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The Cash Flow Advantages of 3PLs as Supply Chain Orchestrators

Xiangfeng Chen

Gangshu (George) Cai
Santa Clara University, gcai@scu.edu

Jing-Sheng Song

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Xiangfeng Chen *   Gangshu (George) Cai †   Jing-Sheng Song ‡

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Abstract

With an increasingly open global economy and advanced technologies, some third-party logistics providers (3PLs), such as Eternal Asia, have emerged as supply chain orchestrators, linking buyers with manufacturers worldwide. In addition to their traditional transportation services, these orchestrators provide procurement and financial assistance to buyers in the supply network, especially small and medium sized enterprises (SMEs) in developing countries. Oftentimes, the 3PLs can obtain payment delay arrangements from the financially stronger manufacturers, which in turn can be partially extended to the SME buyers, alleviating their high costs of capital. To illustrate the efficiency improvements of aforementioned practice, we use a model to explicitly capture the cash-flow dynamics in a supply chain consisting of a manufacturer, a buyer, and a 3PL firm, and explore the conditions under which this innovation benefits all parties in the supply chain so that the business model is sustainable. We characterize these conditions and show that the supply chain profit can be higher under leadership by the 3PL than by the manufacturer. The intermediary role of the 3PL is crucial, in that its benefit may vanish if the manufacturer chooses to directly grant payment delay to the buyers. We demonstrate that the benefit is more likely to occur with more buyers. We further identify the unique Nash bargaining solution for the transportation time and the payment delay grace period.

Key words: third-party logistics (3PL); payment scheme; cash-flow dynamics; cash opportunity cost; supply chain leadership

*School of Management, Fudan University, Email:chenxf@fudan.edu.cn.
†Leavey School of Business, Santa Clara University. Email: gcai@scu.edu.
‡The Fuqua School of Business, Duke University. Email: jssong@duke.edu

1 Introduction

1.1 Background and Research Questions

To better focus on core competencies, firms increasingly rely on third-party logistics providers (3PLs) for logistics tasks, which may include packaging, transportation, and warehousing (see, e.g., Lambert et al., 1999). Indeed, in 2012 the U.S. 3PL market reported a revenue of $142 billion and the world 3PL had a revenue of more than $550 billion. According to *Inbound Logistics*' 2014 3PL Perspectives market research, "92 percent of service providers surveyed said they grew their client base by at least five percent over the past year. This compares with 90 percent in 2013, 88 percent in 2012, and 73 percent four years ago." In addition, "93 percent of 3PLs report they increased sales at least five percent during the past year." There is no question that 3PLs contribute substantially and growingly to supply chain operations.

In most supply chain models, however, the role of a 3PL firm is either insignificant or nonexistent. This phenomenon is in large part due to the fact that the 3PL firm is generally considered as an auxiliary component of the supply chain, delivering purchased products from the vendor (termed the *manufacturer* hereafter) to the buyer. The buyer typically pays the manufacturer when ordering products, and then pays the 3PL when the products are shipped. The only contribution of the 3PL is shipping. However, the situation in the real world is changing. As competition in the 3PL industry intensifies, the shipping service alone no longer generates substantial revenue. Thus, 3PLs seek to expand their businesses beyond their traditional services.

In this movement, an innovative *procurement service* provided by Eternal Asia, a large Asian 3PL firm, stands out. In this new business model, Eternal Asia places orders to a manufacturer on behalf of buyers. Instead of collecting the order payment from each buyer upfront (at the time the order is placed), the manufacturer allows Eternal Asia to pay at a later time via a special trade credit term (ranging from 30-60 days) and/or a letter of credit (typically 30 days). When Eternal Asia delivers the products to a buyer, it collects both the purchase payment and logistics fee from the buyer. In this way, the buyers do not need to communicate directly with the manufacturer; the 3PL serves as the intermediary for both ordering and payments (see, e.g., Chen, 2008; Eternal Asia, 2007a).

Eternal Asia is then able to partially extend the favorable credit terms to the buyers, many of whom are small and medium sized enterprises (SMEs). In other words, the integrated procurement and logistics service includes financing for the SMEs, which is especially important in emerging economies, where access to bank loans by such firms is limited (Eternal Asia, 2007a,b). The company has been offering the procurement service to Chinese SME buyers since 1998, mostly buying components and parts from established manufacturers such as Cisco, GE, Acer, Lenovo, Haier, as well as other smaller suppliers (see, e.g., http://eternal-asia.com/). According to *Sina Finance*, in 2016, Eternal Asia's revenue was 57.91 billion RMB (with estimated market value of $2.7 billion as of June, 2017), its profit growth rate was 14.12%, while the industry average was 8.12%.

With this new business model, the 3PL firm in effect becomes a supply chain organizer, creating additional value for SMEs. Similar integrated service has also been practiced by other 3PL firms. For example, Jianfa Shanghai Logistics Group helps the small-medium buyers purchase raw material from international companies. In this practice, Jianfa pays the upstream firms with Letters of Credit, and collects the payments from the buyers within the period of Letters of Credit (Zhou and Wang, 2009).

While there are many benefits of the integrated services of supply chain orchestrators, this paper is particularly interested in their distinctive cash-flow dynamics as in the practice of Eternal Asia and Jianfa. Our focus is on the efficiency improvement brought by the payment timing arrangement with SMEs in the 3PL's procurement service. This focus allows us to assess the value of enhancing financing operationally, which is especially important in effective distribution of products and services for local demands in developing countries. We consider the question: when can such an innovation benefit all members of the supply chain? Only when all members benefit can this new approach be sustained.

1.2 Main Findings and Contributions

To study the impact of cash-flow dynamics of the aforementioned 3PL's procurement service, we develop a game-theoretic model of a three-player supply chain, consisting of one manufacturer, one 3PL, and one buyer. We later extend the model to include multiple buyers. The buyer faces a single selling season with stochastic demand and a fixed market price. The supply chain is governed by wholesale price-only contracts for both procurement and shipping. We explore two scenarios. In the first, traditional scenario (model T), the 3PL only ships the products from the manufacturer to the buyer. The buyer pays the manufacturer when ordering and the 3PL upon delivery, respectively. In the second, procurement service scenario (model P), the 3PL takes orders from the buyer and procures the items from the manufacturer, allowing the buyer to pay both the purchasing cost and the logistics service fee when the order is delivered. Meanwhile, the 3PL transfers the order payment to the manufacturer after a grace period, according to their agreement. In the majority of the paper, for ease of exposition, we assume the transportation time and the payment grace period are exogenously given. Later, in Section 6, we endogenize these variables and show the qualitative findings of the previous findings sustain. Within each model, with given transportation time and payment grace period, the players decide the wholesale price, the shipping fee, and the order quantity, respectively, each to maximize its own expected profit. We compare the equilibria of the two models.

Model T extends the two-player model of Lariviere and Porteus (2001) in two ways. First, it inserts a third layer – the 3PL – in the supply chain. This new layer induces triple-marginalization. Second, it introduces payment timing and hence the time value of money into the classic newsvendor model. Model P further extends this framework by altering the ordering flow and introducing different, innovative payment timings to reflect the 3PL's new procurement and finance role.

We first assume the same unit cash opportunity cost across all players and compare the equilibria of the resulting base models T and P. Building on these two base models, we further investigate: 1) the effect of supply chain power – whether the manufacturer or the 3PL is the Stackelberg leader; 2) the effect of cash opportunity costs of different players; and 3) the effect of the number of buyers.

Under equal unit cash opportunity costs, we show that, when the 3PL is the Stackelberg leader, model P benefits all firms as long as the manufacturer is willing to provide the 3PL a payment grace period longer than the shipping time (or the physical order leadtime), but not exceeding a threshold. A payment grace period longer than the shipping time allows the 3PL to hold cash for a longer time and hence to reduce the logistics cost for the buyer. Consequently, the buyer orders more, which generates a higher revenue for the supply chain (Proposition 1). However, the situation is more subtle for the manufacturer, because the benefit of a larger order is counterweighed by the delayed payment from the 3PL. If the grace period is too short, the benefit of the extra order quantity is insufficient to fully compensate for the loss due to the payment delay. If the grace period is too long, the cost of the payment delay will be too large. As a result, there exists a Pareto zone, where the payment grace period is neither too short nor too long, such that all firms are better off in model P than in model T (Proposition 2).

The 3PL leadership and finance role is critical and significant in model P, demonstrated in two ways. First, the above advantage of P under the 3PL leadership disappears when the leadership shifts to the manufacturer. Under the manufacturer leadership, the manufacturer pushes up the wholesale price to compensate for the delayed payment in P. Even though the 3PL may lower the shipping fee, the sum of the wholesale price and the shipping fee in P is higher than that in T. Therefore, the Pareto zone in P vanishes (Proposition 3). Second, keeping everything else the same as in model P, but letting the manufacturer directly offer a payment grace period for the buyer (instead of the 3PL), the profits for each player as well as the whole chain stay the same as in model T (Proposition 4). Thus, the 3PL's role in model P is irreplaceable.

In the case of different cash opportunity costs for different firms, the Pareto zone of model P emerges regardless of which firm is the Stackelberg leader. In particular, the superiority of P over T is more evident when the buyer's cash opportunity cost is higher than that of the manufacturer (Proposition 5 (i)). Moreover, a lower-cash-opportunity-cost manufacturer is more willing to extend a payment delay grace period, which leads to an even larger order from the buyer (Proposition 5 (ii)). Finally, model P continues to be superior if the 3PL's cash opportunity cost is sufficiently lower than the other firms' costs. In this situation, the 3PL is even willing to pay the manufacturer up front but collects the payment from the buyers when the products are delivered (Proposition 5 (iii)). Although the Pareto zone in model P with the manufacturer leadership can be larger than that with the 3PL leadership (Proposition 6), the profit of the entire supply chain is higher with the 3PL leadership, as long as the payment delay grace period is sufficiently long (Proposition 7).

We also show that, with more buyers (fixing the market size), the Pareto zone enlarges and the firms' profits increase more significantly in model P (Proposition 9) due to a more significant risk pooling effect in P than in T. However, if the buyer is capital constrained, the firms' profits shrink as the financial market interest rate increases (Proposition 8). In Section 6, we show that, under Nash bargaining negotiation, both the buyer and the 3PL firm prefers to ship the product at the minimal transportation time (Proposition 10), and the manufacturer and the 3PL can identify a unique optimal payment delay grace period (Proposition 11), hence all previous results sustain with endogenous transportation time and grace period.

1.3 Related literature

The operations management literature has not paid much attention to the roles of 3PLs with a few exceptions. Based on a collaborative project with a leading building products manufacturer, Balakrishnan et al. (2000) apply a novel linear programming model to develop a fair and effective payment schedule to 3PLs. Schittekat and Sorensen (2009) describe how Toyota selected 3PLs and designed the corresponding transport network. The closest work to ours is perhaps that by Chen and Cai (2011), who show that a 3PL can benefit from providing a joint logistics and financial service to a capital-constrained buyer. However, these authors do not model the 3PL procurement service nor the cash-flow sequence. Moreover, they assume an exogenous logistics fee, so the triple marginalization effect is not analyzed explicitly.

Our research is related to studies on supply chain intermediation. Sarkar et al. (1995) list a variety of intermediation services that benefit the customers and suppliers. Using a bargaining framework, Wu (2004) identifies transactional and informational advantages offered by supply chain intermediaries. Using a three-tier supply chain model with two buyers, two suppliers, and one intermediary, Belavina and Girotra (2012) argue that, even in the absence of the above two advantages, intermediaries can improve supply chain performance through relational advantages. Yang and Babich (2014) consider a supply chain with one retailer and two suppliers and identify conditions under which the buyer can benefit from engaging services of an intermediary firm due to its better information about supply risk. Adida et al. (2016) study a three-tier supply chain with an intermediary and provide a rationale for the intermediary to thrive in retailer-led supply chains. Our work complements these studies by identifying certain cash-flow advantages of 3PLs as integrated procurement and logistics intermediaries. It is worth noting that, different from this literature, the firms in our model are not necessarily capital-constrained but can still benefit from the 3PL's combined procurement and financial role and the cash flow dynamics.

Our study is also related to the growing literature on financing supply chains. Scholars have considered two types of sources of capital. One type is external to the supply chain, such as financial instruments offered by a bank or other third party financial institutions. The other type is internal to the supply chain, such as the supplier's trade credit to the buyer, or the buyer's advance payment to the suppliers. Some researchers consider the combination of the two types. On one hand, our 3PL financing role can be viewed as an external source in the traditional supplier-buyer supply chain model that is prevalent in the literature. On the other hand, because we explicitly model 3PL's operation with the supplier and the buyer, model P can also be viewed as an internal source.

The majority of papers concerning external financing instruments assume a newsvendor environment. Within this stream, Xu and Birge (2004) analyze how a firm's inventory decisions are impacted by budget limit and capital structure (debt/equity ratio). Buzacott and Zhang (2004) examine the impact of asset-based financing on inventory management and decision-making of a bank and a set of retailers. Dada and Hu (2008) consider a capital-constrained newsvendor who can borrow from a bank and specify conditions where channel coordination can be achieved. Caldentey and Haugh (2009) compare the performance of a supply chain with a producer and a budget-constrained retailer under three types of supply contracts. An exception from the newsvendor setting is Chao et al. (2008), who explore a budget constrained retailer's joint dynamic replenishment and bank loan decisions in a multi-period inventory model with stochastic demand.

There have been discussions on internal financing sources. Petersen and Rajan (1997) demonstrate that most empirical studies support that trade credit plays a substitutable role to bank credit. Burkart and Ellingsen (2004) from the economic literature show that trade credit can be either complementary or substitutable to bank credit and explain why trade credit has shorter maturity. Haley and Higgins (1973) investigate the relationship between inventory policy and trade credit policy and demonstrate that, in general, optimality requires order quantity and payment time decisions to be determined simultaneously. Gupta and Wang (2009) consider a stochastic inventory system in which the trade credit term is modeled as a non-decreasing holding cost rate and prove that a base-stock policy is optimal under mild assumptions. Yang and Birge (2013) study how different priority rules of order payment influence trade credit usage.

The following works consider a combination of both external and internal financing sources. Kouvelis and Zhao (2012) assume the supplier offers different types of financial service to the capital-constrained retailer, and show that the retailer always prefers financing from the supplier rather than the bank. Cai et al. (2014) explore the retailer's financing strategy under moral hazards when using both bank credit and trade credit and support their results using empirical data. Jing et al. (2012) identify the financing equilibrium in a model in terms of production cost with both bank and trade credits. Tang et al. (2015) develop a game-theoretical model that captures the interactions between three parties (a manufacturer, a financially constrained supplier, and a bank), and examine the efficiency of two supplier innovative financing schemes: purchase order financing (i.e., external source) and buyer direct financing (i.e., internal source).

Our paper deviates from the above works on financing supply chain by considering the role of 3PLs and the impact of cash flow timing, which is related to some recent works on the effect of payment schemes on supply chain performance. In a newsvendor model, Chen et al. (2013) compare three different payment schemes: the newsvendor is financed by itself, by the manufacturer through a trade credit, and by consumers through advanced revenue. They show that the payment scheme can lead to different inventory decisions. Luo and Shang (2015) model a centralized dynamic serial supply chain integrating material flows with cash flows and show a firm may stock more even with a higher inventory holding cost in the presence of transaction costs. Tong et al. (2016) develop a framework to trace cash flows triggered by various payment terms under the wholesale price contract in a dynamic serial supply chain. They show that a partial payment delay can coordinate a two-tier chain. These works, however, do not consider the role of 3PLs.

2 The Base Model and Game Formulation

We consider a single-product, single-selling-season model. The retail price of the product is p, which is exogenous. The unit production cost is $c_m < p$. The subscript m refers to the manufacturer. The production time for the product is zero – this does not qualitatively affect the results. The minimal transportation time from the manufacturer to the buyer is $\ell_s > 0$, which is proved to be the optimal Nash bargaining solution between the buyer and the 3PL in Section 6.1. So, in the base

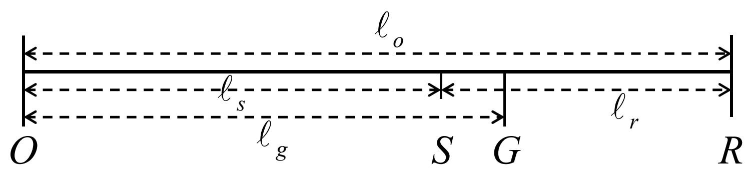

Figure 1: Operations and payment epochs

Table 1: List of Notations

j	Player index; $j = m, l, b$ represents the manufacturer, the 3PL firm, and the buyer, respectively
i	Model index; $i = T, P, B$ represents Model T, P and B, respectively
p	Retail price in the market
q	Ordering quantity
w_j	Wholesale price, $j = m, l$ represents the manufacturer and the 3PL, respectively
w_{ji}	Wholesale price of player j in Model i
c_j	Player j's unit operations cost
a_j	Player j's unit cash opportunity costs
ℓ_s	Transportation time between Epoch O and S
ℓ_o	Payment lead time between Epoch O and R
ℓ_r	Payment lead time between Epoch S and R, and $\ell_r = \ell_o - \ell_s$
ℓ_g	Payment grace period between Epoch O and G
π_{ji}	Player j's profit function in Model i

model, we assume the transportation time is fixed at ℓ_s. As shown in Figure 1, production occurs at time epoch O, shipping at time S, while demand occurs at time R. The 3PL pays the ordering cost to the manufacturer on behalf of the buyer at time G. The time between production epoch O and 3PL payment point G is ℓ_g. The time between the production point O and the demand point R is $\ell_o \geq \ell_s$. For simplicity, except in Section 6, we assume the lead times are fixed and exogenous. See Table 1 for a summary of notation.

At time O, our knowledge about the demand D at the demand point R is through its probability density f, with cumulative distribution F. The corresponding hazard function is $h(x) = f(x)/\bar{F}(x)$ and the generalized failure rate (GFR) is $H(x) = xh(x)$, where $\bar{F}(x) = 1 - F(x)$. We assume F has a convex and increasing generalized failure rate, i.e., $H(x)$ increases in x, which is a common assumption in the supply chain literature, satisfied by many common distributions (see, e.g., Lariviere and Porteus, 2001). There is no salvage value for leftovers after R. Throughout the paper, for any real numbers x and y, we denote $x \wedge y = \min\{x, y\}$.

2.1 Preliminaries

To facilitate understanding of our ideas and contributions, we first review the classic integrated newsvendor model and the decentralized two-player "selling to the newsvendor" model. Then we describe the cash-flow dynamics and the shipping service.

2.1.1 Integrated Newsvendor Model: The Manufacturer

Suppose the manufacturer sells the product itself. The manufacturer's objective is to choose a production quantity q that maximizes its expected profit $\pi(q) = pE[q \wedge D] - c_m q$, which is concave in q. The optimal solution is

$$q_m^* = \bar{F}^{-1}(c_m/p).$$

The main thrust of this model is the effect of demand uncertainty. Keeping the mean demand ($E[D]$) fixed, a less variable demand (or an more accurate demand forecast) increases the optimal profit $\pi(q_m^*)$ (see, e.g., Song, 1994).

2.1.2 Selling to a Newsvendor: The Buyer

Next, suppose the manufacturer sells the product through a retailer, which we call the *buyer*. Before production, the manufacturer first offers the buyer a unit wholesale price w_m ($c_m < w_m < p$), then the buyer determines the order quantity q at time O. After this, the manufacturer produces immediately at O and delivers q units to the buyer at time S. Then the season starts and demand is realized at time R. All demand risk is born by the buyer. In this Stackelberg game, each player maximizes its own expected profit. All costs and the demand distribution are known to both parties. Extending the integrated newsvendor model to this decentralized setting, Lariviere and Porteus (2001) show that the equilibrium wholesale price w_m^* and order quantity q_b^* are the solutions of

$$q_b^*(w_m) = \bar{F}^{-1}(w_m/p), \quad (1)$$

$$w_m = c_m/[1 - H(q_b^*(w_m))]. \quad (2)$$

Also, $q_b^* < q_m^*$, so decentralization decreases the order quantity. Because $\pi(q)$ is increasing in $[q_b^*, q_m^*]$, selling to the newsvendor reduces supply chain profit (or efficiency). The main thrust of this model is to quantify the supply chain inefficiency caused by double marginalization introduced by w and the allocation of the demand risk to the buyer.

2.1.3 Cash Flow Dynamics

Neither the above two classic models considers the impact of the cash flow dynamics. First, both models implicitly assume that the production cost is paid for at O (by the manufacturer or the buyer, respectively), and the shipping delivery and demand consumption occur at the same time (i.e., $S \equiv R$), upon which the revenue is collected from the customers. Second, there is no consideration of the opportunity cost of cash paid at O, even though the revenue is collected at a later time (at R). Therefore, the two models do not explicitly consider the event epochs in Figure 1. In practice, however, the production cost and the order payment must be financed by the manufacturer and the buyer, respectively, which are usually not paid for until the product is sold to the downstream buyer or market. The interest rate paid for the financed amount (from O to R) is the opportunity cost of cash. The first departure of our modeling framework from the literature is to specifically account for the time value of money or the opportunity cost of cash.

2.1.4 Shipping Service: The Shipper

In addition, the above two classic models implicitly assume the manufacturer transports the product to the buyer. In reality, however, this transportation is often done by a third-party logistics (3PL) provider, which charges a unit shipping fee w_l. The second important departure of our model from the literature is to model a 3PL in the supply chain and to study three-player games, as detailed below. As a 3PL enters into the picture, we also must incorporate the associated changes in cash flow dynamics (i.e., when the buyer pays the shipping fee to the 3PL). Because the delivery point is in general not the same as the demand point (i.e., $S < R$), if the buyer pays the shipping fee at S but collects revenue at R, it incurs the opportunity cost of cash for the shipping fee. Therefore, it is essential in our model to explicitly consider the event epochs in Figure 1.

2.2 The Three-Player Model

We now extend the "selling-to-the-newsvendor" model of Section 2.1.2 by explicitly considering the service provided by a 3PL firm. In the base model, there are three players in the supply chain – one manufacturer, one 3PL provider, and one buyer – indexed by subscript $j = m, l, b$, respectively. To focus on the impact of cash flow timing and for tractability, we assume all players have sufficient capital and will not go bankrupt, as in the classic newsvendor model. The manufacturer's unit production cost is c_m, while the 3PL's unit logistics operation cost is c_l. Before the season starts, the manufacturer offers the product at a wholesale price w_m, and the 3PL firm offers shipping service from the manufacturer to the buyer at a unit rate w_l. After the prices are announced, the buyer orders q units of the product from the manufacturer (which the manufacturer is obliged to fulfill) and arranges for the 3PL firm to transport the product. The unfilled demand during the season is lost, and any leftovers at the end of the season have no salvage value. In line with Section 2.1.2, all costs and the demand distribution are common knowledge.

The *unit cash opportunity cost per time unit* (e.g., interest rate) for player j is $a_j \in (0, 1)$, $j = m, l, b$. That is, a unit of cash at time 0 is worth $(1 + a_j)^\ell$ at time ℓ to player j. For tractability, we use $1 + a_j\ell$ to approximate $(1 + a_j)^\ell$, which is accurate for small a_j. Our numerical experiments indicate that this approximation does not affect the qualitative findings of the paper.

2.3 The 3PL Service Types and Payment Timing

We now specify the cash flow dynamics of the supply chain under different roles of the 3PL. Under its traditional role (abbreviated by T), the 3PL provides only the logistics service. The buyer pays the manufacturer $w_m q$ at the time of ordering, epoch O (see Figure 1). The buyer receives the products via the 3PL firm and simultaneously pays logistics service fee $w_l q$ at epoch S. The demand is realized and the buyer collects the revenue $p[D \wedge q]$ at epoch R. We denote the payment lead time between epochs O and R by ℓ_o, and the time between epochs S and R by $\ell_r = \ell_o - \ell_s$.

Under the procurement role (abbreviated by P), the 3PL provides both procurement and logistics services, a case resembling the practice of Eternal Asia mentioned in the Introduction. The 3PL collects the order from the buyer and submits it to the manufacturer at epoch O. The manufacturer gives the 3PL a grace period ℓ_g to pay $w_m q$ at epoch G. That is, the time between O and G is ℓ_g (≥ 0), which typically depends on the relative bargaining power of the 3PL (discussed in Section 6.2). The 3PL ships the products to the buyer and collects both the procurement and logistics payments $(w_m + w_l)q$ at epoch S. (If the 3PL would incur shipping cost at epoch O, the qualitative results sustain because the shipping cost time effect cancels out each other between models T and P.) Finally, the buyer collects the revenue $p[D \wedge q]$ at epoch R. Without loss of generality, we assume $0 \leq \ell_g \leq \ell_o$. This is reasonable, because the 3PL usually pays the manufacturer before the buyer collects all payments from consumers.

We aim to understand how model P impacts each supply chain party's profit, compared with the traditional model T. For consistency, we compare the present cash values of each player at epoch R under both models. Let π_{ji} be the present value of expected profit of player j under model i at epoch R, $j = m, l, b$, $i = T, P$.

Given the wholesale price and service rate, the buyer's expected profit as a function of its order quantity q_{bi} at epoch R is

$$\pi_{bi}(q_{bi}|w_{mi}, w_{li}) = \begin{cases} pE[D \wedge q_{bi}] - [w_{mi}(1 + \ell_o a_b) + w_{li}(1 + \ell_r a_b)]q_{bi}, & \text{if } i = T, \\ pE[D \wedge q_{bi}] - w_i(1 + \ell_r a_b)q_{bi}, & \text{if } i = P, \end{cases} \tag{3}$$

where $w_i = w_{mi} + w_{li}$. Here, the buyer gains an expected revenue of $pE[D \wedge q_{bi}]$ at epoch R, but needs to pay the procurement and logistics costs plus opportunity costs of paying those costs upfront. Since the present money values are calculated at epoch R, in T, the buyer pays the procurement costs $w_{mi}(1 + \ell_o a_b)$ at epoch O and logistics costs $w_{li}(1 + \ell_r a_b)q_{bi}$ at epoch S. In P, the buyer pays both the procurement costs and the logistics costs $w_i q_{bi}(1 + \ell_r a_b)$ at epoch S.

Given the buyer's order quantity, the manufacturer's profit as a function of its own wholesale price is

$$\pi_{mi}(w_{mi}|q_{bi}) = \begin{cases} (w_{mi} - c_m)(1 + \ell_o a_m) q_{bi}, & \text{if } i = T, \\ \left[w_{mi}(1 + (\ell_o - \ell_g)a_m) - c_m(1 + \ell_o a_m)\right] q_{bi}, & \text{if } i = P. \end{cases} \tag{4}$$

Here, in both T and P, the manufacturer obtains negative opportunity costs (positive profits) due to receiving the cash before epoch R. In particular, the manufacturer collects the purchase payment $w_{mP} q_{bP}(1 + (\ell_o - \ell_g)a_m)$ from the 3PL at epoch G.

Finally, given the manufacturer's wholesale price and the buyer's order quantity, the 3PL's profit as a function of its own service fee is

$$\pi_{li}(w_{li}|w_{mi}, q_{bi}) = \begin{cases} (w_{li} - c_l)(1 + \ell_r a_l) q_{bi}, & \text{if } i = T, \\ (w_{li} - c_l)(1 + \ell_r a_l) q_{bi} + w_{mi}(\ell_g - \ell_s) a_l q_{bi}, & \text{if } i = P. \end{cases} \tag{5}$$

In both T and P, the 3PL collects the logistics service fees at epoch S when the shipping is delivered. In P, the 3PL earns an extra profit of $w_{mP}(\ell_g - \ell_s)a_l q_{bP}$, which can be negative, for paying the purchase costs to the manufacturer at epoch G while collecting them from the buyer at epoch S.

2.4 The Game

We study two three-stage Stackelberg games, where either the 3PL or the manufacturer is the Stackelberg leader. To focus on the impact of 3PL leadership on the supply chain, by default, this paper assumes that the 3PL is the Stackelberg leader, although we also explicitly compare 3PL leadership to manufacturer leadership. In the case of 3PL leadership, in stage 1 the 3PL firm determines the logistics service rate. In stage 2, the manufacturer determines the wholesale price. In stage 3, the buyer determines the order quantity. Finally, demand is realized. When the manufacturer is the leader, the above stages 1 and 2 are reversed. In the remainder of this section, we focus on the case where the 3PL is the Stackelberg leader; the case with manufacturer leadership can be described similarly. The game is solved backward.

First, given the wholesale price and service rate, the buyer chooses its order quantity to maximize expected profit as described in Eq. (3). Denote the resulting solution as

$$q^*_{bi}(w_{mi}, w_{li}) = \underset{q_{bi}}{\operatorname{argmax}}\ \pi_{bi}(q_{bi}|w_{mi}, w_{li}), \quad i = T, P. \tag{6}$$

Second, based on the above buyer's best response, the manufacturer chooses his wholesale price to maximize its profit as described in Eq. (4). The resulting solution is denoted as

$$w^*_{mi}(w_{li}) = \underset{w_{mi}}{\operatorname{argmax}}\ \pi_{mi}(w_{mi}|q^*_{bi}(w_{mi}, w_{li})), \quad i = T, P. \tag{7}$$

Third, based on the above two solutions, the 3PL chooses its service fee to maximize its profit as described in Eq. (5), yielding

$$w^*_{li} = \underset{w_{li}}{\operatorname{argmax}}\ \pi_{li}(w_{li}|w^*_{mi}(w_{li}), q^*_{bi}(w^*_{mi}(w_{li}), w_{li})), \quad i = T, P. \tag{8}$$

Finally, the equilibrium wholesale price, service fee and order quantity are $w^*_{mi} \equiv w^*_{mi}(w^*_{li})$, w^*_{li}, and $q_i^* \equiv q_{bi}^*(w_{mi}^*, w_{li}^*)$, respectively.

Definition 1 *We say player j prefers model P to model $\mathfrak{T}$, denoted by $T \prec_j P$, if in equilibrium player j's expected profit under P is no less than that under T. Similarly, $T \prec_{j,k} P$ means that both players j and k prefer P to model T. When all parties prefer P to model T, we write $T \prec P$, and say P is Pareto optimal.*

3 Equal Unit Cash Opportunity Costs

We now analyze the equilibrium strategies of T and P, compare the firms' preferences between the two models, and study the effects of the number of buyers and the 3PL leadership. To highlight the impact of payment timing in model P, this section assumes all players have the same unit cash opportunity costs, that is,

$$a_b = a_l = a_m = a. \tag{9}$$

We will study the impact of different cash opportunity costs in Section 4.

3.1 Firms' Equilibrium Strategies

3.1.1 The buyer's Decision and Triple Marginalization

From Eq. (3), we obtain the following result for the buyer's decision.

Lemma 1 *Assume $a_j = a$ for all j. For any given w_{mi} and w_{li}, we have*

$$q_{bi}^*(w_{li}, w_{mi}) = \begin{cases} \bar{F}^{-1}\left(\frac{w_{mi}(1+\ell_o a)+w_{li}(1+\ell_r a)}{p}\right), & \text{if } i = T, \\ \bar{F}^{-1}\left(\frac{(w_{mi}+w_{li})(1+\ell_r a)}{p}\right), & \text{if } i = P. \end{cases}$$

Both order quantities decrease with the unit cash opportunity cost a. Moreover, under the same wholesale price and service fee (i.e., $w_{ji} = w_j$, $i = T, P$), we have $q_{bT}^*(w_m, w_l) < q_b^*(w_m)$ (defined in (1)) and $q_{bP}^*(w_m, w_l) > q_{bT}^*(w_m, w_l)$.

Note that in Subsection 2.1.2, there is no 3PL, so there is no difference between models T and P, which implies $w_{li} = 0$ and $w_{mi} = w_m$. Moreover, there is no consideration of payment time, so $a = 0$, in which the above order quantities reduce to Eq. (1) (i.e., given $a = 0$, $q_{bT}^*(w_m, 0) = q_{bP}^*(w_m, 0) = q_b^*(w_m)$).

The first inequality of Lemma 1 shows that with a 3PL, the buyer's optimal order quantity is smaller than that without, as long as the 3PL will charge a positive w_{li}, which is a function of c_l in 3PL's decision even if c_l equals zero. Hence, the existence of the 3PL further reduces the supply chain profit, a phenomenon we refer to as the effect of *triple marginalization*. In addition, Lemma

1 shows that this effect is more profound when the cash opportunity cost increases, as $q^*_{bi}(w_m, w_l)$ decreases with a.

The second inequality of Lemma 1 shows the effect of the payment schemes. For any given w_m and w_l, the buyer pays the procurement cost at a later epoch in model P, which reduces its cash conversion cycle from ℓ_o to ℓ_r and hence lowers its total cash opportunity cost, a financial burden in procurement under T. This financial benefit stimulates the buyer to purchase more in P, and thus increases the supply chain profit. The longer the payment period ℓ_r, the more significant the effect.

Thus, the 3PL plays a dual role in P. On the one hand, it induces triple marginalization, which reduces the supply chain profit. On the other hand, it grants the buyer delayed procurement payment, which increases the supply chain profit.

3.1.2 The Manufacturer's Decision

Let

$$\eta_m = \frac{1 + \ell_o a}{1 + (\ell_o - \ell_g)a}.$$

Note that $\eta_m > 1$ as long as $\ell_g > 0$. In addition, η_m increases with both ℓ_g and a. From Eq. (4), we can characterize the manufacturer's optimal wholesale price as follows.

Lemma 2 *Assume $a_j = a$ for all j. For any given w_{li}, the manufacturer's profit is unimodal in w_{mi}, and the unique optimal wholesale price is given by*

$$w^*_{mi}(w_{li}) = \begin{cases} c_m + \frac{pq^*_{bi} f(q^*_{bi}(w^*_{mi}, w_{li}))}{1+\ell_o a}, & \text{if } i = T, \\ \eta_m c_m + \frac{pq^*_{bi} f(q^*_{bi}(w^*_{mi}, w_{li}))}{1+\ell_r a}, & \text{if } i = P. \end{cases}$$

If $\ell_g = \ell_s$, then $q^*_{bT}(w^*_{mT}, w_l) = q^*_{bP}(w^*_{mP}, w_l)$, and $w^*_{mP} > w^*_{mT}$; if $\ell_g < \ell_s$, then $q^*_{bT}(w^*_{mT}, w_l) < q^*_{bP}(w^*_{mP}, w_l)$; otherwise, $q^*_{bT}(w^*_{mT}, w_l) > q^*_{bP}(w^*_{mP}, w_l)$.

Note that if $w_l = 0$ and $a = 0$, the expression of w^*_{mi} in T reduces to Eq. (2). Lemma 2 shows that the manufacturer charges a markup wholesale price on top of the production cost c_m. This markup depends on the payment times, the value of η_m, as well as a. The first term of w^*_{mP} is bigger than that of w^*_{mT} as long as the manufacturer grants the 3PL a grace period for the order payment (i.e., $\ell_g > 0$). The longer the grace period or the larger the unit cash opportunity cost, the bigger the difference is. Given the same logistics service rate, if $\ell_g = \ell_s$, the manufacturer would command a higher wholesale price in P to compensate for a higher opportunity cost. Intuitively, if $\ell_g \geq \ell_s$, the wholesale price in P would be even larger. If $\ell_g < \ell_s$, the manufacturer would be willing to reduce the wholesale price in P. Nevertheless, both the manufacturer and the buyer's decisions hinge on the 3PL's logistics service rate.

3.1.3 The 3PL's Decision

As indicated by Lemma 1, there is a one-to-one mapping between w_{li} and $q^*_{bi}(w^*_{mi}, w_{li})$ because $\bar{F}^{-1}()$ is a monotone, increasing function. The 3PL's optimization problem of finding w_{li} can be transformed into solving the optimal order quantity $q_{li} \equiv q^*_{bi}(w^*_{mi}, w_{li})$. Replacing $q^*_{bi}(w^*_{mi}, w_{li})$ with q_{li} and implementing the manufacturer's first order condition of obtaining $w^*_{mi}(w_{li})$ in Eq. (5) yields

$$\pi_{li}(q^*_{bi}(w^*_{mi}, w_{li})) = \pi_{li}(q_{li})$$
$$= \begin{cases} [p\bar{F}(q_{li})(1 - H(q_{li})) - c_m(1 + \ell_o a) - c_l(1 + \ell_r a)]q_{li}, & \text{if } i = T, \\ \left[p\bar{F}(q_{li})(1 - \eta_l H(q_{li})) - c_m(1 + \ell_o a) - c_l(1 + \ell_r a)\right] q_{li}, & \text{if } i = P, \end{cases} \tag{10}$$

where

$$\eta_l = \frac{1 + (\ell_o - \ell_g)a}{1 + \ell_r a} = \frac{1 + (\ell_o - \ell_g)a}{1 + (\ell_o - \ell_s)a}.$$

Note that $1+(\ell_o-\ell_g)a$ represents the cash value per unit when the 3PL firm disburses the payment to the manufacturer (i.e., cash outflow), whereas $1+(\ell_o-\ell_s)a$ represents the cash value per unit when the 3PL firm collects payment from the buyer (i.e., cash inflow). Thus, η_l can be interpreted as the ratio of 3PL's cash outflow over inflow. Define $G(p, q, \eta) = p\bar{F}(q)[(1-H(q))(1-\eta H(q)) - \eta q H'(q)]$.

Lemma 3 *Assume $a_j = a$ for all j. The 3PL's profit function $\pi_{li}(q_{li})$ is unimodal in q_{li}. The system equilibrium order quantity q^*_i solves the following functions:*

$$\begin{cases} G(p, q^*_i, 1) - c_m(1 + \ell_o a) - c_l(1 + \ell_r a) = 0, & \text{if } i = T, \\ G(p, q^*_i, \eta_l) - c_m(1 + \ell_o a) - c_l(1 + \ell_r a) = 0, & \text{if } i = P. \end{cases}$$

Based on the outcome, we can then deduce the manufacturer's optimal wholesale price and the 3PL's optimal service rate.

Lemma 3 indicates whether $q^*_P \geq q^*_T$ depends on the value of η_l, which is determined by the size of ℓ_g. If $\ell_g > \ell_s$, then $\eta_l < 1$, implying that the 3PL's cash inflow outpaces the outflow and it is intuitive to expect a lower logistics service rate; otherwise, the logistics service rate will go up.

Comparing the optimal order quantities in T and P leads to our first main result:

Proposition 1 *Assume $a_j = a$ for all j and $\ell_g \geq \ell_s$. (i) $q^*_T \leq q^*_P$; (ii) $w^*_{lT} = c_l + \frac{p\bar{F}(q^*_T)(1-H(q^*_T)) - G(p,q^*_T,1)}{1+\ell_r a}$, $w^*_{lP} = \frac{c_l}{\eta_l} + \frac{p\bar{F}(q^*_P)(1-H(q^*_P)) - G(p,q^*_P,\eta_l)/\eta_l}{1+\ell_r a}$, and $w^*_{lP} \leq w^*_{lT}$; (iii) q^*_P increases with ℓ_g, and both π^*_{bP} and π^*_{lP} are convex, increasing functions of ℓ_g; (iv) $T \prec_{l,b} P$, and the preference grows stronger as ℓ_g increases.*

Proposition 1 is a consequence of different payment schemes in models T and P. There are three interactive factors here: the 3PL's logistics service rate, the manufacturer's wholesale price, and the buyer's order quantity. *Ceteris paribus*, the 3PL will reduce the logistics service rate as the length of grace period in P increases; however, the manufacturer will increase the wholesale price to compensate for a larger total cash opportunity cost caused by a lengthier ℓ_g. These two opposite forces intertwine and affect the buyer's order quantity. Although the buyer tends to order more in P than in T, because the buyer does not pay until the product is delivered at epoch S, a substantially higher wholesale price could instead hurt the buyer in P. Therefore, the payment timings will significantly affect the final outcome.

If $\ell_g \geq \ell_s$, we have $\eta_l \leq 1$, so the 3PL's cash inflow outpaces its cash outflow. Accordingly, the 3PL is willing to lower its service rate to the buyer (i.e., $w_{lP}^* < w_{lT}^*$). Although the manufacturer pushes up the wholesale price in P, the buyer manages to order more than in T, because it can pay at delivery rather than at ordering. As ℓ_g grows, the 3PL can further reduce the logistics service rate and stimulate a larger order from the buyer. The larger order, in turn, compensates the manufacturer for its increased total cash opportunity cost and consequently slows down the pace of its wholesale price increase. In this trade-off, the buyer's benefits from a delayed ordering payment and a larger order quantity surpass the disadvantage of a higher wholesale price and, thus, the buyer is better off in P than in T.

For the 3PL, the condition $\ell_g \geq \ell_s$ is critical for two reasons. First, it allows the 3PL to collect the procurement fees from the buyer before paying back to the manufacturer. Second, the 3PL obtains higher logistics revenue owing to a higher order quantity from the buyer. Without the burden of paying a wholesale price to the manufacturer like the buyer, the 3PL benefits more significantly from a larger ℓ_g. Therefore, both the buyer and the 3PL's preferences of P to T become stronger as ℓ_g grows.

3.2 The Pareto Zone of Model P

The next question is, can the manufacturer also benefit from P when $\ell_g \geq \ell_s$? The following proposition provides the answer.

Proposition 2 *Assume $a_j = a$ for all j. (i) The manufacturer's equilibrium profit is unimodal in ℓ_g. (ii) There exists an $\overline{\ell}$, such that $T \prec_m P$ iff $\ell_g \in [\ell_s, \overline{\ell}]$.*

Proposition 2 reveals that the manufacturer prefers P to T if ℓ_g is adequately larger than ℓ_s. While the manufacturer's total cash opportunity cost increases rapidly with ℓ_g, the buyer will not increase the order quantity at the same pace because of a higher wholesale price even if the 3PL can moderately reduce the logistics service rate. Provided with the same market demand, if ℓ_g is too high, the manufacturer can no longer afford the financial burden of delaying the procurement payment.

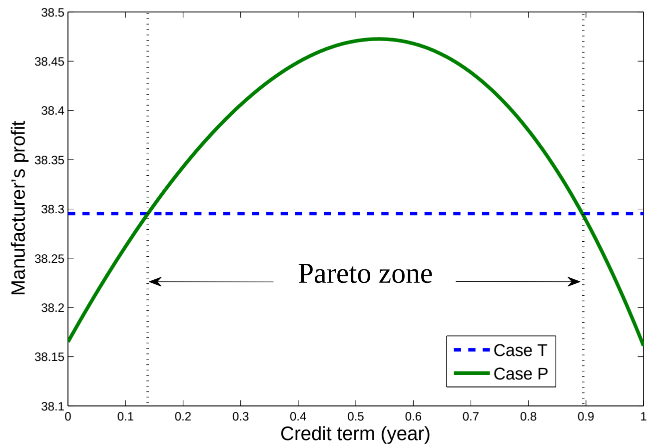

Figure 2: Manufacturer's profit as a function of grace period ℓ_g

Pareto Zone We call the interval $[\ell_s, \overline{\ell}]$ the *Pareto zone* of model P, because from Propositions 1 and 2, all firms are better off under P than under T when $\ell_g \in [\ell_s, \overline{\ell}]$.

Figure 2 provides an example of the Pareto zone, which plots the manufacturer's profit under P and T, respectively, as a function of ℓ_g. The Pareto zone is the interval on the horizontal axis in which the manufacturer's profit is higher in P than in T. Here, we assume there are 360 business days per year, the demand is exponentially distributed with mean 1000, $c_l = 0.1$, $c_m = 0.2$, $p = 1$, $a = 0.3$, $\ell_o = 80/360 = 0.22$, and $\ell_r = 30/360 = 0.08$.

The emergence of the Pareto zone naturally results in higher supply chain profit. Define π_i^{SC} to be the supply chain profit in model i, which is the combined profits of all firms in the supply chain. Let $q^{SC}(a)$ be the supply chain optimal quantity, which solves $p\bar{F}(q) = c_m(1 + \ell_o a) + c_l(1 + \ell_r a)$.

Corollary 1 *Assume $a_j = a$ for all j. (i) $q_P^* \leq q^{SC}(a)$ and (ii) $\pi_T^{SC}(q_T^*) \leq \pi_P^{SC}(q_P^*)$ iff $\ell_g \geq \ell_s$.*

Note that when combining all firms' profits, the internal transactions among firms cancel out. Because the (exogenous) retail price is higher than the (exogenous) product cost, a larger order quantity (when below the centralized order quantity) leads to higher supply chain revenue. Corollary 1 reveals that when $\ell_g \geq \ell_s$, the extra revenue exceeds the additional production cost, so the whole supply chain benefits. In this sense, a lengthier ℓ_g leads to a higher supply chain profit.

Corollary 1 also demonstrates that $\ell_g \geq \ell_s$ is a necessary condition for all firms benefit from P. When ℓ_g is too big, however, the benefits to the buyer and the 3PL come at the expense of the manufacturer. In theory, the buyer and the 3PL can transfer a side payment to the manufacturer in exchange for a lengthier ℓ_g which leads to higher supply chain efficiency. Without coordinating contracts in place, a sustainable procurement service should have a lengthier (but not too long) payment grace period than the shipping time.

3.3 Importance of 3PL Leadership

We now examine the supply chain performance under T and P when the manufacturer is the Stackelberg leader, and compare that with the results under 3PL leadership. For a meaningful comparison, the timing of operations and payment remains the same as under the 3PL leadership.

Proposition 3 *Assume $a_j = a$ for all j. When the manufacturer is the Stackelberg leader, all firms have the same profits in P as in T.*

Proposition 3 is somewhat surprising: in contrast to Propositions 1 and 2, under the manufacturer leadership, the advantage of P under the 3PL leadership disappears. This result can be explained as follows. As the Stackelberg game usually benefits the leader, the mechanism works differently when the leadership shifts. The manufacturer leadership allows the manufacturer to push up the wholesale price to fully compensate for its loss in cash opportunity cost in P, whereas the 3PL leadership prevents the manufacturer from doing so. Under the 3PL leadership, as ℓ_g increases, the manufacturer's concession grows larger, which in turn incentivizes a larger order from the buyer and, as indicated in Proposition 2, benefits the manufacturer as a reciprocation. Therefore, if $\ell_s \le \ell_g \le \bar{\ell}$, all firms can strictly benefit from a Stackelberg leadership from the 3PL. The whole supply chain can also benefit from the 3PL Stackelberg leadership as long as $\ell_s \le \ell_g$. This result demonstrates the value of the 3PL leadership to all firms.

3.4 The 3PL's Supply Chain Finance Role

To demonstrate the 3PL's supply chain finance role, we construct a virtual model, referred to as model B, which is identical to model T except that the manufacturer grants the buyer (instead of the 3PL) a grace period ℓ_g to pay at epoch G. Comparing model P to model B will single out the importance of delaying the payment through the 3PL.

Proposition 4 *With the same cash opportunity cost under either the 3PL leadership or the manufacturer leadership, model T is equivalent to model B.*

Proposition 4 delivers a critical message that manufacturer's payment delay arrangement to the buyer alone does not improve supply chain efficiency, nor the profit for each individual firm. The rationale behind is that in model B the manufacturer increases the wholesale price to fully compensate for its financial loss due to the deferred payment by the buyer. This result demonstrates the irreplaceable importance of the 3PL's role in model P, in which the payment grace period allows the 3PL to lower the logistic service rate to incentivize a larger order size from the buyer.

To summarize, we have obtained four main findings in the scenario with equal unit cash opportunity costs. First, the supply chain's optimal order quantity in P is larger than that in T as long as the manufacturer's payment grace period is sufficiently long (i.e., $\ell_g \geq \ell_s$), so the supply chain efficiency enhances in P. Second, there exists a Pareto zone where all firms are better off in P than in T. Third, P is only beneficial to the supply chain and its partners when the 3PL is the Stackelberg leader. Lastly, the 3PL's supply chain finance role in P is irreplaceable.

4 Effects of Different Unit Cash Opportunity Costs

This section studies how different unit cash opportunity costs affect the findings in Section 3. We first study the case of 3PL leadership, then that of manufacturer leadership, and finally compare them. For convenience, denote $a_{\max} = \max\{a_m, a_l, a_b\}$ and $a_{\min} = \min\{a_m, a_l, a_b\}$.

To facilitate our discussion, we define the following ratios, some of which are the generalization of those defined in Section 3.

$$\eta_l = \frac{1 + (\ell_o - \ell_g)a_l}{1 + \ell_r a_l} = \frac{1 + (\ell_o - \ell_g)a_l}{1 + (\ell_o - \ell_s)a_l}, \tag{11}$$

$$\eta_m = \frac{1 + \ell_o a_m}{1 + (\ell_o - \ell_g)a_m}, \tag{12}$$

$$\eta_b = \frac{1 + \ell_r a_b}{1 + \ell_o a_b} = \frac{1 + (\ell_o - \ell_s)a_b}{1 + \ell_o a_b}. \tag{13}$$

As previously discussed, η_l can be considered as the ratio of 3PL's cash outflow over inflow in P. The 3PL has an advantage if $\eta_l < 1$. η_m measures the ratio of the manufacturer's revenue per unit in T over that in P. The manufacturer incurs a larger opportunity cost as η_m increases. η_b represents the ratio of the buyer's purchasing cost per unit in P over that in T. The buyer gains more financial benefit in P for a smaller η_b. Moreover, η_m increases in ℓ_g; η_b is independent of ℓ_g; and η_l decreases with ℓ_g. Although η_m and η_l are affected by ℓ_g in the opposite direction, we find $\eta_m\eta_l$ increases with ℓ_g if and only if $a_m \geq a_l$.

4.1 3PL Leadership

We now examine the case with the 3PL being the Stackelberg leader. Similar to Section 3, we solve the game backward and obtain the equilibrium order quantity as follows.

Lemma 4 *Given any a_b, a_l, and a_m, when the 3PL is the Stackelberg leader, the equilibrium quantities q_T^* and q_P^* in T and P are solved by the following equations, respectively,*

$$\begin{cases} G(p, q_T^*, 1) - c_l(1 + \ell_r a_b) - c_m(1 + \ell_o a_b) = 0, \\ G(p, q_P^*, \eta_l) - c_l(1 + \ell_r a_b) - c_m(1 + \ell_o a_b)\eta_m\eta_l\eta_b = 0. \end{cases}$$

As Lemma 4 shows, the equilibrium order quantity in P is impacted by η_l and a combined ratio of $\eta_m\eta_l\eta_b$. This outcome is a generalization of Lemma 3 under the same unit opportunity cost, in which the order quantity is impacted only by η_l, because in that case $\eta_m\eta_l\eta_b = 1$.

Proposition 5 *Assume the 3PL is the Stackelberg leader. Consider any a_b, a_l, and a_m. (i) When $\ell_g = \ell_s$, $T \prec P$ iff $a_b \geq a_m$. (ii) When $\ell_g > \ell_s$, $T \prec P$ if $a_m = a_{\min}$. (iii) When $\ell_g < \ell_s$, $T \prec P$ if $a_l \leq \min\{a_m, \hat{a}_l\}$, where $\hat{a}_l$ solves $q_T^*(a_l) = q_P^*(a_l, \ell_g)$.*

Proposition 5 underscores the dependence of the firms' preferences on a_b and a_m. In P, the buyer benefits from the postponed payment that increases with a_b, whereas the manufacturer's financial burden increases with a_m. Compared with T, a higher a_b incentivizes a higher order quantity from the buyer in P. On the other hand, a higher a_m pressures the manufacturer to increase the wholesale price which in turn subdues the order quantity.

When $\ell_g = \ell_s$, $\eta_l = 1$; therefore, the firms' preferences of P or T are indifferent of a_l, but purely rely on the relative values of a_b and a_m. The impact of above two forces on order quantity neutralizes if $a_b = a_m$. If $a_b > a_m$, the impact of a_b outpaces that of a_m such that P has a higher order quantity, which leads to higher profits for all firms; otherwise, T stands out.

When $\ell_g > \ell_s$, according to Propositions 1 and 2, all firms prefer P to T if $a_b = a_m = a_l$. With the impact of different opportunity costs, the lower the manufacturer's cash opportunity cost, the higher incentive for the manufacturer to grand a longer grace period to the 3PL, so as to benefit all firms in P.

When $\ell_g < \ell_s$, $\eta_m\eta_l$ increases with ℓ_g. Provided a_l is smaller than a_m and some quantity related to other parameters, $\hat{a}_b(\ell_g)$ satisfies $q_T^*(a_b) = q_P^*(a_b, \ell_g)$, if $a_l \leq \min\{a_m, \hat{a}_l\}$, the 3PL is willing to pay the manufacturer before collecting payment from the buyer in exchange for a higher order quantity from the buyer, so all firms prefer P.

The observation in Proposition 5 is consistent with the procurement service of Eternal Asia. Eternal Asia is reputed for helping small buyers to purchase goods from large international companies, such as GE, Cisco, and IBM. These small buyers typically have higher cash opportunity costs, such as higher interest rates for borrowing from the bank, because of their small sizes or lack of credit-worth. Therefore, the condition of $a_b \geq a_m$ as required by Proposition 5 is usually satisfied. As Section 5.2 will show, the benefit of P increases when the number of buyers increases. Thus, the 3PL's procurement service in effect plays a supply chain finance role to the small buyers.

It is worth noting that, in practice, some 3PL firms also provide *Distribution Service*, in which the manufacturer distributes the products to the buyers through the 3PL (i.e., a push system). In this process, the 3PL pays the manufacturer upfront but collects the payment from the buyers when the products are delivered. This is a special case of model P with $\ell_g = 0$. Given that the 3PL, such as Eternal Asia and UPS Capital, has sufficient capital support (i.e., its cash opportunity cost is sufficiently small, corresponding to Part (iii) in Proposition 5), all firms can still benefit from this kind of distribution service owing to higher order quantities from the buyers.

4.2 Manufacturer Leadership

Next, consider the case with the manufacturer being the Stackelberg leader. Recall from Proposition 3 that, in this case, under equal unit cash opportunity costs, all firms obtain the same profits in P and T. We now show that it is no longer true under unequal unit cash opportunity costs.

Lemma 5 *Given any a_b, a_l, and a_m, when the manufacturer is the Stackelberg leader, the equilibrium order quantities q_T^* and q_P^* in T and P are solved by the following equations, respectively:*

$$\begin{cases} G(p, q_T^*, 1) - c_l(1 + \ell_r a_b) - c_m(1 + \ell_o a_b) = 0, \\ G(p, q_P^*, 1) - c_l(1 + \ell_r a_b) - c_m(1 + \ell_o a_b)\eta_b\eta_m\eta_l = 0. \end{cases}$$

Lemma 5 shows that in P the equilibrium order quantity is affected by $\eta_m\eta_l\eta_b$. Define

$$\delta = \frac{1 + \ell_r a_l}{1 + \ell_o a_m} = 1/(\eta_m\eta_l), \quad \beta = \frac{(a_b - a_m)\delta}{a_l - a_b}, \quad \xi = \frac{1}{1 + \beta}.$$

Based on Lemma 5, we can compare the firms' profits in P and T as follows.

Proposition 6 *Suppose the manufacturer is the Stackelberg leader. (i) If $\eta_b\eta_m\eta_l \leq 1$, then $q_P^* \geq q_T^*$. (ii) If $\eta_b > \frac{a_m}{a_l}\delta$ and $a_l \geq a_b$, then $T \prec P$ iff $\xi\ell_s \leq \ell_g$; if $\eta_b < \frac{a_m}{a_l}\delta$ and $a_l < a_b$, then $T \prec P$ iff $\ell_g < \xi\ell_s$.*

Proposition 6 indicates that under different unit cash opportunity costs, P can actually outperform T for all firms even if the manufacturer is the Stackelberg leader of the supply chain. This outcome corroborates the result in Proposition 5 by showing all firms can benefit from the 3PL procurement service regardless of whether the 3PL or the manufacturer is the Stackelberg leader.

To dissect the above rather complicated boundary conditions in Proposition 6, we discuss three special cases as follows.

Corollary 2 *Assume the manufacturer is the Stackelberg leader. (i) Case $a_b = a_m$: when $a_l = a_{\min}$, $T \prec P$ iff $\ell_g \leq \ell_s$; when $a_l = a_{\max}$, $T \prec P$ iff $\ell_g \geq \ell_s$. (ii) Case $a_b = a_l$: when $a_m = a_{\min}$, $T \prec P$; when $a_m = a_{\max}$, $P \prec T$; (iii) Case $a_m = a_l$: when $a_b = a_{\max}$, $T \prec P$; when $a_b = a_{\min}$, $P \prec T$.*

Intuitively, the wholesale price w_m increases in the grace period ℓ_g, while the logistics service rate w_l decreases in ℓ_g. With the same cash opportunity costs, the wholesale price increment offsets the logistics service rate decrement and payment delay benefit; as a result, all firms are indifferent of P and T, as stated in Proposition 3. With different cash opportunity costs, the firms' sensitivity to the payment delay length (ℓ_g) varies.

In Case $a_b = a_m$, it can be shown that the logistics service rate w_l increases in a_l if $\ell_g \leq \ell_s$, but decreases in a_l if $\ell_g > \ell_s$. This property leads to a lower wholesale price and a smaller order quantity as a_l grows when $\ell_g \leq \ell_s$, but a higher wholesale price and a higher order as a_l grows when $\ell_g > \ell_s$. As a result, as long as the payment is not delayed for too long (i.e., $\ell_g \leq \ell_s$), the wholesale price will not be substantially higher in P than in T. Consequently, the buyer's purchasing cost will be lower in P than in T, which prompts a higher order quantity, resulting in more profits for all firms iff $\ell_g \leq \ell_s$. In contrast, if $a_b = a_m < a_l$ and $\ell_g \geq \ell_s$, even though the manufacturer has more incentives to increase the wholesale price, the 3PL can substantially reduce its logistics service fee to compensate the buyer for the increased wholesale price because of a longer payment delay (i.e., $\ell_s \leq \ell_g$). As a result, the buyer orders more, which benefits all firms.

In Case $a_b = a_l$, if $a_b = a_l \geq a_m$, the manufacturer does not overwhelmingly increase the wholesale price to fully compensate for the payment delay. The 3PL is willing to reduce the logistics service rate in exchange for a larger order. Therefore, with a lower total cost benefiting from the payment delay, the buyer orders more, so all firms prefer P to T. Otherwise if $a_b = a_l < a_m$, the manufacturer becomes more sensitive to the order payment delay, whereas the 3PL and the buyer are more reluctant to give up their benefits from the payment delay. As a result, the buyer orders less in P as a_m increases, such that all firms prefer T to P.

In Case $a_m = a_l$, the buyer orders less as a_b grows. The downsized order forces the manufacturer to lower its wholesale price and the 3PL to reduce the logistics service rate. So, if $a_m = a_l < a_b$, the manufacturer is more willing to delay the order payment to encourage ordering, while the 3PL is less sensitive to the payment delay. The buyer welcomes the payment delay because of its relatively higher capital opportunity cost and, thus, orders more. Therefore, under this condition, P outperforms T for all firms. In contrast, if $a_m = a_l > a_b$, the manufacturer is more reluctant to delay order payment because of a big order, even though the 3PL is willing to lower the service logistics rate. A less cost sensitive buyer ends up ordering less in P than in T, such that all firms prefer T to P.

4.3 The Impact of Supply Chain Leadership

With heterogeneous unit cash opportunity costs, the above analysis demonstrates that the manufacturer leadership can actually lead to a larger Pareto zone under certain conditions. However, a larger Pareto zone does not automatically warrant more profits for all firms. The following result indicates that the 3PL leadership plays an important role in boosting the supply chain efficiency.

Proposition 7 *Given any a_b, a_l, and a_m, if $\ell_g \geq \ell_s$, the buyer's ordering quantity is larger under 3PL leadership than under manufacturer leadership; otherwise, vice versa.*

Proposition 7 demonstrates that the 3PL leadership role will help the entire supply chain when $\ell_g \geq \ell_s$. The rationale is similar to the discussion after Proposition 3. However, if $\ell_g < \ell_s$, the 3PL loses the advantages for paying the buyer's purchasing cost to the manufacturer earlier than collecting payment from the buyer. Therefore, the 3PL has incentives to increase its logistics service rate to compensate for its financial loss. Under this circumstance, the manufacturer leadership prevents the 3PL from fully compensating itself.

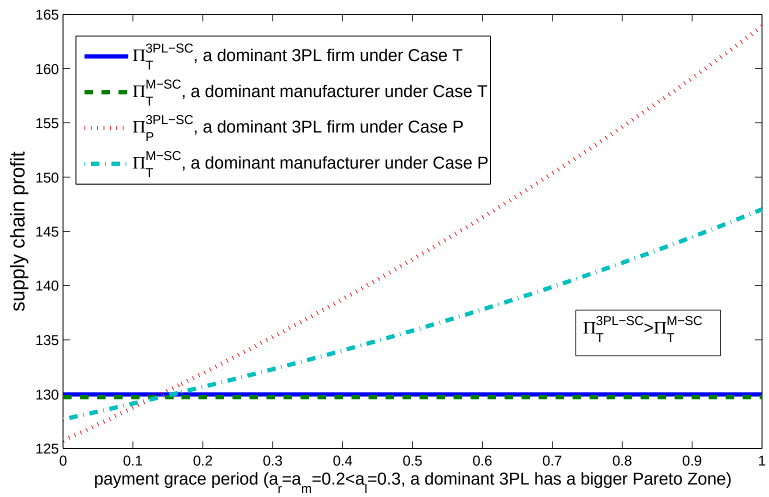

Figure 3: The supply chain efficiency comparison when $a_b = a_m < a_l$.

Figure 3 illustrates Proposition 7 for the case $a_l > a_b = a_m$. As predicted, model P with the 3PL leadership has a higher supply chain efficiency as long as the payment delay length is sufficiently long (i.e., $\ell_g \geq \ell_s$). What is more interesting is that when ℓ_g in model P results in improvement in supply chain efficiency, the improvement under the 3PL leadership grows much faster than that under the manufacturer leadership as ℓ_g increases. Intuitively, if $a_l < a_b = a_m$, the 3PL will gain even more financial benefit in P and the whole supply chain becomes even more efficient compared to that in model T.

When $a_m \leq a_b$, the supply chain efficiency of P can be still higher with the 3PL leadership than that with a dominant manufacturer. This scenario very well describes developing countries where the buyers are generally small with higher cash opportunity costs. Although the Pareto zone is larger with the manufacturer leadership when $a_m \leq a_b$, as long as the 3PL is willing to transfer a fee for the manufacturer to implement P or helps the buyer securing a lower cash opportunity cost such that $a_b < a_m$, the 3PL-leadership scenario can still outperform a manufacturer-leadership scenario for all firms, and emerges as a mutually beneficial choice. Therefore, for developing countries, it is socially significant to support a 3PL leadership role in supply chain procurement service.

5 Impact of Capital Constraint and Number of Buyers

This section investigates the robustness of our results in respect of a capital-constrained buyer and multiple buyers.

5.1 A Capital-constrained Buyer

So far, we have assumed the buyer has no capital constraints to single out the importance of the 3PL's role and the cash flow dynamics. This subsection relaxes this assumption by considering a capital-constrained buyer who has only initial capital v and limited liability. This buyer can borrow from a bank with a loan interest rate r_{bi}. In line with the literature (see, etc., Cai et al. (2014), Jing et al. (2012), Kouvelis and Zhao (2012), and Xu and Birge (2004)), we assume that the bank resides in a competitive financing market with a risk-free interest rate r_f. Now, the buyer's profit function can be written as follows:

$$\pi_{bi} = (pE[D \wedge q_{bi}] - u_i(1 + r_{bi}))^+ - v(1 + \ell_o a_b)(1 + r_f), i = T, P, \tag{14}$$

where $u_i = w_{mi}q_{bi}(1 + \ell_o a_b) + w_{li}q_{bi}(1 + \ell_r a_b) - v(1 + \ell_o a_b)$ is the loan sizes in model i. The bank's decision on the interest rate is made by solving the following equations

$$\min[pE \min[D, q_{bi}], u_i(1 + r_{bi})] = u_i(1 + r_f), i = T, P. \tag{15}$$

Submitting (15) into (14) results in

$$\pi_{bi} = \begin{cases} pE[D \wedge q_{bi}] - [w_{mi}q_{bi}(1 + \ell_o a_b) + w_{lT}q_{bi}(1 + \ell_r a_b)](1 + r_f), & \text{if } i = T, \\ pE[D \wedge q_{bi}] - [(w_{mi} + w_{li})(1 + \ell_r a_b)q_{bi}](1 + r_f). & \text{if } i = P. \end{cases} \tag{16}$$

If $r_f = 0$, (16) reduces to Eq. (3), thus the initial capital level has no impact on our previous results. If $r_f \neq 0$, letting $\hat{p} = \frac{p}{1+r_f}$, we can characterize the optimal ordering q_i^* as follows.

Proposition 8 *Assume $a_j = a$ for all j. The buyer is capital-constrained and raises capital from a competitive financing market with a risk-free interest rate r_f.*

1. *The 3PL's profit function $\pi_{li}(q_{li})$ is unimodal in q_{li}. The system equilibrium order quantity q_i^* solves the following functions:*

$$\begin{cases} G(\hat{p}, q_i^*, 1) - c_m(1 + \ell_o a) - c_l(1 + \ell_r a) = 0, & \text{if } i = T, \\ G(\hat{p}, q_i^*, \eta_l) - c_m(1 + \ell_o a) - c_l(1 + \ell_r a) = 0, & \text{if } i = P. \end{cases}$$

2. As the interest rate r_f increases, the firms' profits and optimal order quantities decrease in both T and P.

Proposition 8 indicates that the optimal order quantity is very similar to our previous one if we replace p with $\hat{p}$. This result suggests that our previous qualitative results likely continue to hold as long as the financial cost to the buyer (i.e., r_f) is not skyrocketing. It is intuitive that all firms' profits are negatively affected if r_f increases, leading to a smaller order quantity. Through numerical studies, we observe that as the interest rate r_f increases, the Pareto zone shrinks.

5.2 Multiple Buyers

In practice, the 3PL typically provides the same service for multiple buyers. Intuitively, more buyers implies a higher business volume and demand risk pooling, which should have positive impact on the supply chain under either P or T. But it is less clear whether more buyers makes the procurement service more or less attractive than T. To answer this question, we expand our baseline model to include multiple buyers. In line with Cachon (2003), to fairly compare with the baseline model, we assume D is the total demand and divide it among n symmetric firms proportional to their stocking quantity. That is, Buyer k's demand is given by $D_k = \left(\frac{q_k}{q}\right) D$, where q_k denotes Buyer k's order quantity, the overall order quantity is $q = \sum_{k=1}^{n} q_k$, and $q_{-k} = q - q_k$. In doing so, we ignore the volume effect by focusing on the risk pooling effect.

Similar to Eq. (3), Buyer k's profit function can be written as

$$\begin{aligned}
\pi_{bik}(q_{bik}) &= \begin{cases} p[\frac{q_{bik}}{q_{bi}} D \wedge q_{bik}] - [w_{mi}(1+\ell_o a) + w_{li}(1+\ell_r a)]q_{bik}, & \text{if } i = T, \\ p[\frac{q_{bik}}{q_{bi}} D \wedge q_{bik}] - w_i q_{bik}(1+\ell_r a), & \text{if } i = P, \end{cases} \qquad (17) \\
&= \begin{cases} [p - (w_{mi}(1+\ell_o a) + w_{li}(1+\ell_r a))]q_{bik} - p\frac{q_{bik}}{q_{bi}} \int_0^{q_{bi}} F(x)dx, & \text{if } i = T, \\ [p - (w_{mi} + w_{li})(1+\ell_r a)]\, q_{bik} - p\frac{q_{bik}}{q_{bi}} \int_0^{q_{bi}} F(x)dx, & \text{if } i = P. \end{cases}
\end{aligned}$$

The profit function is concave, so there is a unique optimal order quantity, $q_{bi}^*(w_{mi}, w_{li})$. For any fixed w_{li}, we define $q_{mi} \equiv q_{bi}^*(w_{mi}, w_{li})$, and the manufacturer's problem can be written as

$$\begin{aligned}
&\pi_{mi}(w_{mi}(q_{bi}^*)) = \pi_{mi}(q_{mi}) \qquad (18) \\
&= \begin{cases} [p\ (1 - M(q_{mi})) - w_{li}(1+\ell_r a) - c_m(1+\ell_o a)]q_{mi}, & \text{if } i = T, \\ [p\eta_l\ (1 - M(q_{mi})) - w_{li}(1+(\ell_o - \ell_g)a) - c_m(1+\ell_o a)]\, q_{mi}, & \text{if } i = P, \end{cases}
\end{aligned}$$

where $M(q_{mi}) = \frac{F(q_{mi})}{n} + \frac{n-1}{n}\frac{1}{q_{mi}}\int_0^{q_{mi}} F(x)dx$. In this way, we have transformed an optimal wholesale price problem to an optimal order quantity problem from the manufacturer's perspective. Let $q_{mi}^*(w_{li})$ be the optimal solution of this problem, we can then similarly transform the 3PL's problem of finding the optimal $w_{li}(q_{mi}^*)$ to solving an optimal q_{li}. In other words, Eq. (5) can be written as

$$\pi_{li}(w_{li}(q^*_{mi})) = \pi_{li}(q_{li}) \tag{19}$$

$$= \begin{cases} \left[\frac{p}{n}\bar{F}(q_{li})[n - H(q_{li})] - c_l(1+\ell_r a) - c_m(1+\ell_o a)\right] q_{li}, & \text{if } i = T, \\ \left[\frac{p}{n}\eta_l \bar{F}(q_{li})[n - H(q_{li})] - c_l(1+\ell_r a) - c_m(1+\ell_o a)\right] q_{li} + \frac{p(\ell_g - \ell_s)a}{1+\ell_r a}(1 - M(q_{li}))q_{li}, & \text{if } i = P, \end{cases}$$

where $M(q_{li}) = \frac{F(q_{li})}{n} + \frac{n-1}{n}\frac{1}{q_{li}}\int_0^{q_{li}} F(x)dx$. Solving q^*_{lP} (i.e, q^*_P) and q^*_{lT} (i.e, q^*_T) leads to the following result.

Proposition 9 *Assume there are n buyers and $a_j = a$ for all j. (i) $q^*_P \geq q^*_T$ and $T \prec_{l,b} P$ iff $\ell_g \geq \ell_s$; there exits $\bar{\ell}$ such that $T \prec P$ iff $\ell_g \in [\ell_s, \bar{\ell}]$. (ii) Both q^*_P and q^*_T increase in n, with q^*_P increasing faster than q^*_T, that is, $\frac{\partial q^*_P}{\partial n} \geq \frac{\partial q^*_T}{\partial n} \geq 0$.*

Proposition 9 (i) indicates that the supply chain performance improves in both T and P as the number of buyers increases. This can be understood as follows. As more buyers serve the same market, each buyer's demand uncertainty risk is sliced into a smaller piece, which brings forth a larger total order quantity to the manufacturer. The benefit of having more buyers applies to both T and P equally when $\ell_g = \ell_s$. Analogous to the single buyer scenario, the profits of the buyers and the 3PL increase with ℓ_g. As a result, the buyers and the 3PL prefer P as long as $\ell_g \geq \ell_s$. On the other hand, the manufacturer would back off if ℓ_g becomes too large. Therefore, the firms' preferences of P or T described in Propositions 1 and 2 for a single buyer continue to hold true for multiple buyers.

Proposition 9 (ii) shows that as n grows the order quantity increases faster in P than in T. This result occurs because with more buyers, both the manufacturer and the 3PL in P are more willing to reduce their price/fee to induce an even higher order quantity from the buyers, hence the supply chain efficiency improves more quickly. Consequently, the Pareto zone with multiple buyers is larger than that with a single buyer. Meanwhile, when ℓ_g is chosen at the maximizer of the manufacturer's profit, the profit disparity between P or T enlarges and the manufacturer enjoys a larger profit as n increases.

Figure 4 illustrates the impact of the number of buyers on the Pareto zone through three subgraphs with $n = 1,\ 5,\ 50$, respectively. Here, the demand follows an exponential distribution with mean 1000, $p = 1$, $c_m = 0.05$, $c_l = 0.02$, $\ell_o = 80/360$, $\ell_r = 30/360$, and $a = 0.1$. To be comparable, the lengths of vertical and horizontal axes are the same for the three sub-figures. As predicted in Proposition 9, the manufacturer's profits in both T and P increase with n. Because the order quantity under P increases faster in n than under T, the manufacturer's profit curve becomes more concave when the number of buyers increases. Moreover, the Pareto zone increases

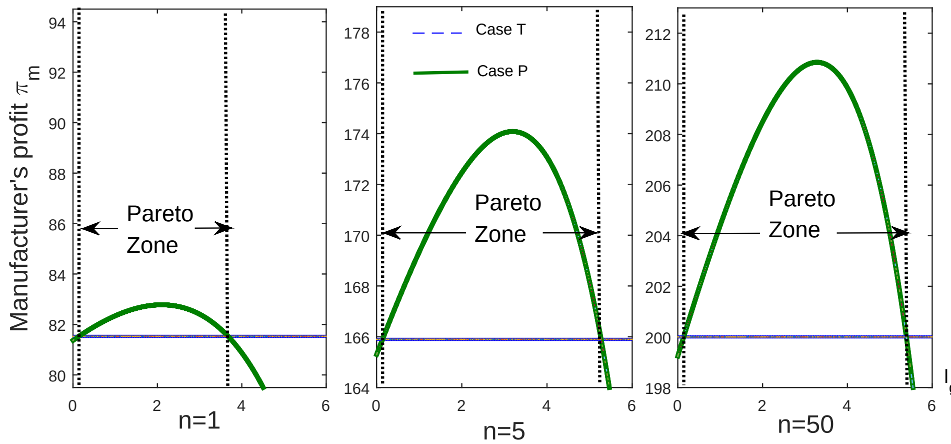

Figure 4: The manufacturer's profits in Cases T and P and the Pareto zone

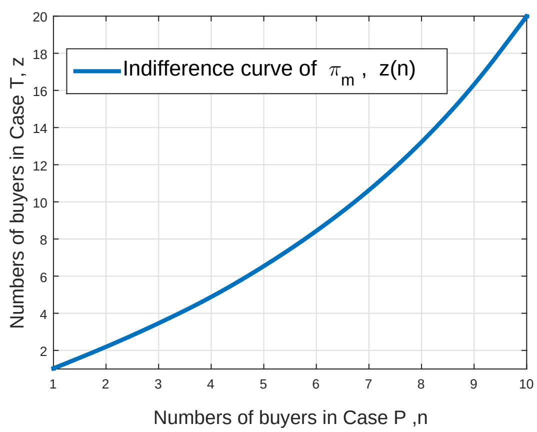

Figure 5: Indifference curve of the manufacturer's profit in Cases T and P in terms of number of buyers, respectively.

as n grows. Nevertheless, the increment of Pareto zone slows down when n is bigger, because the demand risk pooling effect becomes less significant as n grows.

Given that both the number of buyers (procurement size) and model P (procurement type) increase the manufacturer's profit, we use Figure 5 to describe the relative impact of model P in terms of the number of buyers. Suppose the manufacturer can determine the optimal grace period ℓ_g. Define $z(n)$ to be the number of buyers in model T needed in order to obtain the same manufacturer's profit in P with n buyers. Using the same parameter setting as in Figure 4, Figure 5 plots $z(n)$ as a function of n. For example, if there are 10 buyers in P, then it requires about 20 buyers in T to achieve the same manufacturer profit. Figure 5 demonstrates $z(n)$ is convexly increasing in n. That is, as n increases, the manufacturer profit grows faster in P than in T. The same observation holds for the 3PL as well, because 3PL obtains higher profit in P (given the manufacturer's optimal ℓ_g) than in T.

6 Negotiation on Endogenous Leadtimes

For tractability, our baseline model has assumed exogenous transportation leadtime (ℓ_s) and payment grace period (ℓ_g). To demonstrate the robustness of our qualitative results, this section endogenize these two leadtimes. Because the decision on the the payment grace period occurs before the decision on the transportation leadtime, solving the game backward, we first discuss ℓ_s and then ℓ_g. Due to limited space, we focus on the case where $a_b = a_l = a_m = a$ and other scenarios can be analyzed similarly.

6.1 Optimal Transportation Leadtime

To facilitate our discussion, we introduce $L_{si}, i = T, P$, as the decision variable of the transportation leadtime with a lower bound at ℓ_s, the minimal transportation time. The 3PL and the buyer negotiate on L_{si} before the 3PL charges the logistic service fee. We assume the buyer's bargaining power relative to the 3PL is $\theta_b \in [0, 1]$ and the 3PL's is $1 - \theta_b$. For tractability, we also assume that c_l does not depend on L_{si}. To be consistent in both T and P, if the 3PL and the buyer cannot agree on the transportation leadtime, no transaction will occur. Therefore, in line with Nash (1950), for either model, we have

$$L_{si}^* = \arg\max_{L_{si} \geq \ell_s} \left[\pi_{bi}\left(L_{si}\right)\right]^{\theta_b} \left[\pi_{li}\left(L_{si}\right)\right]^{1-\theta_b}, i = T, P,$$

where all profit functions are given in Eq. (3)–(5) with L_{si} replacing ℓ_s. Solving the Nash bargaining solution yields the following property.

Proposition 10 *Assume $a_j = a$. For any given $\theta_b \in [0, 1]$, in either T or P, there exists a unique optimal Nash bargaining solution $L_{si}^* = \ell_s$, $i = T, P$.*

Proposition 10 justifies our assumption in the baseline model that both the buyer and the 3PL have no intention to delay the shipment. Because both the buyer and the 3PL's profits decrease with L_{si}, the Nash bargaining product (i.e., $\left[\pi_{bi}\left(L_{si}\right)\right]^{\theta_b} \left[\pi_{li}\left(L_{si}\right)\right]^{1-\theta_b}$) decreases with L_{si}. Therefore, the optimal Nash bargaining solution is achieved at the lower bound, the minimal transportation time (i.e., $L_{si}^* = \ell_s$, $i = T, P$).

6.2 Optimal Payment Grace Period

In P, the manufacturer and the 3PL firm first determines the payment grace period (ℓ_g) via Nash bargaining. The remaining game follows the baseline model. To investigate the impact of firms' bargaining powers, we assume the manufacturer's bargaining power relative to the 3PL is $\theta_m \in [0, 1]$ and the 3PL's is $1 - \theta_m$. If the negotiation on the procurement service fails, the firms will instead follow model T. The optimal payment grace period satisfies

$$\ell_g^* = \arg\max_{\ell_g} \left[\pi_{mP}(\ell_g) - \pi_{mT}\right]^{\theta_m} \left[\pi_{lP}(\ell_g) - \pi_{lT}\right]^{1-\theta_m}.$$

As discussed previously, there is no closed-form solution in the subsequent subgames, so there is no closed-form solution for ℓ_g^* either. Nevertheless, we can prove the following property.

Proposition 11 *Assume $a_j = a$ with 3PL Stackelberg leadership. For any given $\theta_m \in [0, 1]$, there exists a unique optimal Nash bargaining solution $\ell_g^* \in [\ell_s, \overline{\ell}]$.*

Proposition 11 can be conceptually proved here based on Propositions 1 and 2. Whereas Proposition 1 indicates that the 3PL always has incentives to raise ℓ_g, Proposition 2 shows that the manufacturer's profit first increases and then decreases with ℓ_g. As illustrated in Figure 2, the attractiveness of P to the manufacturer reaches its peak, hereby referred to as $\hat{\ell}$, in $[\ell_s, \overline{\ell}]$. Obviously, both the manufacturer and the 3PL have incentives to set ℓ_g^* at least as lengthy as $\hat{\ell}$. However, because $\pi_{lP}(\ell_g)$ monotonically increases with ℓ_g, the 3PL will push ℓ_g across $\hat{\ell}$. But, the manufacturer's profit monotonically decreases with ℓ_g when $\ell_g > \hat{\ell}$ and will not participate in the game when $\ell_g > \overline{\ell}$ because of negative profit. Consequently, for any given $\theta_m \in [0, 1]$, the product of $[\pi_{mP}(\ell_g)]^{\theta_m} [\pi_{lP}(\ell_g)]^{1-\theta_m}$ first increases with ℓ_g and then decreases with ℓ_g; therefore, there exists a unique optimal Nash bargaining solution $\ell_g^* \in [\hat{\ell}, \overline{\ell}]$.

Based on the above discussion, we can conclude that, if the manufacturer can decide the grace period by itself (i.e., $\theta_m = 1$), then it will set $\ell_g^* = \hat{\ell}$. If the 3PL can decide the grace period by itself (i.e., $\theta_m = 0$), then it will set $\ell_g^* = \overline{\ell}$. For any $\theta_m \in (0, 1)$, $\ell_g^* \in (\hat{\ell}, \overline{\ell})$. According to Proposition 2, $\hat{\ell}$ locates inside $[\ell_s, \overline{\ell}]$; thus, ℓ_g^* must be larger than ℓ_s. For other scenarios, such as manufacturer leadership and $a_j \neq a$, ℓ_g^* can be determined similarly as long as there exists a Pareto zone for the firms in P compared with T.

7 Conclusion

Motivated by Eternal Asia's recent success in helping the SME buyers in Asia through an innovative 3PL procurement service (P), this paper makes a first attempt to theoretically analyze the value of the efficiency improvement brought by model P, compared with a 3PL's traditional shipping service (model T). In the classic supply chain models and in practice, third-party logistics is usually considered only an auxiliary component of a supply chain. By explicitly including a 3PL as an active player in the supply chain and by capturing the cash-flow dynamics characterized by the practice of Eternal Asia and Jianfa, we demonstrate that P can be a mutually beneficial solution for all firms. Triple marginalization does hurt efficiency, but this effect is significantly softened with the 3PL'a procurement service. The 3PL's supply chain finance role cannot be simply replaced by a payment delay to the buyer (instead of the 3PL) at product delivery. The benefits to all firms are higher with more buyers, but lower when the buyer is capital constrained with an elevated financial cost. With equal cash opportunity costs or when the payment delay is sufficiently long, 3PL leadership outperforms manufacturer leadership. In the framework of Nash bargaining, both the buyer and the 3PL firm prefer the minimal transportation time, and the manufacturer and the 3PL can establish a unique optimal payment delay grace period. If implemented properly, the procurement service under 3PL leadership can be socially significant, especially by creating extra value for small businesses in developing countries.

As a first step, consistent with our motivating example, the current study focuses on a supply chain with a financially strong manufacturer and 3PL, and only the buyers may have capital constraints. The model and theory may be enriched in several directions. For example, the choice of finance mode and financing terms critically hinges upon the risk profile of the supply chain parties. Given the complexity of heterogeneous risk profiles of various firms, it is intriguing to investigate the impact of different risk profiles on firms' operations decisions and financing terms. In addition, asymmetric information on demand and costs will also affect firms' operations decision and performance. Hence it would be interesting to study whether the 3PL has incentives to act as a distributor by procuring from the manufacturer, storing the products, and then reselling to the buyers.

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Online Supplements for "The Cash Flow Advantages of 3PLs as Supply Chain Orchestrators"

Xiangfeng Chen * Gangshu (George) Cai † Jing-Sheng Song ‡

Appendix A: Exogenous Service Rate

This appendix assumes that the service rate (w_l) is exogenous. This corresponds to the case in Lemma 2 where the manufacturer decides the optimal solution given the 3PL's service rate.

As Lemma 2 shows, if all firms' opportunity costs are the same, neither P nor T can strictly outperform the other. Compared with results in Section 3, this observation indicates that fixing the 3PL's service rate minimizes the 3PL's capability to facilitate a mutually beneficial outcome for all firms in P.

With different cash opportunity costs, we find that P can reemerge as a preferred choice for all firms. From Lemma 2, we have ${q_P^*}' = \frac{dq_P^*}{d\ell_g} < 0$. Define $A(\ell_g) = (\frac{p\bar{F}(q_P^*)}{1+\ell_r a_l} - w_l)q_P^* + [(w_l - c_l)\ell_r + \frac{c_m(1+\ell_o a_m)(\ell_g-\ell_s)}{1+(\ell_o-\ell_g)a_m}]{q_P^*}'$. Comparing firms' profits between T and P yields the following property.

Corollary 1 *Suppose w_l is exogenous and the firms have different cash opportunity costs. Let $\tilde{\ell} = \frac{(1+\ell_o a_m a_b)(\ell_o-\ell_r)}{a_m(1+\ell_o a_b)}$ and $\check{\ell}$ be the solution to $\pi_{lT}(q_T^*) = \pi_{lP}(q_P^*, \ell_g)$.*

1. *If $a_l \geq \frac{(c_l-w_l){q_P^*}'}{A(\ell_g)}$ and $\frac{d\pi_{lP}(q_P^*)}{d\ell_g} \geq 0$ then $\check{\ell} < \tilde{\ell}$, and $T \prec P$ for all $\ell_g \in [\check{\ell}, \tilde{\ell}]$;*

2. *If $\frac{d\pi_{lP}(q_P^*)}{d\ell_g} < 0$, then $T \prec P$ when $\ell_g \leq \min[\check{\ell}, \tilde{\ell}]$.*

Thus, the qualitative findings with endogenous w_l and different cash opportunity costs still hold for exogenous w_l.

Appendix B

Proof of Lemma 1: Taking first derivative of Eq. (3) with respect to (w.r.t.) q_{bi}, the first-order-condition (FOC) yields

$$q_{bi}^*(w_{li}, w_{mi}) = \begin{cases} \bar{F}^{-1}\left(\frac{w_{mi}(1+\ell_o a)+w_{li}(1+\ell_r a)}{p}\right), & \text{if } i = T, \\ \bar{F}^{-1}\left(\frac{(w_{mi}+w_{li})(1+\ell_r a)}{p}\right), & \text{if } i = P. \end{cases}$$

We have $q_{bT}^*(w_m, 0) = w_{bP}^*(w_m, 0) = q_b^*(w_m) = \bar{F}^{-1}(w_m/P)$. If $w_{li} \not\equiv 0$ and $a \not\equiv 0$, we have $w_{mT}(1+\ell_o a) + w_{lT}(1+\ell_r a) > w_m$, and then $q_{bT}^*(w_m, w_l) < q_b^*(w_m)$.

Let $w_{mT} = w_{mP} = w_m$ and $w_{lT} = w_{lP} = w_l$. Since $\ell_o > \ell_r$, we have $w_{mT}(1+\ell_o a)+w_{lT}(1+\ell_r a) > (w_{mP} + w_{lP})(1 + \ell_r a)$, and then obtain $q_{bP}^*(w_m, w_l) > q_{bT}^*(w_m, w_l)$. Q.E.D.

*School of Management, Fudan University, Email:chenxf@fudan.edu.cn.
†Leavey School of Business, Santa Clara University. Email: gcai@scu.edu.
‡The Fuqua School of Business, Duke University. Email: jssong@duke.edu

Proof of Lemma 2: We first prove that $\pi_{mi}(w_{mi})$ is unimodal w.r.t. $w_{mi} \geq 0$. The first derivative of $\pi_{mi}(w_{mi})$ w.r.t. w_{mi} is $\frac{d\pi_{mi}(w_{mi})}{dw_{mi}} = \frac{\partial \pi_{mi}}{\partial q^*_{bi}} \frac{dq^*_{bi}}{dw_{mi}} + \frac{\partial \pi_{mi}}{\partial w_{mi}}$, and we have,

$$\frac{d\pi_{mi}(w_{mi})}{dw_{mi}} = \begin{cases} \frac{1+\ell_o a}{ph(q^*_{bi})}\left[pH(q^*_{bi}) + \frac{c_m(1+\ell_o a)}{\bar{F}(q^*_{bi})} - \frac{w_{mi}(1+\ell_o a)}{\bar{F}(q^*_{bi})}\right] & \text{if } i = T, \\ \frac{1+(\ell_o-\ell_g)a}{ph(q^*_{bi})}\left[pH(q^*_{bi}) + \frac{(1+\ell_o a)(1+\ell_r a)}{1+(\ell_o-\ell_g)a}\frac{c_m}{\bar{F}(q^*_{bi})} - \frac{w_{mi}(1+\ell_r a)}{\bar{F}(q^*_{bi})}\right] & \text{if } i = P. \end{cases}$$

It is straightforward that $pH(q^*_{bi})$ and $\frac{1}{\bar{F}(q^*_{bi})}$ decrease with w_{mi} since $\frac{dq^*_{bi}}{dw_{mi}} < 0$. And $\frac{w_{mi}}{\bar{F}(q^*_{bi})}$ increases with w_{mi} since $\left[\frac{w_{mi}}{\bar{F}(q^*_{bi})}\right]' = \frac{\bar{F}(q^*_{bi}) - w_{mi}(1+\ell_r a)}{\bar{F}^2(q^*_{bi})} > 0$. Hence, $\frac{d\pi_{mi}(w_{mi})}{dw_{mi}}$ decreases with w_{mi}. Then, if $w_{mi} = 0$, we have $\frac{d\pi_{mi}(w_{mi})}{dw_{mi}} > 0$; if $w_{mi} \to \propto$, $\frac{d\pi_{mi}(w_{mi})}{dw_{mi}} < 0$. Therefore, $\pi_{mi}(w_{mi})$ is unimodal w.r.t. w_{mi}. Solving the first order condition $\frac{d\pi_{mi}(w_{mi})}{dw_{mi}} = 0$ results in

$$w^*_{mi}(w_{li}) = \begin{cases} c_m + \frac{pq^*_{bT}(w_{lT}, w^*_{mT}) f(q^*_{bT}(w_{lT}, w^*_{mT}))}{1+\ell_o a}, & \text{if } i = T, \\ \eta_m c_m + \frac{pq^*_{bP}(w_{lP}, w^*_{mP}) f(q^*_{bP}(w_{lP}, w^*_{mP}))}{1+\ell_r a}, & \text{if } i = P, \end{cases}$$

where $\eta_m = \frac{1+\ell_o a}{1+(\ell_o-\ell_g)a}$. Given $\ell_g \geq 0$, we get $\eta_m \geq 1$.

For a fixed $w_l = w_{lT} = w_{lP}$, from Lemma 1, we have, $p\bar{F}(q^*_{bT}) = w_{mT}(1+\ell_o a) + w_l(1+\ell_r a)$ and $p\bar{F}(q^*_{bP}) = (w_{mP} + w_l)(1+\ell_r a)$. Submitting w^*_{mi} into these two equations, we can obtain the optimal order quantities $q_{bi}(w^*_{mi}, w_l)$ solving the following equations

$$\begin{aligned} p\bar{F}(q^*_{bT}) - pq^*_{bT} f(q^*_{bT}) - w_l(1+\ell_r a) &= c_m(1+\ell_o a), \\ p\bar{F}(q^*_{bP}) - pq^*_{bP} f(q^*_{bP}) - w_l(1+\ell_r a) &= c_m \frac{1+\ell_o a}{\eta_l}. \end{aligned}$$

Note that $qf(q) = H(q)\bar{F}(q)$ increases with q, since $[H(q)\bar{F}(q)]' = \bar{F}(q)[h(q)[1-H(q)] + qh'(q)] > 0$. Then, we have $p\bar{F}(q) - pqf(q)$ decreases in q. Let $\ell_g = \ell_s$, we have $q^*_{bT}(w^*_{mT}, w_l) = q^*_{bP}(w^*_{mP}, w_l)$, since $\eta_l = 1$. Since $\eta_m > 1$ and $\ell_o > \ell_r$, we have $w^*_{mP} \geq w^*_{mT}$. If $\ell_g < \ell_s$, we have $\eta_l > 1$ and $q^*_{bT}(w^*_{mT}, w_l) < q^*_{bP}(w^*_{mP}, w_l)$; otherwise, $q^*_{bT}(w^*_{mT}, w_l) \geq q^*_{bP}(w^*_{mP}, w_l)$. Q.E.D.

Proof of Lemma 3: From Eq. (10),

$$\frac{d\pi_{li}(q^*_{bi}(w_{li}))}{dq^*_{bi}(w_{li})} = \begin{cases} p\bar{F}(q^*_{bT})[(1-H(q^*_{bT}))^2 - q^*_{bT}H'(q^*_{bT})] - c_m(1+\ell_o a) - c_l(1+\ell_r a), \\ p\bar{F}(q^*_{bP})\left[(1-H(q^*_{bP}))(1-\eta_l H(q^*_{bP})) - \eta_l q^*_{bP}H'(q^*_{bP})\right] - c_m(1+\ell_o a) - c_l(1+\ell_r a). \end{cases}$$

Define $G(p, q_{li}, 1) = p\bar{F}(q_{li})[(1-H(q_{li}))^2 - q_{li}H'(q_{li})] > 0$. Since $H(q_{li})$ and $q_{li}H'(q_{li})$ increase with q_{li}, and $\bar{F}(q_{li})$ decreases with q_{li}, $G(p, q_{li}, 1)$ decreases with q_{li}. Similarly, define $G(p, q_{li}, \eta_l) \equiv p\bar{F}(q_{li})[(1-H(q_{li}))(1-\eta_l H(q_{li})) - \eta_l q_{li}H'(q_{li})]$, which decreases with q_{li}. In addition, we have $\frac{d\pi_{li}(q_{li})}{dq_{li}}|_{q_{li}=0} = p - c_m(1+\ell_o a) - c_l(1+\ell_r a) > 0$, and $\frac{d\pi_{li}(q_{li})}{dq_{li}}|_{q_{li}\to\propto} = -c_m(1+\ell_o a) - c_l(1+\ell_r a) < 0$. Thus, $\pi_{li}(q_{li})$ is unimodal in $q_{li} \geq 0$. Solving the FOC leads to the result in Lemma 3. Q.E.D.

Proof of Proposition 1: **Part (i):** From Lemma 3, we get $G(p, q^*_{lP}, \eta_l) = G(p, q^*_{lT}, 1) \leq G(p, q^*_{lT}, \eta_l)$ since $\ell_g \geq \ell_s$ and $\eta_l \leq 1$. Since $G(p, q, \eta_l)$ decreases in q, we obtain $q^*_{lT} \leq q^*_{lP}$.

Part (ii): Based on the proof of Part (iv), we have $w^*_{lT}(1+\ell_r a) = p\bar{F}(q^*_{lT})[1-H(q^*_{lT})] - c_m(1+\ell_o a)$ and $w^*_{lP}(1+\ell_r a) = p\bar{F}(q^*_{lP})[1-H(q^*_{lP})] - c_m\frac{(1+\ell_o a)}{\eta_l}$. Since $\bar{F}(q)[1-H(q)]$ decreases with q and $q^*_{lT} \leq q^*_{lP}$, we have $p\bar{F}(q^*_{lP})[1-H(q^*_{lP}) \leq p\bar{F}(q^*_{lT})[1-H(q^*_{lT})]$. And since $\ell_g \geq \ell_s$, we have $\frac{c_m(1+\ell_o a)}{\eta_l} \geq c_m(1+\ell_o a)$. Thus, we obtain $w^*_{lP} \leq w^*_{lT}$.

Part (iii): According to Lemma 3, q^*_{lP} (i.e., q^*_P) solves the following equation.

$$G(p, q^*_P, \eta_l) = c_m(1+\ell_o a) + c_l(1+\ell_r a).$$

It is easy to see that the left hand side (LHS) of the above equation increases with ℓ_g, but decreases with q. Since the value of LHS decreases with q, we should increase q to make the equation satisfied when ℓ_g increases. Thus, given that the right hand side is fixed, we have q^*_{lP} increases with ℓ_g.

We can rewrite the buyer's profit as $\pi^*_{bP} = p\mathbb{E}[D \wedge q^*_{lP}] - q\bar{F}(q^*_{lP})]$. Taking first order and second order derivatives with respect to q^*_{lP}, we have $\frac{d\pi_{bP}(q^*_{lP})}{dq^*_{lP}} = p\bar{F}(q^*_{lP})H(q^*_{lP}) > 0$ and $\frac{d^2\pi_{lP}(q^*_{lP})}{d{q^*_{lP}}^2} =$

$p\bar{F}(q_{lP}^*)[h(q_{lP}^*)(1-H(q_{lP}^*))+q_{lP}^*h'(q_{lP}^*)]>0$. Therefore, $\pi_{bP}(q_{lP}^*)$ convexly increases with q_{lP}^*. Since q_{lP}^* increases with ℓ_g, $\pi_{bP}(q_{lP}^*)$ convexly increases with ℓ_g.

The profit of 3PL is: $\pi_{lP}(q_{lP}^*)=\left[p\bar{F}(q_{lP}^*)\left(1-\eta_l H(q_{lP}^*)\right)-c_m(1+\ell_o)-c_l(1+\ell_r a)\right]q_{lP}^*$. We then have $\frac{d\pi_{lP}(q_{lP}^*)}{d\ell_g}=\frac{p\ a}{1+\ell_r a}q_{lP}^*\bar{F}(q_{lP}^*)H(q_{lP}^*)>0$ and $\frac{d^2\pi_{lP}(q_{lP}^*)}{d\ell_g{}^2}=\frac{pa}{1+\ell_r a}\bar{F}(q_{lP}^*)[H(q_{lP}^*)(1-H(q_{lP}^*))+q_{lP}^*H'(q_{lP}^*)]\frac{dq_{lP}^*}{d\ell_g}>0$. Thus we can show that $\pi_{lP}(q_{lP}^*)$ convexly increases with ℓ_g.

Part (iv): Given q_{li}^*, we have

$$w_{mi}^*=\begin{cases}\frac{pq_{lT}^*f(q_{lT}^*)+c_m(1+\ell_o a)}{1+\ell_o a}, & \text{if } i=T,\\ \frac{pq_{lP}^*f(q_{lP}^*)+c_m(1+\ell_o a)/\eta_l}{1+\ell_r a}, & \text{if } i=P,\end{cases}$$

and

$$w_{li}^*(1+\ell_r a)=\begin{cases}p\bar{F}(q_{lT}^*)[1-H(q_{lT}^*)]-c_m(1+\ell_o a), & \text{if } i=T,\\ p\bar{F}(q_{lP}^*)[1-H(q_{lP}^*)]-c_m(1+\ell_o a)/\eta_l, & \text{if } i=P.\end{cases}$$

Submitting $w_{li}^*(q_{li}^*)$ and $w_{mi}^*(q_{li}^*)$ into Eq. (3), we have

$$\pi_{bi}(q_{li}^*)=\begin{cases}p\ \mathbb{E}[D\wedge q_{lT}^*]-q_{lT}^*\bar{F}(q_{lT}^*), & \text{if } i=T,\\ p\ \mathbb{E}[D\wedge q_{lP}^*]-q_{lP}^*\bar{F}(q_{lP}^*), & \text{if } i=P.\end{cases}$$

Since $\frac{d\pi_{bi}(q_{li}^*)}{dq_{li}^*}=q_{li}^*f(q_{li}^*)\geq 0$, we have $\pi_{bT}(q_{lT}^*)\leq\pi_{bP}(q_{lP}^*)$ because $q_{lT}^*\leq q_{lP}^*$ conditional on $\ell_g\geq\ell_s$ according to Proposition 1. Similarly, we have $\pi_{lT}(q_{lT}^*)\leq\pi_{lP}(q_{lP}^*)$.

Since the buyer and the 3PL's profits in P increase with ℓ_g but those in T are independent of ℓ_g, the buyer and the 3PL firm's preference of P to T grows stronger as ℓ_g increases. Q.E.D.

Proof of Proposition 2 Part (i): Note that $\pi_{mP}(q_{lP}^*,\ell_g)=p\eta_l q_{lP}^*\bar{F}(q_{lP}^*)H(q_{lP}^*)$. $\frac{d\pi_{mP}(q_{lP}^*,\ell_g)}{d\ell_g}=p\eta_l\bar{F}(q_{lP}^*)[H(q_{lP}^*)(1-H(q_{lP}^*))+q_{lP}^*H'(q_{lP}^*)]\frac{dq_{lP}^*}{d\ell_g}-p\frac{a}{1+\ell_r a}q_{lP}^*\bar{F}(q_{lP}^*)H(q_{lP}^*)$. And

$$\frac{dq_{lP}^*}{d\ell_g}=\frac{a}{1+\ell_r a}\frac{H(q_{lP}^*)-H^2(q_{lP}^*)+q_{lP}^*H'(q_{lP}^*)}{\frac{h(q_{lP}^*)(c_m'+c_l')}{p\bar{F}(q_{lP}^*)}+H'(q_{lP}^*)+\eta_l[2H'(q_{lP}^*)(1-H(q_{lP}^*))+q_{lP}^*H''(q_{lP}^*)]}\geq 0.$$

According to Lemma 3, q_{lP}^* solves the following equation $p\bar{F}(q_{lP}^*)[(1-H(q_{lP}^*))(1-\eta_l H(q_{lP}^*))-\eta_l q_{lP}^*H'(q_{lP}^*)]=c_m'+c_l'$, where $c_m'=c_m(1+\ell_o a)$ and $c_l'=c_l(1+\ell_r a)$.

Furthermore, we have, $p\eta_l\bar{F}(q_{lP}^*)[H(q_{lP}^*)(1-H(q_{lP}^*))+q_{lP}^*H'(q_{lP}^*)]=p\bar{F}(q_{lP}^*)(1-H(q_{lP}^*))-c_m'-c_l'$. Then, we have,

$$\frac{d\pi_{mP}(q_{lP}^*,\ell_g)}{d\ell_g}=[p\bar{F}(q_{lP}^*)(1-H(q_{lP}^*))-c_m'-c_l']\left[-\frac{p\ a}{1+\ell_r a}\frac{p\bar{F}(q_{lP}^*)q_{lP}^*H(q_{lP}^*)}{p\bar{F}(q_{lP}^*)(1-H(q_{lP}^*))-c_m'-c_l'}+\frac{dq_{lP}^*}{d\ell_g}\right].$$

Define $M1(\ell_g)=\frac{p\bar{F}(q_{lP}^*)q_{lP}^*H(q_{lP}^*)}{p\bar{F}(q_{lP}^*)(1-H(q_{lP}^*))-c_m'-c_l'}$. As ℓ_g increases, $q(\ell_g)$ increases, $p\bar{F}(q_{lP}^*)q_{lP}^*H(q_{lP}^*)$ increases, but $p\bar{F}(q_{lP}^*)(1-H(q_{lP}^*))-c_m'-c_l'$ decreases. Thus, $M1(\ell_g)$ increases with ℓ_g.

Let $M2(\ell_g)=\frac{dq_{lP}^*}{d\ell_g}$.

Case I: $\frac{dM2(\ell_g)}{d\ell_g}\geq 0$

Therefore, both $M1(\ell_g)$ and $M2(\ell_g)$ are monotonic increasing with ℓ_g.

If we can show that $M2(\ell_g)-M1(\ell_g)>0$ when ℓ_g is a very small quantity making $q_{bP}\to 0$. $M2(\ell_g)-M1(\ell_g)<0$ when ℓ_g is a very large value making $q_{lP}^*\to\check{q}$, which solves $p\bar{F}(1-H(q))=c_m'+c_l'$. Let $q_{lP}^*(\ell_g)\to 0$, we have, $\frac{d\pi_{mP}(q_{lP}^*,\ell_g)}{d\ell_g}|_{q_{lP}^*(\ell_g)=0}=\frac{a}{[1+(\ell_o-\ell_g)a]p\bar{F}(q_{lP}^*)}\frac{[p\bar{F}(q_{lP}^*)-c_m'-c_l']^2}{\frac{h(q_{lP}^*)(c_m'+c_l')}{p\bar{F}(q_{lP}^*)}}>0$.

Let $q_{lP}^*(\ell_g)\to\check{q}$, we have, $\frac{d\pi_{mP}(q_{lP}^*,\ell_g)}{d\ell_g}|_{q_{lP}^*(\ell_g)=\check{q}}=-p\frac{a}{1+\ell_r a}q_{lP}^*\bar{F}(q_{lP}^*)H(q_{lP}^*)<0$. Let $\check{q}$ solves $p\bar{F}(q_{lP}^*)(1-H(q_{lP}^*))-c_m'-c_l'=0$, we get $\frac{\pi_{mP}(q_{lP}^*,\ell_g)}{d\ell_g}|_{q_{lP}^*(\ell_g)\to\check{q}}<0$. Thus, $\pi_{mP}(q_{lP}^*(\ell_g))$ is a unimodal function ℓ_g, and $q_{lP}^*(\ell_g^*)$ satisfying $\frac{d\pi_{mP}^*(q_{lP}^*(\ell_g))}{d\ell_g}=0$.

Case II: $\frac{dM2(\ell_g)}{d\ell_g}<0$

$\frac{d\pi_{mP}(q_{lP}^*,\ell_g)}{d\ell_g}=[p\bar{F}(q_{lP}^*)(1-H(q_{lP}^*))-c_m'-c_l'][-M1(\ell_g)+M2(\ell_g)]$. According to Case I, we show that $M1(\ell_g)$ increases with ℓ_g, and $-M1(\ell_g)+M2(\ell_g)$ decreases with ℓ_g. Thus, $\frac{d\pi_{mP}(q_{lP}^*,\ell_g)}{d\ell_g}$ decreases in ℓ_g. As shown in Case I, $\frac{d\pi_{mP}(q_{lP}^*,\ell_g)}{d\ell_g}|_{q_{lP}^*(\ell_g)=0} > 0$ and $\frac{d\pi_{mP}(q_{lP}^*,\ell_g)}{d\ell_g}|_{q_{lP}^*(\ell_g)=\check{q}} < 0$. Therefore, $\pi_{mP}(q_{lP}^*,\ell_g)$ is a unimodal function in ℓ_g.

Part (ii): Part (ii) is a direct result of Part (i). Since $\pi_{mP}(q_{lP}^*(\ell_g))$ is a unimodal function in ℓ_g, there must exist a unique $\overline{\ell} > \ell_o - \ell_r$ such that $\pi_{mP}(q_{lP}^*(\ell_g)) = \pi_{mT}(q_{lT}^*(\ell_g))$ and $\pi_{mP}(q_{lP}^*(\ell_g)) < \pi_{mT}(q_{lT}^*(\ell_g))$ if $\ell_g > \overline{\ell}$. Q.E.D.

Proof of Corollary 1: **Part (i):** We have $G(p,q_{lP}^*,\eta_l) = c_m(1+\ell_o a)+c_l(1+\ell_r a) = p\bar{F}(q^C(a)) > G(p,q^C(a),\eta_l)$ since $\eta_l \le 1$ and $\ell_g \ge \ell_s$. Since $G(p,q,\eta_l)$ decreases with q, we have $q_{lP}^* < q^C(a)$.
Part (ii): We have $\pi_T^{SC}(q_{lT}^*) \le \pi_P^{SC}(q_{lP}^*) < \pi_T^{SC}(q^C(a)) = \pi_P^{SC}(q^C(a))$ because of $q_{lT}^* \le q_{lP}^* < q^C(a)$. Similarly, we can show that $q_{lT}^* > q_{lP}^*$ and $\pi_T^{SC}(q_{lT}^*) > \pi_P^{SC}(q_{lP}^*)$ when $\ell_g < \ell_s$. Q.E.D.

Proof of Proposition 3: We now prove that, when the manufacturer is the Stackelberg leader, $\pi_{mP}(q_{mP}^*) = \pi_{mT}(q_{mT}^*)$, $\pi_{lP}(q_{mP}^*) = \pi_{lT}(q_{mT}^*)$, and $\pi_{bP}(q_{mP}^*) = \pi_{bT}(q_{mP}^*)$. We solve the game backward. Given the manufacturer and the 3PL's decisions, the buyer problem is the same as in Lemma 1 where the 3PL is the Stackelberg leadership. From Lemma 1, we have $w_{lT} = \frac{p\bar{F}(q_{bT}^*)-w_{mT}(1+\ell_o a)}{1+\ell_r a}$, and $w_{lP} = \frac{p\bar{F}(q_{bP}^*)-w_{mP}(1+\ell_r a)}{1+\ell_r a}$. Submitting $w_{li}(q_{bi}^*)$ into Eq. (5), rewriting the 3PL firm's problem, and taking the first order derivative w.r.t. q_{li}, we have,

$$\frac{d\pi_{li}(q_{li})}{dq_{li}} = \begin{cases} p\bar{F}(q_{lT})[1-H(q_{lT})] - w_{mT}(1+\ell_o a) - c_l(1+\ell_r a), & \text{if } i=T,\\ p\bar{F}(q_{lP})[1-H(q_{lP})] - w_{mP}(1+(\ell_o-\ell_g)a) - c_l(1+\ell_r a), & \text{if } i=P.\end{cases}$$

Denoting q_{li}^* that solves $\frac{d\pi_{li}(q_{li})}{dq_{li}} = 0$, we then obtain

$$w_{mi}(q_{li}^*) = \begin{cases} \frac{p\bar{F}(q_{lT}^*)[1-H(q_{lT}^*)]-c_l(1+\ell_r a)}{1+\ell_o a}, & \text{if } i=T,\\ \frac{p\bar{F}(q_{lP}^*)[1-H(q_{lP}^*)]-c_l(1+\ell_r a)}{1+(\ell_o-\ell_g)a}, & \text{if } i=P.\end{cases}$$

Submitting $w_{mi}(q_{li}^*)$ into Eq. (4), we can rewrite the manufacturer's profit function as below:

$$\pi_{mi}(w_{mi}(q_{li}^*)) = \pi_{mP}(q_{mi}) = \begin{cases} (p\bar{F}(q_{mT})[1-H(q_{mT})] - c_l(1+\ell_r a) - c_m(1+\ell_o a))q_{mT}, & \text{if } i=T,\\ (p\bar{F}(q_{mP})[1-H(q_{mP})] - c_l(1+\ell_r a) - c_m(1+\ell_o a))q_{mP}, & \text{if } i=P.\end{cases}$$

Therefore, we have $q_{mT}^* = q_{mP}^*$, which solves $\frac{d\pi_{mP}(q_{mP})}{dq_{mP}} = 0$. As a result, $\pi_{mP}(q_{mP}^*) = \pi_{mT}(q_{mT}^*)$. The buyer's profit is: $\pi_{bi}(q_{mi}^*) = p\mathbb{E}[D\wedge q_{mi}^*] - p\, q_{mi}^*\bar{F}(q_{mi}^*)$. Since $q_{mT}^* = q_{mP}^*$, we have $\pi_{bT}(q_{mT}^*) = \pi_{bP}(q_{mP}^*)$. Similarly, we can show that $\pi_{lT}(q_{mT}^*) = \pi_{lP}(q_{mP}^*)$. Q.E.D.

Proof of Proposition 4: The buyer, the manufacturer, and the 3PL's profit functions under Case T and Case B can be written as follows.

$$\pi_{bi}(q_{bi}) = \begin{cases} p\mathbb{E}[D\wedge q_{bi}] - [w_{mi}(1+\ell_o a_b) + w_{li}(1+\ell_r a_b)]q_{bi}, & \text{if } i=T,\\ p\mathbb{E}[D\wedge q_{bi}] - w_{mi}(1+(\ell_o-\ell_g)a_b)q_{bi} - w_{li}q_{bi}(1+\ell_r a_b), & \text{if } i=B,\end{cases} \quad (1)$$

$$\pi_{mi}(w_{mi}) = \begin{cases} (w_{mi}-c_m)(1+\ell_o a_m)q_{bi}, & \text{if } i=T,\\ [w_{mi}(1+(\ell_o-\ell_g)a_m) - c_m(1+\ell_o a_m)]\,q_{bi}, & \text{if } i=B.\end{cases} \quad (2)$$

$$\pi_{li}(w_{li}) = \begin{cases} (w_{li}-c_l)(1+\ell_r a_l)q_{bi}, & \text{if } i=T,\\ (w_{li}-c_l)\,(1+\ell_r a_l)q_{bi}, & \text{if } i=B.\end{cases} \quad (3)$$

Case one: the 3PL leadership. We solve the game backward. First, we solve the optimal ordering level q_{bT}^* and q_{bB}^*, and get $w_{mT}(q_{bT}^*,w_{lT})$ and $w_{mB}(q_{bB}^*,w_{lB})$; Next, submitting $w_{mT}(q_{bT}^*,w_{lT})$ and $w_{mB}(q_{bB}^*,w_{lB})$ into manufacturer's profit function, we change $\pi_{mT}(w_{mT})$ and $\pi_{mB}(w_{mB})$ into $\pi_{mT}(q_{mT},w_{lT})$ and $\pi_{mB}(q_{mB},w_{lB})$, solve the optimal problems of q_{mT}^* and q_{mB}^* instead of w_{mT}^* and w_{mB}^*, and obtain $w_{lT}(q_{mT}^*)$ and $w_{lB}(q_{mB}^*)$; Finally, submitting $w_{lT}(q_{mT}^*)$ and $w_{lB}(q_{mB}^*)$ into 3PL's profit functions, we change the optimal problems of $\pi_{lT}(w_{lT})$ and $\pi_{lB}(w_{lB})$ into $\pi_{lT}(q_{lT})$ and $\pi_{lB}(q_{lB})$, and obtain q_{lT}^*(i.e., q_T^*) and q_{lB}^*(i.e., q_B^*) solving the following equations, respectively.

$$\begin{cases} G(p,q_T^*,1) - c_l(1+\ell_r a_b) - c_m(1+\ell_o a_b) = 0,\\ G(p,q_B^*,1) - c_l(1+\ell_r a_b) - c_m(1+\ell_o a_b)\frac{1+(\ell_o-\ell_g)a_b}{1+(\ell_o-\ell_g)a_m}\frac{1+\ell_o a_m}{1+\ell_o a_b} = 0.\end{cases}$$

As a result, the firms' profits can be written as:

$$\begin{cases} \pi_{bT}(q_T^*) = p\mathbb{E}[D\wedge q_T^*] - p\bar{F}(q_T^*)q_T^*,\\ \pi_{bB}(q_B^*) = p\mathbb{E}[D\wedge q_B^*] - p\bar{F}(q_B^*)q_P^*.\end{cases}$$

$$\begin{cases} \pi_{mT}(q_T^*) = p\frac{1+\ell_o a_m}{1+\ell_o a_b} q_T^* \bar{F}(q_T^*) H(q_T^*), \\ \pi_{mB}(q_B^*) = p\frac{1+(\ell_o-\ell_g)a_m}{1+(\ell_o-\ell_g)a_b} q_B^* \bar{F}(q_B^*) H(q_B^*). \end{cases}$$

$$\begin{cases} \pi_{lT}(q_T^*) = pq_T^* \frac{1+\ell_r a_l}{1+\ell_r a_b} \bar{F}(q_T^*)[1-H(q_T^*)] - \frac{(1+\ell_o a_b)(1+\ell_r a_l)}{1+\ell_r a_b} c_m q_T^* - (1+\ell_r a_l) c_l q_T^*, \\ \pi_{lB}(q_B^*) = pq_B^* \frac{1+\ell_r a_l}{1+\ell_r a_b} \bar{F}(q_B^*)[1-H(q_B^*)] - \frac{(1+\ell_o a_m)(1+\ell_r a_l)(1+(\ell_o-\ell_g)a_b)}{(1+\ell_r a_b)(1+(\ell_o-\ell_g)a_m)} c_m q_B^* - (1+\ell_r a_l) c_l q_B^*. \end{cases}$$

With the same cash opportunity cost, we have $q_T^* = q_B^*$, and then obtain that $\pi_{bT}(q_T^*) = \pi_{bB}(q_B^*)$, $\pi_{mT}(q_T^*) = \pi_{mB}(q_B^*)$, and $\pi_{lT}(q_T^*) = \pi_{lB}(q_B^*)$.

Case two: the manufacturer leadership. The profit functions of players are the same as those in Case one. Similar to Case one, we have $q_T^* = q_B^*$ if the cash opportunity cost is equal, and get $\pi_{bT}(q_T^*) = \pi_{bB}(q_B^*)$, $\pi_{lT}(q_T^*) = \pi_{lB}(q_B^*)$, and $\pi_{mT}(q_T^*) = \pi_{mB}(q_B^*)$. Q.E.D.

Proof of Lemma 4: We solve the game in its general form backward. For the buyer, solving $\frac{d\pi_{bi}(q_{bi})}{dq_{bi}} = 0$ in Eq. (3), we have the buyer's optimal ordering solution q_{bi}^*. Since q_{bi}^* monotonically decreases with w_{mi}, there exists a one-by-one mapping between q_{bi}^* and w_{mi}, that is,

$$w_{mi}(q_{bi}^*) = \begin{cases} \frac{p\bar{F}(q_{bi}^*) - w_{li}(1+\ell_r a_b)}{1+\ell_o a_b}, & i = T, \\ \frac{p\bar{F}(q_{bi}^*) - w_{li}(1+\ell_r a_b)}{1+\ell_r a_b}, & i = P. \end{cases}$$

Submitting $w_{mi}(q_{bi}^*)$ into the manufacturer's profit function, and solving $\frac{d\pi_{mi}(q_{mi})}{dq_{mi}} = 0$, we then have

$$w_{li}(q_{mi}^*) = \begin{cases} \frac{p\bar{F}(q_{mi}^*)[1-H(q_{mi}^*)] - c_m(1+\ell_o a_b)}{1+\ell_r a_b}, & i = T, \\ \frac{p\bar{F}(q_{mi}^*)[1-H(q_{mi}^*)]}{1+\ell_r a_b} - \frac{c_m(1+\ell_o a_m)}{1+(\ell_o-\ell_g)a_m}, & i = P. \end{cases}$$

For P, submitting $w_{lP}(q_{mi}^*)$ into $w_{mP}(q_{mi}^*)$, we have, $w_{mP}(q_{mP}^*) = \frac{p\bar{F}(q_{mP}^*)H(q_{mP}^*)}{1+\ell_r a_b} + \frac{c_m(1+\ell_o a_m)}{1+(\ell_o-\ell_g)a_m}$. Then, submitting $w_{li}(q_{mi}^*)$ and $w_{mP}(q_{mi}^*)$ into 3PL's profit function, we have

$$\pi_{li}^l(q_{li}) = \begin{cases} p\ q_{li} \frac{1+\ell_r a_l}{1+\ell_r a_b} \bar{F}(q_{li})[1-H(q_{li})] - \frac{(1+\ell_o a_b)(1+\ell_r a_l)}{1+\ell_r a_b} c_m q_i - c_l(1+\ell_r a_l) q_{li}, & i = T, \\ p\ q_{li} \frac{1+\ell_r a_l}{1+\ell_r a_b} \bar{F}(q_{li})[1 - \frac{1+(\ell_o-\ell_g)a_l}{1+\ell_r a_l} H(q_{li})] - \frac{(1+\ell_o a_m)[1+(\ell_o-\ell_g)a_l]}{1+(\ell_o-\ell_g)a_m} c_m q_i - c_l(1+\ell_r a_l) q_{li}, & i = P. \end{cases}$$

Define $\eta_m = \frac{1+\ell_o a_m}{1+(\ell_o-\ell_g)a_m}$, $\eta_l = \frac{1+(\ell_o-\ell_g)a_l}{1+\ell_r a_l}$ and $\eta_b = \frac{1+\ell_r a_b}{1+\ell_o a_b}$. Taking the first derivative of $\pi_{li}(q_{li})$ w.r.t. q_{li}, and letting $\frac{d\pi_{li}(q_{li})}{dq_{li}} = 0$, we can solve the 3PL's optimal solution q_{lT}^*(i.e., q_T^*) and q_{lP}^* (i.e., q_P^*) by the following equations.

$$\begin{cases} G(p, q_T^*, 1) - c_l(1+\ell_r a_b) - c_m(1+\ell_o a_b) = 0, \\ G(p, q_P^*, \eta_l) - c_l(1+\ell_r a_b) - c_m(1+\ell_o a_b)\eta_m\eta_l\eta_b = 0. \end{cases} \quad (4)$$

Q.E.D.

Proof of Proposition 5: We first show that, if $a_m \geq a_l$, $\eta_m\eta_l$ increases in ℓ_g and vise versa. We obtain that $\frac{d\eta_m\eta_l}{d\ell_g} = \frac{1+\ell_o a_m}{(1+\ell_r a_l)[1+(\ell_o-\ell_g)a_m]^2}(a_m - a_l)$. Therefore, if $a_m \geq a_l$, we have $\frac{d\eta_m\eta_l}{d\ell_g} \geq 0$, and vise versa.

Part (i): *First:* $\pi_{lP} \geq \pi_{lT}$ *iff* $a_b \geq a_m$. Given $\ell_g = \ell_o - \ell_r$, $\eta_l = 1$. In Eq. (4), we observe $\eta_m\eta_b \leq 1$ when $a_b \geq a_m$,, which suggests $q_P^* \geq q_T^*$ because $p\bar{F}(q)[(1-H(q))^2 - qH'(q)]$ decreases with q. Consequently,

$$\begin{cases} \pi_{lT}(q_T^*) = pq_T^* \frac{1+\ell_r a_l}{1+\ell_r a_b} \bar{F}(q_T^*)[1-H(q_T^*)] - \frac{(1+\ell_o a_b)(1+\ell_r a_l)}{1+\ell_r a_b} c_m q_T^* - (1+\ell_r a_l) c_l q_T^*, \\ \pi_{lP}(q_P^*) = pq_P^* \frac{1+\ell_r a_l}{1+\ell_r a_b} \bar{F}(q_P^*)[1-H(q_P^*)] - \frac{(1+\ell_g a_m)(1+\ell_r a_l)}{1+\ell_r a_m} c_m q_P^* - (1+\ell_r a_l) c_l q_P^*. \end{cases} \quad (5)$$

Since $a_b \geq a_m$, we have $\frac{(1+\ell_r a_l)(1+\ell_o a_b)}{1+\ell_r a_b} \geq \frac{(1+\ell_r a_l)(1+\ell_o a_m)}{1+\ell_r a_m}$, then we show $\pi_{lT}(q_T^*) \leq \pi_{lP}(q_T^*)$.

Second: $\pi_{mP} \geq \pi_{mT}$ *if* $a_b \geq a_m$.

Submitting q_{lT}^* and q_{lP}^* into the manufacture's profit, we have,

$$\begin{cases} \pi_{mT}(q_T^*) = p\frac{1+\ell_o a_m}{1+\ell_o a_b} q_T^* \bar{F}(q_T^*) H(q_T^*), \\ \pi_{mP}(q_P^*) = p\frac{1+\ell_r a_m}{1+\ell_r a_b} q_P^* \bar{F}(q_P^*) H(q_P^*). \end{cases} \quad (6)$$

Following our proof in Proposition 1, in which we show that $\bar{F}(q)H(q)$ increases in q, we have $q_T^*\bar{F}(q_T^*)H(q_T^*) \leq q_P^*\bar{F}(q_P^*)H(q_P^*)$ since $q_T^* \leq q_P^*$. Because $a_b \geq a_m$ and $\ell_o \geq \ell_r$, we have $\frac{1+\ell_o a_m}{1+\ell_o a_b} \leq$

$\frac{1+\ell_r a_m}{1+\ell_r a_b}$. Consequently, if $a_b \geq a_m$, $\pi_{mT}(q_T^*) \leq \pi_{mP}(q_P^*)$.
Third: $\pi_{bP} \geq \pi_{bT}$ *if* $a_b \geq a_m$.

Similarly, the buyer's profit can be rewritten as

$$\begin{cases} \pi_{bT}(q_T^*) = p\mathbb{E}[D \wedge q_T^*] - p\bar{F}(q_T^*)q_T^*, \\ \pi_{bP}(q_P^*) = p\mathbb{E}[D \wedge q_P^*] - p\bar{F}(q_P^*)q_P^*. \end{cases} \quad (7)$$

We can show $\frac{d\pi_{bi}(q_{li}^*)}{dq_{li}^*} = p\bar{F}(q_{li}^*)H(q_{li}^*) \geq 0$. Therefore, $\pi_{bT}(q_T^*) \leq \pi_{bP}(q_P^*)$ given $q_T^* \leq q_P^*$ if $a_b \geq a_m$.

Based on the above analysis, we can infer that $q_{lT}^* = q_{lP}^*$ if $a_b = a_m$ regardless of the value of a_l. Thus, all firms are indifferent of P and T if $a_b = a_m$. For tie-breaking, our analysis assumes P is taken when firms are indifferent of both cases.

Part (ii):

If $\ell_g > \ell_s$, we have $\eta_l < 1$. Furthermore, when $a_l > a_m$, $\eta_m\eta_l$ decreases in ℓ_g. As we show in Part (i), $\ell_g = \ell_s$, $\eta_m\eta_b < 1$ if $a_b \geq a_m$. Then, we can show that if $a_m \leq a_l, a_b$, $\eta_m\eta_l\eta_b < 1$, we have $q_P^* > q_T^*$. In Eq. (5) and (6), if $a_b \geq a_m$ and $q_P^* > q_T^*$, we have $\pi_{lP}(q_P^*) > \pi_{lT}(q_T^*)$ and $\pi_{mP}(q_P^*) > \pi_{mT}(q_T^*)$. In Eq. (7), if $q_P^* > q_T^*$, we have $\pi_{bP}(q_P^*) > \pi_{bT}(q_T^*)$. As a result, the proof of Part (ii) is well done.

Part (iii):

Since $a_m > a_l$, we have $\eta_m\eta_l$ increases in ℓ_g. If $\ell_g < \ell_s$, we have $\eta_m\eta_l < 1$. And we already have $\eta_b < 1$. Then, we obtain $\eta_m\eta_l\eta_b < 1$.

According to Eq. (4), we have q_T^* and q_P^*, further q_P^* decreases in η_l. If $\ell_g < \ell_s$, we obtain η_l increases in a_l. Let $\hat{a}_l$ solve $q_T^*(a_l) = q_P^*(a_l)$. Then, we have $q_P^* \geq q_T^*$ if $a_l \leq \hat{a}_l$.

Similar to the proof in Part (ii), we can show that all firms prefer P to T if $a_b > a_m > a_l$ and even $a_b \geq \hat{a}_b(\ell_g)$. Q.E.D.

Proof of Lemma 5: Solving Eq. (3), we have $w_{lT} = \frac{p\bar{F}(q) - w_{mT}(1+\ell_o a_b)}{1+\ell_r a_b}$, and $w_{lP} = \frac{p\bar{F}(q) - w_{mP}(1+\ell_r a_b)}{1+\ell_r a_b}$. Submitting w_{lT} and w_{lP} into Eq. (5), we have $w_{mT} = \frac{p\bar{F}(q)[1-H(q)] - c_l(1+\ell_r a_b)}{1+\ell_o a_b}$ and

$$w_{mP} = \frac{p\bar{F}(q)[1-H(q)](1+\ell_r a_l)}{(1+\ell_r a_b)[1+(\ell_o-\ell_g)a_l]} - c_l\frac{1+\ell_r a_l}{1+(\ell_o-\ell_g)a_l}.$$

Submitting w_{mT} and w_{mP} into Eq. (4), we can solve q_{mT}^* and q_{mP}^* as described in Lemma 5. Q.E.D.

Proof of Proposition 6: Part (i): From Lemma 5, we know q_{mT}^* (i.e., q_T^*) and q_{mP}^* (i.e., q_P^*) solving the following equations, respectively,

$$\begin{cases} G(p, q_T^*, 1) - c_l(1+\ell_r a_b) - c_m(1+\ell_o a_b) = 0 \\ G(p, q_P^*, 1) - c_l(1+\ell_r a_b) - \eta_m\eta_l\eta_b c_m(1+\ell_o a_b) = 0. \end{cases} \quad (8)$$

As we know, $G(p, q, 1)$ decreases with q. Thus, if $\eta_m\eta_l\eta_b \leq 1$, we immediately have $q_P^* \geq q_T^*$.

Part (ii): We first prove that $\pi_{bT}(q_T^*) \leq \pi_{bP}(q_P^*)$, $\pi_{lT}(q_T^*) \leq \pi_{lP}(q_P^*)$, and $\pi_{mT}(q_T^*) \leq \pi_{mP}(q_P^*)$, when $q_T^* \leq q_P^*$. Similar to the proof of Lemma 5, we submit $w_{lT}(q_T^*)$, $w_{mT}(q_T^*)$, $w_{lP}(q_P^*)$, and $w_{mP}(q_P^*)$ into all firms' profit functions. And we obtain $\pi_{bT}(q_T^*) = p\mathbb{E}[D \wedge q_T^*] - p\bar{F}(q_T^*)q_T^*$ and $\pi_{bP}(q_P^*) = p\mathbb{E}[D \wedge q_P^*] - p\bar{F}(q_P^*)q_P^*$; $\pi_{lT}(q_T^*) = p\frac{1+\ell_r a_l}{1+\ell_r a_b}\bar{F}(q_T^*)H(q_T^*)q_T^*$ and $\pi_{lP}(q_P^*) = p\frac{1+\ell_r a_l}{1+\ell_r a_b}\bar{F}(q_P^*)H(q_P^*)q_P^*$; $\pi_{mT}(q_T^*) = \frac{1+\ell_o a_m}{1+\ell_o a_b}[p\bar{F}(q_T^*)(1-H(q_T^*)) - c_l(1+\ell_r a_b)] - c_m(1+\ell_o a_m)q_T^*$ and

$$\pi_{mP}(q_P^*) = \frac{1+\ell_r a_l}{1+\ell_r a_b}\frac{1+(\ell_o-\ell_g)a_m}{1+(\ell_o-\ell_g)a_l}[p\bar{F}(q_P^*)(1-H(q_P^*)) - c_l(1+\ell_r a_b)] - c_m q_P^*(1+\ell_o a_m).$$

We can show that $\frac{d\pi_{bi}(q)}{dq} > 0$ and $\frac{d\pi_{li}(q)}{dq} > 0$. Therefore, $\pi_{bT}(q_{mT}^*) \leq \pi_{bP}(q_{mP}^*)$ and $\pi_{lT}(q_{mT}^*) \leq \pi_{lP}(q_{mP}^*)$ when $q_{mT}^* \leq q_{mP}^*$.

Consider the manufacturer. The inequality of $1+\ell_o a_b \geq \frac{(1+\ell_o a_m)(1+\ell_r a_b)[1+(\ell_o-\ell_g)a_l]}{(1+\ell_r a_l)[1+(\ell_o-\ell_g)a_m]}$ (i.e., $\eta_m\eta_l\eta_b \leq 1$) is equivalent to $\frac{1+\ell_o a_m}{1+\ell_o a_b} \leq \frac{1+\ell_r a_l}{1+\ell_r a_b}\frac{1+(\ell_o-\ell_g)a_m}{1+(\ell_o-\ell_g)a_l}$. Therefore, for any given q, we have $\pi_{mP}(q) \geq \pi_{mT}(q)$. Let $q = q_T^*$, we obtain $\pi_{mP}(q_T^*) \geq \pi_{mT}(q_T^*)$. Given $\eta_m\eta_l\eta_b \leq 1$, we also have $q_P^* \geq q_T^*$. Since q_P^* is the optimal solution for the manufacturer in P, we must have

$\pi_{mP}(q_P^*) \geq \pi_{mP}(q_T^*)$. Therefore, the manufacturer is better off as long as $\eta_m\eta_l\eta_b \leq 1$, because $\pi_{mP}(q_P^*) \geq \pi_{mP}(q_T^*) \geq \pi_{mT}(q_T^*)$.

If $\eta_b \geq \frac{a_m}{a_l}\delta$ and $a_l \geq a_b$, we can rewrite $\eta_m\eta_l\eta_b \leq 1$ as $\ell_g \geq \frac{(1+\ell_o a_m)(a_l-a_b)(\ell_o-\ell_r)}{(1+\ell_o a_m)(a_l-a_b)+(1+\ell_r a_l)(a_b-a_m)} = \frac{1}{1+\beta}\ell_s = \xi\ell_s$. Therefore, all firms prefer P to T, as long as $\ell_g \in [\xi\ell_s, \ell_o]$. Here, without loss of generality, we assume P is preferred if there is a tie between P and T.

Otherwise if $\eta_b < \frac{a_m}{a_l}\delta$ and $a_l < a_b$, we can rewrite $\eta_m\eta_l\eta_b \leq 1$ as $\ell_g \leq \frac{(1+\ell_o a_m)(a_b-a_l)(\ell_o-\ell_r)}{(1+\ell_o a_m)(a_b-a_l)+(1+\ell_r a_l)(a_m-a_l)} = \frac{1}{1+\beta}\ell_s = \xi\ell_s$. $\ell_g \leq \xi\ell_s$. That is, all firms prefer P to T, as long as $\ell_g \in [0, \xi\ell_s]$. Q.E.D.

Proof of Corollary 2: Corollary 2 is a special case of Proposition 6 and can be obtained immediately by plugging the corresponding conditions. Thus, due to limited space, the proof is omitted. Q.E.D.

Proof of Proposition 7: The proof can be obtained from comparing Eq. (4) with Eq. (8), then thus omitted because of limited space. Q.E.D.

Proof of Proposition 8: Part (1): When the buyer borrows capital from the bank, we have

$$\pi_{bi}(q_{bi}) = \begin{cases} p\mathbb{E}[D \wedge q_{bi}] - [w_{mi}(1+\ell_o a_b) + w_{li}(1+\ell_r a_b)]q_{bi}(1+r_f), & \text{if } i = T, \\ p\mathbb{E}[D \wedge q_{bi}] - [(w_{mi}+w_{li})(1+\ell_r a_b)]q_{bi}(1+r_f), & \text{if } i = P. \end{cases} \tag{9}$$

Taking derivative of q_{bi} in Eq.(9), we have $w_{mT} = \frac{p\bar{F}(q) - w_{lT}(1+\ell_r a_b)(1+r_f)}{(1+\ell_o a_b)(1+r_f)}$ and $w_{mP} = \frac{p\bar{F}(q) - w_{lP}(1+\ell_r a_b)(1+r_f)}{(1+\ell_r a_b)(1+r_f)}$. Submitting w_{mT} and w_{mP} into manufacturer's profit function, and taking derivative of q_{mi}, we have $w_{lT} = \frac{\frac{p}{1+r_f}\bar{F}(q)[1-H(q)] - c_m(1+\ell_o a_m)}{1+\ell_r a_b}$, $w_{lP} = \frac{\frac{p}{1+r_f}\frac{1+(\ell_o-\ell_g)a_m}{1+\ell_r a_b}\bar{F}(q)[1-H(q)] - c_m(1+\ell_o a_m)}{1+(\ell_o-\ell_g)a_m}$, and $w_{mP} = \frac{p\bar{F}(q)H(q)}{(1+r_f)(1+\ell_r a_b)} + \frac{c_m(1+\ell_o a_m)}{1+(\ell_o-\ell_g)a_m}$. Let $a_i = a$, where $i = b, m, l$. Then, we submit w_{lT}, w_{lP} and w_{mP} into 3PL's profit function, and rewrite it as the following. $\pi_{lT} = \frac{p}{1+r_f}\bar{F}(q_{lT})[1 - H(q_{lT})]q_{lT} - c_m(1+\ell_o a)q_{lT} - c_l(1+\ell_r a)q_{lT}$, and $\pi_{lP} = \frac{p}{1+r_f}\bar{F}(q_{lP})[1 - \frac{1+(\ell_o-\ell_g)a}{1+\ell_r a}H(q_{lP})]q_{lP} - c_m(1+\ell_o a)q_{lP} - c_l(1+\ell_r a)q_{lP}$. Solving $\frac{d\pi_{lT}}{dq_{lT}} = 0$ and $\frac{d\pi_{lP}}{dq_{lP}} = 0$ results in Proposition 8.

Part (2): For any given r_f, q_i^* solves the following equations:

$$\begin{cases} G(\frac{p}{1+r_f}, q_i^*, 1) = c_m(1+\ell_o a) + c_l(1+\ell_r a), & \text{if } i = T, \\ G(\frac{p}{1+r_f}, q_i^*, \eta_l) = c_m(1+\ell_o a) + c_l(1+\ell_r a), & \text{if } i = P. \end{cases}$$

The RHS in the above equations is fixed for any given ℓ_s and ℓ_g. The LHSs decrease with q_i^*. If r_f increases, $\frac{p}{1+r_f}$ decreases. To keep the equations hold, both $G(\frac{p}{1+r_f}, q_i^*, 1)$ and $G(\frac{p}{1+r_f}, q_i^*, \eta_l)$ must increase. Consequently q_T^* and q_P^* must decrease with r_f.

Next, we prove that $\frac{d\pi_{li}(q_i^*)}{dr_f} < 0$, $\frac{d\pi_{mi}(q_i^*)}{dr_f} < 0$, and $\frac{d\pi_{bi}(q_i^*)}{dr_f} < 0$. The buyer's profit is $\pi_{bi}(q_i^*) = p[\mathbb{E}[D \wedge q_i^*] - \bar{F}(q_i^*)q_i^*]$. Since $\frac{d\pi_{bi}(q_i^*)}{dq_i^*} > 0$ and $\frac{dq_i^*}{dr_f} < 0$, we have $\frac{d\pi_{bi}(q_i^*)}{dr_f} < 0$. The manufacturer's profit functions are $\pi_{mT}(q_T^*) = \frac{p}{1+r_f}q_T^*\bar{F}(q_T^*)H(q_T^*)$ and $\pi_{mP}(q_P^*) = \frac{p}{1+r_f}\eta_l q_P^*\bar{F}(q_P^*)H(q_P^*)$, in T and P, respectively. We have $q\bar{F}(q)H(q)$ increases with q, and then $q_i^*\bar{F}(q_i^*)H(q_i^*)$ decreases with r_f, because $\frac{dq_i^*}{dr_f} < 0$. Because $\frac{p}{1+r_f}$ decreases with r_f, both $\pi_{mT}(q_T^*)$ and $\pi_{mP}(q_P^*)$ decrease with r_f. For the 3PL, we have $\frac{d\pi_{li}(q_i^*)}{dr_f} = \frac{\partial\pi_{li}(q_i^*)}{\partial q_i^*}\frac{dq_i^*}{dr_f} + \frac{\partial\pi_{li}(q_i^*)}{\partial r_f} = \frac{\partial\pi_{li}(q_i^*)}{\partial r_f}$. We then obtain $\frac{d\pi_{lT}(q_T^*)}{dr_f} = -\frac{p}{(1+r_f)^2}\bar{F}(q_T^*)[1-H(q_T^*)]q_T^* < 0$ and $\frac{d\pi_{lP}(q_P^*)}{dr_f} = -\frac{p}{(1+r_f)^2}\bar{F}(q_P^*)[1-\frac{1+(\ell_o-\ell_g)a}{1+\ell_r a}H(q_P^*)]q_P^* < 0$. Q.E.D.

Proof of Proposition 9: Part (i): Similar to the proof in Lemma 3, the first order conditions of Eq. (19) in T and P, respectively, give us

$$\frac{p}{n}\bar{F}(q_{lT})[(1 - H(q_{lT}))(n - H(q_{lT})) - q_{lT}H'(q_{lT})] = c_l' + c_m', \tag{10}$$

$$\frac{p}{n}\bar{F}(q_{lP})[(1 - \eta_l H(q_{lP}))(n - H(q_{lP})) - \eta_l q_{lP}H'(q_{lP})] = c_l' + c_m'. \tag{11}$$

For a fixed q_{lT}, as n increases, $(p - \frac{p}{n}H(q_{lT}))$ and $-\frac{p}{n}q_{lT}H'(q_{bT})$ increases, and then $\bar{F}(q_{lT})[(1 -$

$H(q_{lT}))(p - \frac{p}{n}H(q_{lT})) - \frac{p}{n}q_{lT}H'(q_{lT})]$ increases. Given that the right hand sides are constant, as n increases, q_{li} must increase to satisfy the first equation. The same logic applies to the second equation. Then, we can show that $\frac{\partial q_T^*}{\partial n} > 0$ and $\frac{\partial q_P^*}{\partial n} > 0$.

The proof is similar to that of Proposition 1. Given any n, the firms' preference is independent of n but hinges on the values of ℓ_g and ℓ_s. We have $q_{lP}^* \geq q_{lT}^*$ iff $\ell_g \geq \ell_s$; otherwise $q_{lP}^* < q_{lT}^*$.

Part (ii): We use the contradiction approach to prove this. Assume that $\frac{\partial q_P^*}{\partial n} \leq \frac{\partial q_T^*}{\partial n}$ if $\ell_g \geq \ell_s$. Let $G1(n) = \frac{p}{n}\bar{F}(q_{lT})[(1 - H(q_{lT}))(n - H(q_{lT})) - q_{lT}H'(q_{lT})]$ in Eq. (10), and $G2(n) = \frac{p}{n}\bar{F}(q_{lP})[(1 - \eta_l H(q_{lP}))(n - H(q_{lP})) - \eta_l q_{lP}H'(q_{lP})]$ in Eq. (11). And $G1(n)$ and $G2(n)$ increase in n. Since $\frac{\partial q_P^*}{\partial n} \leq \frac{\partial q_T^*}{\partial n}$, we should find an n equal to $\tilde{n}$ satisfying $q_T^*(\tilde{n}) > q_P^*(\tilde{n})$. If $\ell_g \geq \ell_s$, we have $\eta_l \leq 1$. Then for any given n, we have $G1(n) \leq G2(n)$. Consequently, from Eq. (10) and (11), we have $q_T^* \leq q_P^*$, which contradicts the previous result. Therefore, we have $\frac{\partial q_P^*}{\partial n} \geq \frac{\partial q_T^*}{\partial n}$. Q.E.D.

Proof of Proposition 10: For limited space, we focus only on the 3PL leadership game. The result is the same for manufacturer leadership. According to Lemma 3, the following FOC conditions must hold.

$$\begin{cases} G(p, q_T^*, 1) = c_m(1 + \ell_o a) + c_l(1 + \ell_r a), \\ G(p, q_P^*, \eta_l) = c_m(1 + \ell_o a) + c_l(1 + \ell_r a). \end{cases}$$

As proved in Lemma 3, for any given ℓ_g, LHS decreases with q in the above equations. Note that $\ell_o = L_{si} + \ell_r$. When L_{si} increases, RHS increases in both Models T and P. To keep the equations hold, LHS must increase and, therefore, q_i^* must decrease with L_{si}. Meanwhile, for the buyer, its profit is $\pi_{bi}(q_i^*) = p[\mathbb{E}[D \wedge q_i^*] - q_i^*\bar{F}(q_i^*)]$. It can be proved that $\frac{d\pi_{bi}(q_i^*)}{dq_i^*} > 0$. Since $\frac{dq_i^*}{dL_{si}} < 0$, we have $\frac{d\pi_{bi}(q_i^*)}{dL_{si}} < 0$. For the 3PL, its profit in Model T is $\pi_{lT}(q_T) = p\bar{F}(q_T)[1 - H(q_T)]q_T - c_m(1 + \ell_o a)q_T - c_l(1 + \ell_r)q_T$. $\frac{d\pi_{lT}(q_T^*)}{dL_{sT}} = \frac{\partial \pi_{lT}(q_T^*)}{\partial q_T^*}\frac{dq_T^*}{dL_{sT}} + \frac{\partial \pi_{lT}(q_T^*)}{\partial L_{sT}}$. From Lemma 3, we have $\frac{\partial \pi_{lT}(q_T^*)}{\partial q_T^*} = 0$, and $\frac{d\pi_{lT}(q_T^*)}{dL_{sT}} = \frac{\partial \pi_{lT}(q_T^*)}{\partial L_{sT}} = -c_m q_T^* a < 0$. The 3PL's profit in Model P is $\pi_{lP}(q_P) = p\bar{F}(q_P)[1 - \eta_l H(q_P)]q_P - c_m(1 + \ell_o a)q_P - c_l(1 + \ell_r)q_P$. Similarly, we have $\frac{d\pi_{lT}(q_P^*)}{dL_{sP}} = -\frac{pa}{1+\ell_r a}q_P^*\bar{F}(q_P^*)H(q_P^*) - c_m q_P^* a < 0$.

Because both Π_{bi} and Π_{li}, $i = T, P$, decrease with L_{si}, the Nash bargaining product for any given $\theta_b \in [0, 1]$ decreases with L_{si}. Therefore, the optimal Nash bargaining solution is achieved at the lower bound $L_{si}^* = \ell_s$, $i = T, P$. Q.E.D.

Proof of Proposition 11: The proof is provided conceptually right after the proposition. Q.E.D.

Proof of Corollary 3: Let $\tilde{\ell} = \frac{(1+\ell_o a_m a_b)(\ell_o - \ell_r)}{a_m(1+\ell_o a_b)}$ and $\check{\ell}$ solve $\pi_{lT}(q_T^*) = \pi_{lP}(q_P^*, \ell_g)$. If $\ell_g \leq \tilde{\ell}$, we have $\frac{1+(\ell_o - \ell_g)a_m}{1+\ell_r a_b} \geq \frac{1+\ell_o a_m}{1+\ell_o a_m}$. From Lemma 2, we obtain $q_P^* \geq q_T^*$. To prove $T \prec P$, we need to prove $\pi_{bT}(q_T^*) \leq \pi_{bP}(q_P^*)$, $\pi_{mT}(q_T^*) \leq \pi_{mP}(q_P^*)$, and $\pi_{lT}(q_T^*) \leq \pi_{lP}(q_P^*)$. Submitting q_T^* and q_P^* into buyer's profit function, we have $\pi_{bT}(q_T^*) = p\min[D \wedge q_T^*] - p\bar{F}(q_T^*)q_T^*$ and $\pi_{bP}(q_P^*) = p\min[D \wedge q_P^*] - p\bar{F}(q_P^*)q_P^*$. Then we have $\pi_{bT}(q_T^*) \leq \pi_{bP}(q_P^*)$ if $q_T^* \leq q_P^*$ or $\ell_g \leq \tilde{\ell}$. Similarly, $\pi_{mT}(q_T^*) \leq \pi_{mP}(q_P^*)$ when $\ell_g \leq \tilde{\ell}$.

Since $\frac{d\pi_{lP}(q_P^*)}{d\ell_g} = \frac{\partial \pi_{lP}(q_P^*)}{\partial q_P^*}\frac{dq_P^*}{d\ell_g} + \frac{\partial \pi_{lP}(q_P^*)}{d\ell_g}$, we have $\frac{d\pi_{lP}(q_P^*)}{d\ell_g} = \{(\frac{p\bar{F}(q_P^*)}{1+\ell_r a_l} - w_l)q_P^* + [(w_l - c_l)\ell_r + \frac{c_m(1+\ell_o a_m)(\ell_g - \ell_s)}{1+(\ell_o - \ell_g)a_m}]q_P^{*\prime}\}a_l + (w_l - c_l)q_P^{*\prime}$, where $q_P^{*\prime} = \frac{dq_P^*}{d\ell_g} < 0$. Let $A(\ell_g) = (\frac{p\bar{F}(q_P^*)}{1+\ell_r a_l} - w_l)q_P^* + [(w_l - c_l)\ell_r + \frac{c_m(1+\ell_o a_m)(\ell_g - \ell_s)}{1+(\ell_o - \ell_g)a_m}]q_P^{*\prime}$, we get $\frac{d\pi_{lP}(q_P^*)}{d\ell_g} = A(\ell_g)a_l + (w_l - c_l)q_P^{*\prime}$, then $\frac{d\pi_{lP}(q_P^*)}{d\ell_g}$ is a linear function of a_l. If $a_l < \frac{(c_l - w_l)q_P^{*\prime}}{A(\ell_g)}$, $\pi_{lP}(q_P^*)$ decreases in ℓ_g. Then $\ell_g \leq [\check{\ell}, \tilde{\ell}]$, $T \prec P$. Otherwise, $\pi_{lP}(q_P^*)$ increases in ℓ_g. If $\check{\ell} < \tilde{\ell}$, we have $T \prec P$. Q.E.D.