}(\theta) = \frac{4(3 - 4\theta^2 + \theta^4)}{\theta(3 - 2\theta^2)}.$$

The feasible domain for θ is $\theta \in [0, 0.876]$. Define $\Delta\Pi_{m1}^{MR-NR}$ as Manufacturer 1's profit in MR minus the one in NR and $\Delta\Pi_{r1}^{MR-NR}$ as Retailer 1's profit in MR minus the one in NR, which compute to

$$\begin{aligned}
\Delta\Pi_{m1}^{MR-NR} &= -\left(3 - 2\theta^2\right)^2\left(1 - \theta^2\right) \left.\frac{1}{\left(72 - 198\theta^2 + 183\theta^4 - 66\theta^6 + 8\theta^8\right)^2} - \frac{1}{\left(63 - 180\theta^2 + 172\theta^4 - 64\theta^6 + 8\theta^8\right)^2}\right) \\
&\quad \times \left(2\left(3 - 4\theta^2 + \theta^4\right)A_1 - A_2\theta\left(2 - \theta^2\right)\right)^2, \\
\Delta\Pi_{r1}^{MR-NR} &= \frac{1}{4}\left(\frac{63 - 144\theta^2 + 92\theta^4 - 16\theta^6}{\left(63 - 180\theta^2 + 172\theta^4 - 64\theta^6 + 8\theta^8\right)^2} - \frac{4\left(3 - 2\theta^2\right)}{\left(6 - 9\theta^2 + 2\theta^4\right)\left(12 - 15\theta^2 + 4\theta^4\right)^2}\right)
\end{aligned}$$

$$\times \left(2\left(3-4\theta^2+\theta^4\right)A_1-\theta\left(2-\theta^2\right)A_2\right)^2.$$

Graphing shows that

$$\frac{1}{\left(72-198\theta^2+183\theta^4-66\theta^6+8\theta^8\right)^2}-\frac{1}{\left(63-180\theta^2+172\theta^4-64\theta^6+8\theta^8\right)^2}<0$$

and

$$\frac{63-144\theta^2+92\theta^4-16\theta^6}{\left(63-180\theta^2+172\theta^4-64\theta^6+8\theta^8\right)^2}-\frac{4\left(3-2\theta^2\right)}{\left(6-9\theta^2+2\theta^4\right)\left(12-15\theta^2+4\theta^4\right)^2}>0$$

for any $\theta \in [0, 0.876]$. Thus $\Delta\Pi_{m1}^{MR-NR} > 0$ and $\Delta\Pi_{r1}^{MR-NR} > 0$ in the common feasible area, meaning that Manufacturer 1 and Retailer 1 always prefer MR to NR. Similarly, $\Delta\Pi_{r2}^{RM-RN} > 0$ and $\Delta\Pi_{m2}^{RM-RN} > 0$ for any $\theta \in [0, 0.876]$. This completes the proof of Lemma 6.

Lemma 6 suggests that both Manufacturer 1 and Retailer 1 would have incentives to switch from MM to RM, if their supply chain has a larger base demand than the other, and these incentives become stronger with higher product substitutability. This occurs because retailer advertising intensifies competition relative to manufacturer advertising (i.e., the levels of retailer advertising are higher in equilibrium, whose impact plays out through the demand function in Eq. (1)). However, an area exists (i.e., $\hat{\Omega}_{m1}^{RM-MM}(\theta) < \Omega < \hat{\Omega}_{r1}^{RM-MM}(\theta)$) within which Manufacturer 1 and Retailer 1 cannot agree on whether to use MM or RM. A similar situation arises with regards to RM and NM. Between RR and MR, Manufacturer 1 and Retailer 1 both prefer MR to RR if the supply chain's base market is the smaller one, but reverse their preference if the base market is the larger, especially when product substitutability is high. Manufacturer 1 and Retailer 1 always prefer MR to NR, because the manufacturer advertising yields significantly more demand for the supply chain but without greatly intensifying the supply chain competition. Similar sentiments govern the preferences of Manufacturer 2 and Retailer 2 as they consider switching from MM to MR, RR to RM, MR to MN, and RM to RN. To summarize, both manufacturer and both retailers prefer retailer advertising when product substitutability is low; when product substitutability is high, manufacturer advertising has some appeal. Lemma 6 also indicates that RN/NR are inferior to RM/MR. These findings, along with Theorems 1 and 2, indicate that MM, RR, RM, and MR are more stable than the other advertising structures.

A state is a strong channel equilibrium if no coalition of players in the same supply chain can profitably deviate from the current state. It can be shown that other advertising structures, including hybrid approaches MN, NM, RN, and NR, are dominated by MM, RR, RM, and MR. So the following will focus on evaluating MM, RR, RM, and MR for the manufacturers and retailers. We continue to consider only the common feasible domain established in Lemma 6.

We start with MM. The proof of Lemma 6 established that Manufacturer 1 prefers RM to MM if and only if $\Omega > \hat{\Omega}_{m1}^{RM-MM}(\theta)$ and Retailer 1 prefers RM to MM if and only if $\Omega > \hat{\Omega}_{r1}^{RM-MM}(\theta)$. Given that $\hat{\Omega}_{r1}^{RM-MM}(\theta) > \hat{\Omega}_{m1}^{RM-MM}(\theta)$, the coalition of Manufacturer 1 and Retailer 1 would never switch from MM to RM as long as $\Omega < \max\{\hat{\Omega}_{r1}^{RM-MM}(\theta), \hat{\Omega}_{m1}^{RM-MM}(\theta)\} = \hat{\Omega}_{r1}^{RM-MM}(\theta)$, because at least one of Manufacturer 1 and Retailer 1 will be worse off switching from MM to RM. On the other hand, for Manufacturer 2, MR outperforms MM if and only if $\Omega < \hat{\Omega}_{m2}^{MR-MM}(\theta)$; whereas for Retailer 2, MR outperforms MM if and only if $\Omega < \hat{\Omega}_{r2}^{MR-MM}(\theta)$. Given that $\hat{\Omega}_{r2}^{MR-MM}(\theta) < \hat{\Omega}_{m2}^{MR-MM}(\theta)$, similarly, the coalition of Manufacturer 1 and Retailer 1 would never switch from MM to MR as long as

$$\Omega > \min\{\hat{\Omega}_{r2}^{MR-MM}(\theta), \hat{\Omega}_{m2}^{MR-MM}(\theta)\} = \hat{\Omega}_{r2}^{MR-MM}(\theta).$$

Therefore, MM is a strong channel equilibrium if $\hat{\Omega}_{r2}^{MR-MM}(\theta) < \Omega < \hat{\Omega}_{r1}^{RM-MM}(\theta)$.

Consider RR. Manufacturer 1 prefers MR to RR if and only if $\Omega < \hat{\Omega}_{m1}^{MR-RR}(\theta)$ and Retailer 1 prefers MR to RR if and only if $\Omega < \hat{\Omega}_{r1}^{MR-RR}(\theta)$. Given that $\hat{\Omega}_{r1}^{MR-RR}(\theta)$ and $\hat{\Omega}_{m1}^{MR-RR}(\theta)$ cross in the common feasible domain, it is conceivable that the coalition of Manufacturer 1 and Retailer 1 would never switch from RR to MR as long as $\Omega > \min\{\hat{\Omega}_{r1}^{MR-RR}(\theta), \hat{\Omega}_{m1}^{MR-RR}(\theta)\}$. On the other hand, Manufacturer 2 prefers RM to RR if and only if $\Omega > \hat{\Omega}_{m2}^{RM-RR}(\theta)$ and Retailer 2 prefers RM to RR if and only if $\Omega > \hat{\Omega}_{r2}^{RM-RR}(\theta)$. Given that $\hat{\Omega}_{r2}^{RM-RR}(\theta)$ crosses $\hat{\Omega}_{m2}^{RM-RR}(\theta)$ at $\theta = 0.802$, it is conceivable that no coalition of both Manufacturer 2 and Retailer 2 would switch from RR to RM as long as $\Omega < \max\{\hat{\Omega}_{r2}^{RM-MM}(\theta), \hat{\Omega}_{m2}^{RM-MM}(\theta)\}$. So RR is a strong channel equilibrium if and only if

$$\min\{\hat{\Omega}_{r1}^{MR-RR}(\theta), \hat{\Omega}_{m1}^{MR-RR}(\theta)\} < \Omega < \max\{\hat{\Omega}_{r2}^{RM-MM}(\theta), \hat{\Omega}_{m2}^{RM-MM}(\theta)\}.$$

And since $\hat{\Omega}_{m2}^{RM-RR}(\theta)$ bypasses $\hat{\Omega}_{m1}^{MR-RR}(\theta)$ at $\theta = 0.775$ before reaching $\hat{\Omega}_{r1}^{MR-RR}(\theta)$ and $\hat{\Omega}_{r2}^{RM-MM}(\theta)$, we conclude that RR is a strong channel equilibrium if $\hat{\Omega}_{m1}^{MR-RR}(\theta) < \Omega < \hat{\Omega}_{m2}^{RM-RR}(\theta)$ in $\theta \in [0, 0.775]$.

Consider MR. Manufacturer 1 prefers RR to MR if and only if $\Omega > \hat{\Omega}_{m1}^{MR-RR}(\theta)$ and Retailer 1 prefers RR to MR if and only if $\Omega > \hat{\Omega}_{r1}^{MR-RR}(\theta)$. Given that $\hat{\Omega}_{r1}^{MR-RR}(\theta)$ and $\hat{\Omega}_{m1}^{MR-RR}(\theta)$ cross in the common feasible domain, it is conceivable that the coalition of both Manufacturer 1 and Retailer 1 would never switch from MR to RR as long as $\Omega < \max\{\hat{\Omega}_{r1}^{MR-RR}(\theta), \hat{\Omega}_{m1}^{MR-RR}(\theta)\}$. On the other hand, for Manufacturer 2 MM outperforms MR if and only if $\Omega > \hat{\Omega}_{m2}^{MR-MM}(\theta)$, whereas for Retailer 2, MM outperforms MR if and only if $\Omega > \hat{\Omega}_{r2}^{MR-MM}(\theta)$. Given that $\hat{\Omega}_{r2}^{MR-MM}(\theta) < \hat{\Omega}_{m2}^{MR-MM}(\theta)$, the coalition of Manufacturer 1 and Retailer 1 would never switch from MR to MM as long as $\Omega < \max\{\hat{\Omega}_{r2}^{MR-MM}(\theta), \hat{\Omega}_{m2}^{MR-MM}(\theta)\} = \hat{\Omega}_{m2}^{MR-MM}(\theta)$. Therefore, MR is a strong channel equilibrium as long as $\Omega < \min\{\hat{\Omega}_{m2}^{MR-MM}(\theta), \max\{\hat{\Omega}_{r1}^{MR-RR}(\theta), \hat{\Omega}_{m1}^{MR-RR}(\theta)\}\}$.

Consider RM. Manufacturer 1 prefers MM to RM if and only if $\Omega < \hat{\Omega}_{m1}^{RM-MM}(\theta)$ and Retailer 1 prefers MM to RM if and only if $\Omega < \hat{\Omega}_{r1}^{RM-MM}(\theta)$. Given that $\hat{\Omega}_{r1}^{RM-MM}(\theta) > \hat{\Omega}_{m1}^{RM-MM}(\theta)$, it is conceivable that the coalition of Manufacturer 1 and Retailer 1 would never switch from MM to RM as long as

$$\Omega > \min\{\hat{\Omega}_{r1}^{RM-MM}(\theta), \hat{\Omega}_{m1}^{RM-MM}(\theta)\} = \hat{\Omega}_{m1}^{RM-MM}(\theta).$$

On the other hand, Manufacturer 2 prefers RR to RM if and only if $\Omega < \hat{\Omega}_{m2}^{RM-RR}(\theta)$ and Retailer 2 prefers RR to RM if and only if $\Omega < \hat{\Omega}_{r2}^{RM-RR}(\theta)$. Given that $\hat{\Omega}_{r2}^{RM-RR}(\theta)$ crosses $\hat{\Omega}_{m2}^{RM-RR}(\theta)$ at $\theta = 0.802$, it is conceivable that the coalition of Manufacturer 2 and Retailer 2 would never switch from RM to RR as long as

$$\Omega > \min\{\hat{\Omega}_{r2}^{RM-MM}(\theta), \hat{\Omega}_{m2}^{RM-MM}(\theta)\}.$$

Therefore, RM is a strong channel equilibrium as long as

$$\Omega > \max\{\hat{\Omega}_{m1}^{RM-MM}(\theta), \min\{\hat{\Omega}_{r2}^{RM-MM}(\theta), \hat{\Omega}_{m2}^{RM-MM}(\theta)\}\}.\square$$

Proof of Lemma 5: Solving the Nash game gives

$$e_{m1} = \frac{(3-2\theta^2)\left(15-2\theta\left(3+\theta\left(13-2\theta-4\theta^2\right)\right)\right)}{45(-1+6k_{m1})+12(9-77k_{m1})\theta^2+4(-19+260k_{m1})\theta^4+16(1-28k_{m1})\theta^6+64k_{m1}\theta^8}.$$

Differentiating this yields

$$\frac{\partial e_{m1}}{\partial k_{m1}} = -\frac{(3-2\theta^2)\left(270-924\theta^2+1040\theta^4-448\theta^6+64\theta^8\right)\left(15-2\theta\left(3+\theta\left(13-2\theta-4\theta^2\right)\right)\right)}{\left(45(-1+6k_{m1})+12(9-77k_{m1})\theta^2+4(-19+260b)\theta^4+16(1-28k_{m1})\theta^6+64k_{m1}\theta^8\right)^2},$$

which is nonpositive if and only if $15 - 2\theta\left(3 + \theta\left(13 - 2\theta - 4\theta^2\right)\right) \geq 0$, which is true under the assumptions that keep demand nonnegative. Therefore, Manufacturer 1's advertising effort decreases with k_{m1}. $\square$