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Impact of In-store Promotion and Spillover Effect onPrivate Label Introduction

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Abstract

This paper investigates the impact of in-store promotion and its spillover effect on private label introduction. We study different retail supply chain scenarios where the retailer carrying a national brand may introduce its own private label product and promote either the national brand or the private label inside the store. The in-store promotion on one product has a positive spillover effect on the other product. Without in-store promotion and spillover effect, the conventional wisdom indicates that, in a retail supply chain, the national brand manufacturer will be negatively impacted by the introduction of a private label product. With in-store promotion and spillover effect, however, the national brand manufacturer can actually benefit from the private label introduction. When the spillover from national brand to private label is high, the retailer prefers to promote the national brand product. When the spillover from private label to national brand is high, promoting the private label product can also benefit the national brand manufacturer. With symmetric spillover rate, the national brand manufacturer can still benefit from the private label introduction, as long as the retailer promotes the national brand product, the horizontal competition is not intense or the private label product quality is sufficiently low.

Key words: private label; in-store promotion; spillover effect; game theory

1 Introduction

The penetration of private labels has been a notable trend in the retail industry. For example, Wal-Mart has introduced a wide variety of products under its own private label *Great Value* to challenge the dominating position of national brands. According to the Private Label Manufacturer's Association, in 2014 one of every four products sold was a private label, \$1 of every \$5 of sales was generated by private labels, and the annual revenue of private labels was \$112 billion. Thus, private labels have attracted growing attentions from both retailers and manufacturers.

Private label introduction benefits the retailer in many ways. The direct benefit is that private label products bring higher gross margin by diminishing double marginalization (Narasimhan and Wilcox 1998, Sachon and Martinez 2009). Private labels also benefit retailers by increasing their bargain power with manufacturers (Pauwels and Srinivasan 2004, Groznik and Heese 2010), increasing customer loyalty to the retailer (Vahie and Paswan 2006), and enhancing a unique store image for the retailer (Ailawadi and Keller 2004).

However, national brand manufacturers are at odds with the private label introductions. On one hand, it is considered conventional wisdom that the introduction of a private label encroaches on the national brand's existing market share. A number of theoretical studies show that the introduction of a private label forces the manufacturer to lower the wholesale price, and thus hurts the manufacturer's profit (please refer to Choi and Coughlan (2006), Mills (1995; 1999), Raju et al. (1995), Sayman et al. (2002), Sayman and Raju (2004) and the references therein). On the other hand, a few empirical studies show that the wholesale price of a national brand product might increase and the manufacturer might benefit when a competing private label debuts (Ailawadi and Harlam (2004), Chintagunta et al. (2002), Pauwels and Srinivasan (2004)). However, the latter argument is rarely backed by any theoretical literature, except Ru et al. (2015), who show that in a retailer-led Stackelberg game with markup pricing, the retailer may lower the retail markup of the national brand when a competing private label is introduced, and the wholesale price and the demand of the national brand may both increase.

In this paper, we show that presence of in-store promotion and its spillover may alter a national brand manufacturer's preference of the private label introduction. In-store promotion is a norm nowadays for almost all major retailers, see, e.g., Shaffer and Zettelmeyer (2009), Dukes and Liu

(2010), Schultz and Block (2011), and Nordfalt and Lange (2013), and it is typically accompanied with the spillover effect. In a market with multiple brands of similar products, when one product gets promoted, demand for other products will likely be impacted. The impact could be positive if the promotion is "cooperative," or be negative if the promotion is "predatory" (Piga 2000). Inside the same store, in-store promotional efforts exerted by the retailer to increase the sales for a particular product, such as prominent locations, eye-level shelf space, special decoration or lighting, and in-store media (Dukes and Liu 2010), are typically cooperative, such that all products in its store can have enhanced demand from the promotion.

The spillover between two products can be either symmetric or asymmetric, according to Cellini et al. (2008), Giannakas et al. (2012), Lei et al. (2008) and Norman et al. (2008). For example, the spillover rates could be symmetric if both products are placed in the same shelf level, adjacent to each other. Consumers will have the same chance of finding both products after viewing the promotion advertisement through the in-store media. On the other hand, the spillover rates could be asymmetric, for example, if one product is located at the eye-level shelf space while the other at the bottom level (Valenzuela and Raghubir 2015). If the distance between two products is small, the spillover rate is high; otherwise, the spillover rate is low.

To explore the impact of in-store promotion and its spillover rate on the national brand manufacturer's preference of the private label introduction, we establish a Stackelberg game framework to examine the competition between a private label product and a national brand product carried by a common retailer. The national brand manufacturer decides the wholesale price of its product. The retailer decides the retail prices of the two products, the in-store promotional effort level, and which product to promote.

First, if there is no spillover effect, we confirm the conventional wisdom that the introduction of a private label is not preferable for the national brand manufacturer, even though there is an in-store promotion. However, if the spillover effect is symmetric and the private label has the same quality as the national brand, the national brand manufacturer can actually benefit from the introduction of a private label product if the spillover effect is high and the product substitutability is low. This result occurs because, while the competition from the private label product is not significant, the retailer will exert a high promotional effort to increase the sales. Nevertheless, with the symmetric spillover effect, the national brand (NB) manufacturer and the retailer always conflict on the in-store promotion type: the retailer prefers to promote the private label (PL) product other than the NB product, whereas NB manufacturer prefers otherwise.

Second, conditional on symmetric quality, we find that the spillover effect asymmetry has significant impact on both firms' preferences. When the spillover from national brand to private label is significantly higher than that from private label to the national brand, both firms prefer to promote the national brand product to utilize the benefit of more efficient promotional effort. Nevertheless, when the spillover from national brand to private label is significantly lower than that from private label to the national brand, both firms prefer to promote the private label product. This demonstrates that, if the retailer can maneuver the spillover effect asymmetrically, the national brand manufacturer would actually prefer to let the retailer introduce and promote the private label.

Third, assuming symmetric spillover rate, we find that, under asymmetric quality, the national brand manufacturer can still benefit from the private label introduction. The manufacturer prefers PL introduction, as long as the retailer promotes NB when the horizontal competition is not intense or PL's quality is sufficiently low. This is because the disadvantage of the PL introduction to the manufacturer is smaller than the advantage of reduced double marginalization. This finding is consistent with the empirical result shown by Pauwels and Srinivasan (2004) that premium-brand manufacturers, but not second-tier brand manufacturers, can benefit from private label introduction.

In addition, we observe that a high spillover rate may not always benefit the whole supply chain. When the retailer introduces a low quality private label but promotes a high quality national brand, a high spillover rate from the national brand product to the private label product may intensify the channel conflict, and thus the whole supply chain profit does not monotonically increase with the spillover rate. This finding may explain why some retailers do not put their private label products immediately next to their national brand counterparts.

Finally, our results show that the predatory effect the manufacturer's out-store promotion may dampen the retailer's interest to introduce the PL product, but the predatory effect of the out-store promotion can be mitigated by the spillover effect of the in-store promotion.

This paper contributes to the extant literature on private labels by investigating the interaction between a retailer and a national brand manufacturer in the presence of the retailer's in-store promotion and the associated spillover effect. Our findings suggest that a national brand manufacturer may actually prefer the private label introduction as long as either the spillover effect is substantially asymmetric or the quality level of the private label is sufficiently lower than that of the national brand.

The remaining of the paper is organized as follows. We review the related literature in Section 2 and establish the model in Section 3. The main analysis and numerical studies are provided in Section 4 and Section 5, respectively. We conclude in Section 6 and list all proofs in the Appendix.

2 Literature Review

Private labels have drawn a lot of attention from both academia and practice. Steiner (2004) provides a retrospective of the history of competition between national brands and private labels, and analyzes the advantage of using private labels to balance the market power between retailers and national brand manufacturers. Consumer welfare is usually improved when the competition becomes intense. Kumar and Steenkamp (2007) offer a comprehensive analysis on the private label topic. They describe the common strategies that retailers use to introduce private labels, and propose strategies for national brand manufactures to compete against or collaborate with private labels. To bridge between academic research and business practices, Sethuraman (2009) assesses the external validity of 44 analytical results that appeared in literature and their applicabilities in practice.

There is a stream of literature studying the impacts of private labels on retailers and supply chains. Narasimhan and Wilcox (1998) demonstrate that when retailers introduce private labels, they not only profit directly but also use the private label as a strategic tool to gain market power against the national brand manufacturers. Sachon and Martinez (2009) point out that a supply chain's total profit increases from a private label introduction only when the competition between the private label and the national brand is not intense. Groznik and Heese (2010) analyze how private labels cause channel conflicts in both single-retailer and multi-retailer channels. Chen et al. (2011) study the role of private label introduction in supply chain coordination. They characterize the conditions under which the retailer will introduce the private label, and the conditions under which the introduction is beneficial or detrimental to the overall supply chain. The above papers focus on the decisions of the retailer, but the reactions of the national brand manufacturer are not considered.

Another stream of literature study the impacts of private labels on national brand manufactures and their reactions. Wedel and Zhang (2004) study the competition between national brands and private labels across the subcategories, and they show asymmetrical price competition exists both within and across subcategories: the cross-subcategory impact of national brands on store brands is greater than that of store brands on national brands. Pauwels and Srinivasan (2004) empirically show that private label penetration benefits the retailer, the consumers, and premium-brand manufacturers, but it hurts second-tier brand manufacturers. Geyskens et al. (2010) examine the impact of economy and premium private labels on mainstream-quality and premium-quality national brands and existing private labels. They show that both economy and premium private labels cannibalize incumbent private labels, and economy private label introductions benefit mainstream-quality national brands because the latter become a middle option in the retailer's assortment. Gielens (2012) investigates how new product introduction helps national brand manufacturers boot their market shares. They suggest that, to fight economy private labels successfully, national brands should maintain a smaller price gap, while offering products that focus less on intrinsic and usage benefits. The above papers focus on the reactions of national brand manufacturers. The interactions between the manufacturer and the retailer are still rarely examined in the literature. Differently, our paper considers the national brand manufacturer and the retailer in an interactive scenario where they contemplate each other's strategy and take actions accordingly.

The in-store promotional efforts have attracted interest from a large group of researchers, who examine the issues from a variety of aspects. Shaffer and Zettelmeyer (2009) study the problem of comparative advertising and in-store displays. They explain why manufacturers may or may not want to engage in comparative advertising, especially in regard to in-store displays, in the channel perspective. Dukes and Liu (2010) study the effects of in-store media, which allows manufacturers to advertise their products. They show that in-store media plays an important role in coordinating a distribution channel and the competition between suppliers. Schultz and Block (2011) study many types of in-store promotion to find which promotion techniques influence consumers' purchase decisions the most. They develop models to predict consumers' response to different combinations of promotional efforts. Nordfalt and Lange (2013) perform two large field experiments to show that in-store promotions are powerful tools to increase sales. They find the effectiveness of in-store promotions varies widely depending on when and how the promotions are executed. Those papers study the promotional effort without considering the spillover effect. We establish scenarios to examine the impacts of in-store promotion along with its spillover effect to find new insights.

The perceived quality of products may affect retailers' decisions on promotional efforts. In this paper, perceived quality is used as a measure of the product's attractiveness by itself, exclusive of price and promotion effect. Besides the product's physical quality, perceived quality also includes the brand's reputation. Many private label products have lower perceived quality compared to their national brand counterparts because of the lack of reputation, which takes time to accumulate (Heese 2010).

The spillover effect of promotion is analyzed separately by many scholars. Cellini and Lambertini (2003) illustrate a Cournot oligopoly game where firms sell similar goods and invest in promotion activities with spillover effects. They find the social welfare of a centralized firm will be larger than that of two oligopoly firms. Norman et al. (2008) investigate the promotion activities in homogeneous goods markets where one firm's promotional effort tends to spill over to rival firms. Since such a phenomenon discourages the promotion investment, they suggest collecting mandatory fees for all firms to support a joint advertising effort. Dharmasena et al. (2010) study the spillover effects of promotions in the U.S. non-alcoholic beverage market. They find asymmetric spillover effects where that the promotional effort on one product can positively affect one group of products but negatively affect another group. Therefore, one firm needs to pay attention to the promotional efforts of other firms even if they do not produce the same type of products. Giannakas et al. (2012) develop a theoretical framework to analyze the effect of advertising spillover on firms' productivity. They use the data of meat processing firms in Greece during 1983-2008 and find the spillover effect is one of the important drivers to improve firms' productivity. Those papers do not investigate the spillover effect in a private label context as our research.

It is noticeable that the competition between private label and national brand with both in-store promotion and spillover effect has not been fully studied in the literature. Our work will contribute to fill this void.

3 The Model

We investigate a two-echelon supply chain where a national brand (NB) manufacturer sells its product through a retailer. The retailer has an option to introduce a private label (PL) product, which will inevitably compete with NB product. Inside its own store, the retailer can utilize its in-store media to promote either product. We use subscripts N and P to denote NB and PL products, respectively. As illustrated in Figure 1, there are three possible scenarios:

1. Case P: The retailer introduces PL and promotes it;
2. Case N: The retailer introduces PL, but promotes NB;
3. Case B: This is a baseline case. The retailer does not introduce the PL while promoting NB.

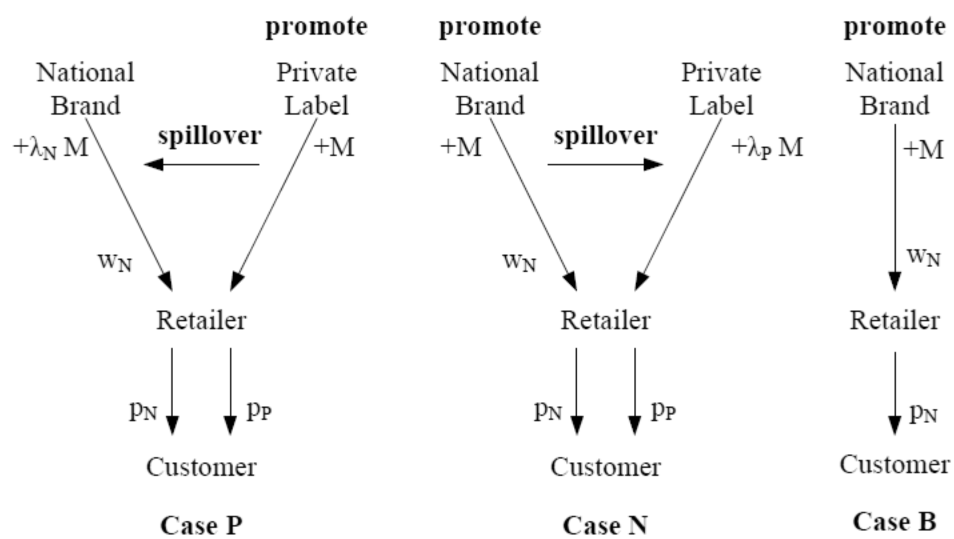

Figure 1: Channel structures

As shown in Figure 1, w_N represents the wholesale price of NB. The retail prices of NB and PL are denoted by p_N and p_P, respectively. The retailer's in-store promotional effort is M, which incurs a cost of θM^2. We normalize θ to 1 without affecting our qualitative results. Similar simplification has also been adopted by Choi and Coughlan (2006), Chen et al. (2009), and Liu et al. (2014).

The quality of NB is denoted by Q_N and normalized to 1. PL's quality is Q_P, which can be either lower or higher than 1. For tractability, both products' quality are assumed to be exogenous. We assume the production costs to be zero for the purpose of analytical tractability. Our numerical analysis later shows that including quality-related production costs will not change the structure of the major results.

Given that both NB and PL are located inside the same store, the promotion of one product will have a spillover effect on the other product. We let λ_N, $0 \leq \lambda_N \leq 1$, denote the spillover rate to the NB when PL is promoted, and λ_P, $0 \leq \lambda_P \leq 1$, denote the spillover rate to PL when NB is promoted.

The sequence of events in this Stackelberg game is as follows. In Stage 1, the manufacturer decides the wholesale price w_N. In Stage 2, the retailer decides both products' retail prices $p_{N/P}$, and the promotion level M on PL or NB. We discuss three scenarios as follows.

3.1 Case P: The retailer introduces PL and promotes it

When a product is promoted, its demand will increase, the demand of the other product will also increase if the spillover effect is high and the substitutability is low. In line with Sayman et al. (2002) and Choi and Coughlan (2006) on modeling a private label and a national brand, we adopt the following quadratic and strictly concave function to describe the utility of a representative customer group who purchase a certain mixture of substitutable products, which is widely used in similar research (Cai et al. 2012, Singh and Vives 1984, Hackner 2003, Ingene 2004).

$$U(D_N, D_P) = D_N(Q_N + \lambda_N M) + (D_P(Q_P + M) - (D_N^2 + 2\gamma D_N D_P + D_P^2)/2 - p_P D_P - p_N D_N,$$

where $D_{N/P}$ is the demand of the NB/PL product and $Q_{N/P}$ is the physical quality of the NB/PL product. The retailer's promotion effort on the PL product increases the perceived quality from the physical quality by M and $\lambda_N M$ for the PL and NB products, respectively. The parameter γ is the product substitutability between the two products. The third term represents the fact that the value of using both substitutable products is less than the sum of the separate values of using each product by itself (Samuelson 1974). The consumer utility decreases as products become more substitutable, i.e., as γ increases, everything else held constant. A more complex function of the initial base demand based on the quality level will not change our main results qualitatively, but quickly leads to intractability.

Maximizing the above utility function yields the following demand functions.

$$\begin{cases} D_N = \dfrac{1}{1-\gamma^2}(Q_N - \gamma Q_P + (\lambda_N - \gamma)M - p_N + \gamma p_P); \\ D_P = \dfrac{1}{1-\gamma^2}(Q_P - \gamma Q_N + (1 - \gamma\lambda_N)M - p_P + \gamma p_N). \end{cases} \tag{1}$$

Let $\Pi^P_{M/R}$ denote the profit of the manufacturer/retailer in Case P. The profit functions are as follows.

$$\begin{cases} \Pi^P_M = w_N D_N; \\ \Pi^P_R = (p_N - w_N)D_N + p_P D_P - M^2. \end{cases} \tag{2}$$

Both the manufacturer and the retailer attempt to maximize their respective profits, which leads to the following result (please refer to the Appendix for all the solutions and proofs in this paper).

Lemma 1 *There exists a unique equilibrium solution of (w_N, p_N, p_P, M) in Case P.*

Note the above model is built in a Bertrand setting in which the retailer decides the prices and then the utility-maximizing demands are derived. Alternatively, we can build the model in a Cournot setting in which the retailer decides both products' ordering quantities $D_{N/P}$ and then the customers pay the utility-maximizing prices as follows.

$$\begin{cases} p_N = Q_N + M\lambda_N - D_N - \gamma D_P; \\ p_P = Q_P + M - \gamma D_N - D_P. \end{cases}$$

The above price functions are inverse functions of Equation (1). After solving the Cournot model, we find the solutions of $\{w_N, p_N, p_P, M\}$ are the same as those in the Bertrand model because the common retailer determines both retail prices or both order quantities. Therefore, this paper focuses on the Bertrand setting, and the same results also apply for the Cournot setting.

3.2 Case N: The retailer introduces PL but promotes NB

In Case N, the promotion increases the initial base demand of NB, Q_N, and the spillover effect of promotion enhances the initial base demand of PL, Q_P, by $M\lambda_P$. The representative customer' utility function is correspondingly described as follows.

$$U = (D_N(Q_N + M - p_N) - \frac{{D_N}^2}{2}) + (D_P(Q_P + M\lambda_P - p_P) - \frac{{D_P}^2}{2}) - \gamma D_N D_P. \tag{3}$$

Maximizing the above utility function results in the demand functions as follows:

$$\begin{cases} D_N = \frac{1}{1-\gamma^2}(Q_N - \gamma Q_P + (1-\gamma\lambda_P)M - p_N + \gamma p_P); \\ D_P = \frac{1}{1-\gamma^2}(Q_P - \gamma Q_N + (\lambda_P - \gamma)M - p_P + \gamma p_N). \end{cases}$$

The profit functions $\{\Pi_M^N, \Pi_R^N\}$ take the same forms as in Equation (2). Similarly, we have

Lemma 2 *There exists a unique equilibrium solution of (w_N, p_N, p_P, M) in Case N.*

3.3 Case B: The retailer does not introduce PL and promotes NB

Case B serves as a baseline case to compare with Cases P and N. The utility function of Case B is similar to that in Case N. Because there is no PL, the demand of PL is zero, that is, $D_P = 0$. Plugging this constraint into Equation (3) and maximizing the utility function results in

$$D_N = Q_N + M - p_N.$$

The profit functions are the same as in Equation (2) (with $D_P = 0$). Similarly, there exists a unique equilibrium solution.

Lemma 3 *There exists a unique equilibrium solution of (w_N, p_N, M) in Case B.*

To ensure the products' demands are non-negative in all scenarios for meaningful discussion, we make two more assumptions: $\gamma < Q_P < min(1/\gamma, 2)$ and $0 < \gamma \leq 0.8$. The reasons are as below. First, without the pricing and promotion issues, the basic demands for both products should be positive, so the non-negative conditions are $Q_N - \gamma Q_P > 0$ and $Q_P - \gamma Q_N > 0$. Considering the normalization of $Q_N = 1$, the above non-negative condition can be rewritten as $\gamma < Q_P < 1/\gamma$. However, the above upper bound $1/\gamma$ can be very large if γ is small. Since PL is generally designed to imitate NB product and Q_P will not be significantly different from Q_N, we limit $\gamma < Q_P <$

$min(1/\gamma, 2)$, which means PL's quality will not be twice as good as the NB product. Second, for γ, as the private label is a substitutable product for the national brand, we have the lower bound as $\gamma > 0$. It is easy to see γ should be less than a certain value to ensure non-negative demand in Equation (1). We find that $\sqrt{3}/2 = 0.866$ is the largest allowed value of γ in the no-spillover model (see the proof of Lemma 1). We tighten the upper bound to its first digit as $\gamma <= 0.8$ for simplicity.

4 Analytical Results

This section serves two major purposes. The first is to study the retailer's two strategic decisions: (1) whether or not to introduce PL; and (2) if PL is introduced, whether to promote PL or NB. The second purpose is to investigate whether or not the manufacturer would benefit from the retailer's PL introduction and in-store promotion. In the following, we first introduce the benchmark case without spillover, then we proceed to analyze the impact of spillover effect.

4.1 No-spillover: Preliminaries

Many of the studies on PL introduction do not consider the in-store promotion as well as the spillover effect of promotion. Normally, channel conflicts arise as the retailer introduces PL, which hurts the manufacturer (see, e.g., Groznik and Heese (2010), Heese (2010), Chen et al. (2011)). The following lemma confirms the same message.

Lemma 4 *Without promotion, the retailer prefers to introduce PL, which always hurts the NB manufacturer.*

Conventional wisdom tells us that the retailer can benefit from selling its own PL product, which encroaches into NB's market. The retailer gains a higher marginal profit in selling PL than selling NB. In contrast, the manufacturer suffers from losing its monopoly of NB in the market and has to reduce its wholesale price because of the horizontal competition from the PL.

Without the spillover effect of promotion, would in-store promotion upon either PL or NB change the manufacturer's preference regarding the introduction of PL? The following lemma suggests a negative answer.

Lemma 5 *With promotion but no spillover, the manufacturer's preference on the three cases are Case B $\succ$ Case N $\succ$ Case P.*

This result is not surprising, because the manufacturer's best scenario is to maintain its monopoly (i.e., Case B). If PL is launched, the manufacturer certainly prefers its own product to be promoted as compared with PL being promoted (i.e., Case N $\succ$ Case P). The retailer's preference is different from the manufacturer's, as demonstrated below.

Lemma 6 *With promotion but no spillover, the retailer's preference on the three cases are as follows:*

- *Case N $\succ$ Case B;*
- *Case P $\succ$ Case B if and only if $Q_P > \bar{Q}_P^{PB}(\gamma) = \dfrac{3\gamma + \sqrt{3}\sqrt{16\gamma^4 - 16\gamma^2 + 3}}{12(1-\gamma^2)}$;*
- *Case P $\succ$ Case N if and only if $Q_P > \bar{Q}_P^{PN}(\gamma) = \dfrac{1}{2\sqrt{1-\gamma^2}}$, where $\bar{Q}_P^{PB}(\gamma) < \bar{Q}_P^{PN}(\gamma)$.*

Lemma 6 confirms that the retailer can be better off by introducing PL (i.e., Case N $\succ$ Case B). Provided that the NB is promoted in both scenarios, introducing PL reduces the double marginalization and hence increases the total demand of both products for the retailer. Therefore, if the manufacturer demands the retailer to promote NB, the retailer will choose to introduce PL.

The nuance comes when the retailer is determined to promote its own PL in Case P. When PL's quality is low, Case B outperforms Case P for the retailer, because promoting a low quality PL leads to lower profit margin than promoting the higher quality NB in a monopoly market (i.e., Case B). This result indicates that introducing and promoting a PL does not always benefit the retailer. As the PL's quality improves, the benefit of having more demand from selling both products outweighs the relatively higher profit margin of selling only NB in a monopoly market; as a result, Case P outperforms Case B for the retailer.

We can also infer from Lemma 6 that both thresholds $\bar{Q}_P^{PN}(\gamma)$ and $\bar{Q}_P^{PB}(\gamma)$ increase with γ. It means that, as the competition between the two products becomes more intense, for the retailer, PL must have a sufficiently high quality to make Case P more preferable than the other two cases.

As illustrated in Figure 2, if PL's quality is low, i.e., $Q_P < \bar{Q}_P^{PN}$, the retailer will promote NB to enlarge the market. If PL's quality is high, i.e., $Q_P > \bar{Q}_P^{PB}$, the retailer will instead promote

PL to capture a higher profit margin. Therefore, the retailer has the incentive to introduce a PL, although whether to promote either NB or PL depends on whether PL's quality is high or low. The "N/A" areas in Figure 2 do not satisfy the non-negativity conditions stipulated after Lemma 3.

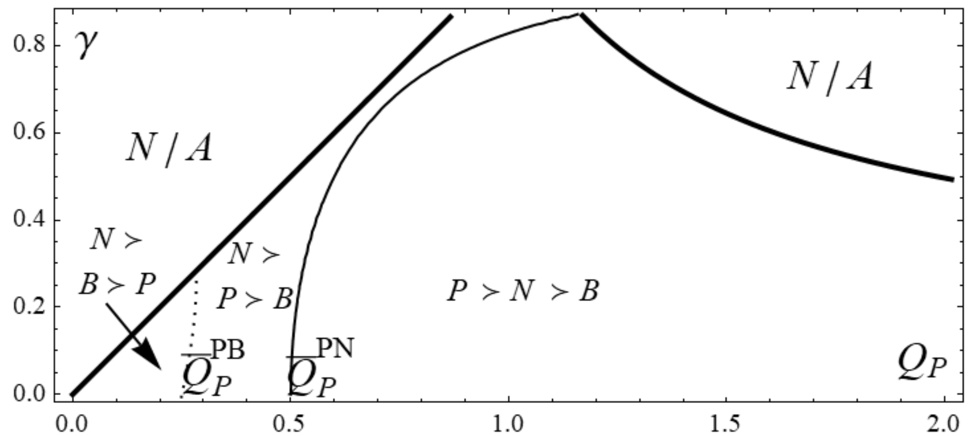

Figure 2: The retailer's preference without spillover.

Comparing Lemma 6 with Lemma 5, we can conclude that the manufacturer will be at odds with the retailer when a PL is introduced, especially when PL's quality is high and Case P is chosen over Case N in in-store promotion. Note that this result is obtained under no spillover effect. With spillover considered, however, will the conflict between the manufacturer and the retailer over PL introduction and promotion be lessened? To answer this question, in the next sections, we explore the cases of symmetric spillover and asymmetric spillover.

4.2 Impact of Symmetric Spillover Rate

To single out the impact of spillover on the two firms' preferences, we start with a simple case with symmetric spillover ($\lambda_N = \lambda_P = \lambda \in [0, 1]$) between the two products and keep both products' qualities equal ($Q_P = Q_N = 1$).

Impact on the retailer

When PL's quality is equal to NB product, we find that the retailer's preference is the same as that in the no-spillover scenario (Lemma 6) with a sufficiently high quality PL product.

Proposition 1 *For the retailer, when the spillover rates between the two products are symmetric and the product qualities are equal, Case P $\succ$ Case N $\succ$ Case B.*

This result reveals that for the retailer, if PL's quality is as high as NB product, the magnitude of a symmetric spillover will not change its preference. In other words, the impact of PL product's quality dominates the impact of spillover on the retailer's preference.

Impact on the NB manufacturer

In the no-spillover scenario, we find the NB manufacturer never prefers introducing PL. With spillover effect, however, the NB manufacturer can benefit from the introduction of a competing PL product when the spillover rate is high, which deviates from the conventional wisdom.

Proposition 2 *For the NB manufacturer, when the spillover rates between the two products are symmetric and the product qualities are equal, its preference on the three cases, N, P, and B, are as follows.*

- *Case N $\succ$ Case B if and only if $\lambda > \bar{\lambda}^M_{NB}(\gamma) = 4\gamma$;*
- *Case P $\succ$ Case B if and only if $\lambda > \bar{\lambda}^M_{PB}(\gamma) = (2\sqrt{3}\sqrt{-8\gamma^2 + 6\gamma + 2} + 8\gamma - 3)/5$ where $\bar{\lambda}^M_{NB}(\gamma) < \bar{\lambda}^M_{PB}(\gamma)$;*
- *Case N $\succ$ Case P.*

To interpret these results, we use Figure 3 to more vividly demonstrate how the NB manufacturer's preference changes in term of the spillover rate λ. First, given any product substitutability level, when the spillover rate λ is low, the NB manufacturer does not favor the introduction of PL. However, when λ is sufficiently high, the NB manufacturer can actually benefit from PL introduction as long as the retailer promotes NB. There are two drivers behind this phenomenon. First, the NB manufacturer benefits from the in-store promotion of the NB product. Second, the spillover effect boosts the horizontal competition between the two products and hence reduces double marginalization to generate higher demand for the manufacturer. Although the PL introduction encroaches into NB's market share, the benefit of a greater market size to the NB manufacturer outweighs its loss, such that Case N is more preferable to Case B for the manufacturer.

As λ continues to grow higher, Case P can be even better than Case B for the NB manufacturer, as long as γ is sufficiently low. In comparison to Case N, the retailer will exert more promotional effort in Case B because of a higher profit margin in PL. Provided that the spillover effect is sufficiently high (i.e., λ is high), the NB manufacturer can also significantly benefit from a larger

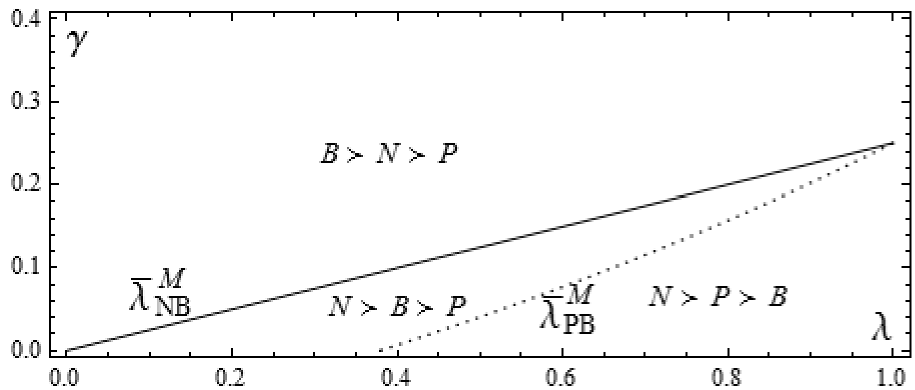

Figure 3: The NB manufacturer's preference with the same spillover/quality.

market size and reduced double marginalization. Overall, a high spillover rate alleviates the negative impact of intense competition on the manufacturer. Nevertheless, conditional on the symmetric spillover rates, if NB manufacturer can determine the promotion type, it always prefers its product, instead of PL, to be promoted.

From Proposition 2, one can also infer that both thresholds $\bar{\lambda}_{NB}^{M}(\gamma)$ and $\bar{\lambda}_{PB}^{M}(\gamma)$ rise as γ increases. It means when the competition between the two products becomes more intense, the manufacturer will more likely prefer a PL introduction, regardless of the promotion type, if and only if the spillover rate becomes sufficiently higher.

Equivalently, the above results can be described from the perspective of product substitutability level γ. For the NB manufacturer, when the spillover rates between the two products are symmetric and the product qualities are equal, its preference on the three cases are:

- Case N $\succ$ Case B if and only if $\gamma < \bar{\gamma}_{NB}^{M}(\lambda) = \lambda/4$;
- Case P $\succ$ Case B if and only if $\gamma < \bar{\gamma}_{PB}^{M}(\lambda) = (-\sqrt{3}\sqrt{5-2\lambda^2} + 2\lambda + 3)/8$ where $\bar{\gamma}_{PB}^{M}(\lambda) < \bar{\gamma}_{NB}^{M}(\lambda)$;
- Case N $\succ$ Case P.

Given the same spillover rate, when the substitutability level is sufficiently low, PL introduction can be beneficial to the NB manufacturer. As γ increases, Case N may still be better than Case B, but Case P will be worse than Case B, because the NB manufacturer will suffer from overly intense horizontal competition. When γ is even higher, PL introduction will hurt the NB manufacturer.

Combining Proposition 1 for the retailer and Proposition 2 for the NB manufacturer, we can conclude that the spillover effect makes the introduction of PL more preferable for both firms. However, with the symmetric spillover rates, the NB manufacturer and the retailer are always at odds with the in-store promotion type: the retailer prefers Case P to Case N, whereas the NB manufacturer prefers Case N to Case P.

4.3 Impact of Asymmetric Product Quality

To single out the impact of PL's quality, we hereby assume symmetric spillover rates, that is $\lambda_N = \lambda_P = \lambda \in [0, 1]$.

Impact on the retailer

We first extend the result from the no-spillover case in Lemma 6 by explicitly including symmetric spillover rates for both firms and obtain the following result.

Proposition 3 *For the retailer, when the spillover rates between the two products are symmetric, its preference on the three cases is as follows:*

- *Case N $\succ$ Case B;*
- *Case P $\succ$ Case B if and only if*

$$Q_P > \bar{Q}_P^{PB}(\lambda, \gamma) = \frac{2\sqrt{3}\sqrt{64\gamma^4 - 64\gamma^3\lambda + 52\gamma^2\lambda^2 - 64\gamma^2 - 18\gamma\lambda^3 + 32\gamma\lambda + 5\lambda^4 - 19\lambda^2 + 12} + 12\gamma - 3\lambda}{3(-16\gamma^2 + 8\gamma\lambda - 5\lambda^2 + 16)};$$

- *Case P $\succ$ Case N if and only if*

$$Q_P > \bar{Q}_P^{PN}(\lambda, \gamma) = \frac{\sqrt{\lambda^4 - 5\lambda^2 + 4}}{\sqrt{16\gamma^2\lambda^2 - 16\gamma^2 - 8\gamma\lambda^3 + 8\gamma\lambda + 5\lambda^4 - 21\lambda^2 + 16}};$$

- $\bar{Q}_P^{PB}(\lambda, \gamma) < \bar{Q}_P^{PN}(\lambda, \gamma)$.

Similar to the result in Lemma 6 without spillover, the retailer is always better off by introducing PL (Case N $\succ$ Case B). With the symmetric spillover rate, the threshold values of Q_P in Lemma 6 are altered accordingly to include $\{\lambda, \gamma\}$. Similar to Lemma 6, the retailer's preference can be categorized as below:

1. When Q_P is low, Case N $\succ$ Case B $\succ$ Case P;

2. When Q_P is medium, Case N $\succ$ Case P $\succ$ Case B;

3. When Q_P is high, Case P $\succ$ Case N $\succ$ Case B.

Proposition 3 confirms that the retailer does not always prefer to promote PL. Instead, when PL's quality is low and medium, the retailer can benefit from promoting NB rather than PL. This is different from Proposition 1 with symmetric quality levels. Given that the retailer has stakes in both products, promoting the higher quality product can lead to a higher profit margin. If PL's quality is sufficiently low, the retailer should promote NB; otherwise, PL will be promoted.

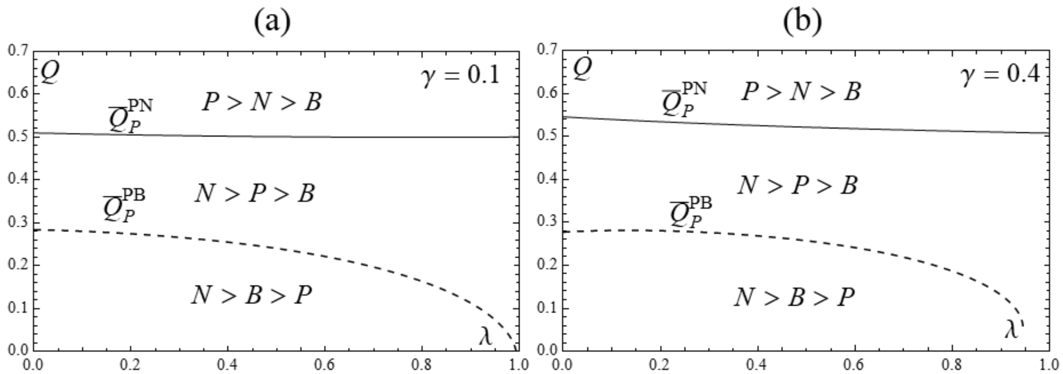

Figure 4: The trends of $\bar{Q}_P^{PN}$ and $\bar{Q}_P^{PB}$ for the retailer when λ increases.

Note that both thresholds, $\bar{Q}_P^{PB}(\lambda,\gamma)$ and $\bar{Q}_P^{PN}(\lambda,\gamma)$, decrease with λ. This suggests, as λ grows, it is more likely for the retailer to introduce and promote PL, which is illustrated in Figure 4. Figure 4 also shows that the lines of $\bar{Q}_P^{PB}(\lambda,\gamma)$ and $\bar{Q}_P^{PN}(\lambda,\gamma)$ shift up as γ increases from 0.1 to 0.4, which indicates that it is more likely for the retailer to introduce PL and promote NB as product substitutability increases.

Impacts on the NB manufacturer

We now extend the result from Proposition 2 with symmetric quality to asymmetric quality as follows.

Proposition 4 *For the NB manufacturer, when the spillover rates between the two products are symmetric, its preference on the three cases is as follows:*

- *Case N $\succ$ Case P;*

- *Case N $\succ$ Case B if and only if*
  - $\gamma < \lambda/4$ *(low competition); or*
  - $\gamma > \lambda/4$ *(high competition), and*

$$Q_P < \bar{Q}_P^{NB}(\lambda, \gamma) = \frac{-2\sqrt{3}\sqrt{4\gamma^2\lambda^2 - 16\gamma^2 - 2\gamma\lambda^3 + 8\gamma\lambda + \lambda^4 - 7\lambda^2 + 12} - 3\lambda^2 + 12}{3(4\gamma - \lambda)};$$

- *Case P $\succ$ Case B if and only if*

$$Q_P < \bar{Q}_P^{PB}(\lambda, \gamma) = \frac{-\sqrt{(6\lambda - 24\gamma)^2 - 4(16\gamma^2 - 8\gamma\lambda + \lambda^2)(16\gamma^2 - 8\gamma\lambda + 4\lambda^2 - 3)} + 24\gamma - 6\lambda}{2(16\gamma^2 - 8\gamma\lambda + \lambda^2)};$$

- $\bar{Q}_P^{PB}(\lambda, \gamma) < \bar{Q}_P^{NB}(\lambda, \gamma)$.

Similar to Proposition 2, we find the same result that the NB manufacturer always prefers its own product to be promoted regardless of PL quality level. Because of the asymmetric quality, however, it is more likely for the manufacturer to accept the introduction of PL. Compared to Proposition 2, in addition to the original condition (i.e., $\gamma < \lambda/4$), the manufacturer might prefer Case N to Case B if $\gamma > \lambda/4$ and $Q_P < \bar{Q}_P^{NB}(\lambda, \gamma)$. In other words, the manufacturer is tolerant of PL introduction, as long as the retailer promotes NB when the horizontal competition is not intense or the PL quality is sufficiently low. In this situation, the disadvantage of PL introduction to the manufacturer is smaller than the advantage of reduced double marginalization. If PL's quality is even lower, the benefit of reduced double marginalization can be higher, such that the manufacturer might even prefer Case P to Case B.

Combining Proposition 3 and Proposition 4, we illustrate the two firms' preferences in Figure 5. On the one hand, the retailer prefers Case P when Q_P is high (to the right of the dashed line), or Case N when Q_P is low (to the left of the dashed line). On the other hand, the NB manufacturer prefers Case N if the competition is low (below the lower solid line), or the competition is high but Q_P is low (to the left of the upper solid line); otherwise the manufacturer prefers Case B. As a result, both the retailer and the NB manufacturer can prefer the same Case N when Q_P is sufficiently low. This is a deviation from the scenario with symmetric spillover rate and symmetric quality levels where the retailer and NB manufacturer always conflict over the in-store promotion type. In summary, PL introduction with in-store promotion of NB can be a Pareto choice for both the retailer and the manufacturer.

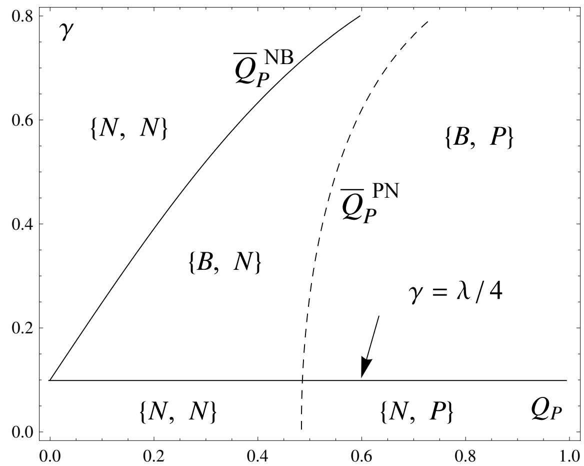

Figure 5: The thresholds of the {Manufacturer's, Retailer's} promotion preference as PL's quality varies where $\lambda = 0.4$.

4.4 Impact of Asymmetric Spillover Rates

We now focus on the two firms' preference for Case N or Case P under a general asymmetric spillover setting ($\{\lambda_N, \lambda_P\} \in [0, 1]$). For tractability, we again assume the products' qualities are equal. Case B is not needed in this analysis for the following two reasons. First, for the NB manufacturer, the thresholds of Case P and Case N over Case B have been described in Proposition 2, which are functions of the substitutability level γ. Case B is inferior to Case P or Case N as long as γ is not substantially large. Second, for the retailer, Case B is always dominated by Case P or Case N as Proposition 1 shows. As a result, it is reasonable to assume PL product has been introduced and, thus, we preclude Case B in this analysis and focus on the comparison between Case N and Case P.

Proposition 5 *When the two products have asymmetric spillover effects and the product qualities are equal,*

- *for the retailer, Case P $\succ$ Case N if and only if $\lambda_N > \bar{\lambda}_N^R(\lambda_P, \gamma)$;*
- *for the NB manufacturer, Case P $\succ$ Case N if and only if $\lambda_N > \bar{\lambda}_N^M(\lambda_P, \gamma)$;*

- $\bar{\lambda}_N^R(\lambda_P, \gamma) < \bar{\lambda}_N^M(\lambda_P, \gamma)$.

Proposition 5 is described in terms of λ_N. In terms of the spillover rate from NB to PL, λ_P, Proposition 5 can be rewritten as follows[1]:

- For the retailer, Case N $\succ$ Case P if and only if $\lambda_P > \bar{\lambda}_P^R(\lambda_N, \gamma)$;
- For the NB manufacturer, Case N $\succ$ Case P if and only if $\lambda_P > \bar{\lambda}_P^M(\lambda_N, \gamma)$;
- $\bar{\lambda}_P^M(\lambda_N, \gamma) < \bar{\lambda}_P^R(\lambda_N, \gamma)$.

We use Figure 6 to graphically illustrate Proposition 5. The two thresholds, $\bar{\lambda}_N^R(\lambda_P, \gamma)$ and $\bar{\lambda}_N^M(\lambda_P, \gamma)$, segment the feasible region $\{\lambda_N, \lambda_P\} \in [0, 1]$ based on the two firms' preferences. First, when λ_N and λ_P are not sufficiently different, the NB manufacturer prefers Case N and the retailer prefers Case P, which is similar to the result with the symmetric spillover rates.

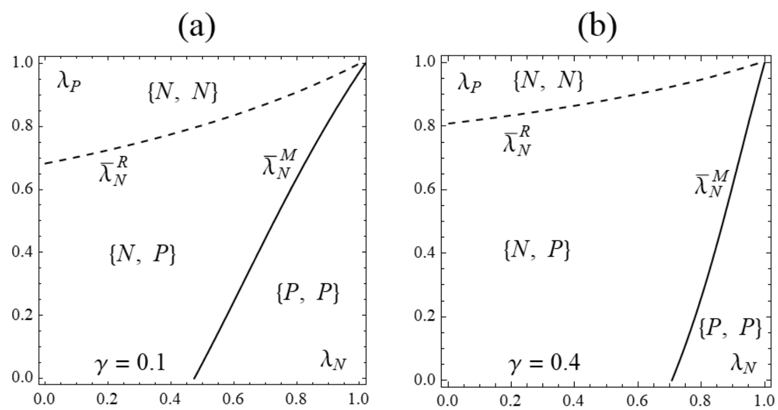

Figure 6: The {Manufacturer's, Retailer's} preferences of Case P or Case N with asymmetric spillover rates.

When λ_P is significantly greater than λ_N (the spillover effect from NB to PL is much larger than from PL to NB), both firms prefer Case N (upper left corner). This occurs because the same amount of promotional effort in Case N leads to a larger overall market size for both firms than in Case P. It is intuitive that the promotion decision has a higher impact on the NB manufacturer than the retailer, because the retailer still keeps a portion of the sales revenue of NB product, but NB manufacturer earns nothing from the sales of PL. Although the retailer has to sacrifice some profit for promoting

[1]From the proof of Proposition 5, we can see that the λ_P-based thresholds $(\bar{\lambda}_P^R(\lambda_N, \gamma), \bar{\lambda}_P^M(\lambda_N, \gamma))$ can be expressed as the inverse functions of the λ_N-based ones $(\bar{\lambda}_N^R(\lambda_P, \gamma), \bar{\lambda}_N^M(\lambda_P, \gamma))$. We hereby skip the complex functions for parsimony.

NB instead of the PL, it benefits from the more significantly reduced double marginalization caused by more intense horizontal competition resulting from higher spillover rates. Similarly, when λ_N is significantly larger than λ_P (the spillover effect from PL to NB is much larger than from NB to PL), both firms prefer Case P to Case N (lower right corner). Comparing Propositions 4 and 5, it shows that the manufacturer's preference sequence is not affected by asymmetric quality levels, but it changes when asymmetric promotion spillover effects are considered. In contrast, the retailer's preference sequence changes when either the quality levels or the spillover effects become asymmetric.

As the product substitutability (γ) grows, the area of {N,P} enlarges, whereas those of {N,N} and {P,P} shrink. These results indicate that the manufacturer is more likely to prefer Case N while the retailer is more likely to prefer Case P, when products become more substitutable. The more intense horizontal competition reduces the benefit of lessened double marginalization, therefore, the benefit of direct in-store promotion becomes more critical to both firms.

In summary, Proposition 5 delivers an unconventional message that both the retailer and the NB manufacturer can actually prefer PL introduction and the same in-store promotion type, that is {N,N} and {P,P}, conditional on the asymmetric spillover effects. In other words, the retailer can actually benefit from promoting NB product rather than promoting its own PL product, whereas the NB manufacturer can be better off from PL introduction with a positive spillover effect in either type of in-store promotions.

5 Extended Numerical Analysis

For analytical tractability, our previous analysis is limited to either only asymmetric spillover rates or only asymmetric quality. For simplicity, we assumed normalize the production cost to zero, and we did not include the NB manufacturer's out-store promotions in the model. In this section, we conduct numerical tests to examine the impact of asymmetric spillover and asymmetric quality simultaneously. Since the spillover rates are controllable in practice, we also study the firms' preferences of spillover rates. We also conduct numerical analysis to examine the impacts of quality-related production costs and the NB manufacturer's out-store promotions.

5.1 Impacts with Asymmetric Spillover and Asymmetric Quality

5.1.1 Improvement of the profits when competition is low

We start with the case of low competition assuming $\gamma = 0.1$. We examine the firms' profits under spillover rates $\lambda = 0.1, 0.5, 0.9$, respectively. We find that both firms' profits increase monotonically when λ increases, because the firms benefit more from the market expansion effect of spillover when the horizontal competition is less of a concern. Due to different game settings, the magnitudes of those profit increases are different in Cases P and N. In Table 1, we underscore each firm's profit increase in each scenario when λ increases from 0.1 to 0.9 (e.g., in Case P when $Q_P = 1$, for the manufacturer, the profit increase is $0.219 - 0.102 = 0.117$).

		γ = 0.1			
		Case P (λ_N)		Case N (λ_P)	
		Π_M	Π_R	Π_M	Π_R
Q_P = 1	λ = 0.1	0.102	0.384	0.144	0.322
	λ = 0.5	0.142	0.405	0.176	0.355
	λ = 0.9	0.219	0.443	0.229	0.428
	Increase	**0.117**	**0.058**	**0.085**	**0.106**
Q_P = 0.7	λ = 0.1	0.109	0.218	0.151	0.198
	λ = 0.5	0.140	0.233	0.173	0.217
	λ = 0.9	0.201	0.264	0.211	0.259
		0.091	**0.046**	**0.060**	**0.061**
Q_P = 0.4	λ = 0.1	0.116	0.112	0.158	0.119
	λ = 0.5	0.137	0.122	0.170	0.128
	λ = 0.9	0.183	0.145	0.193	0.147
		0.067	**0.033**	**0.035**	**0.028**

Table 1: The increase of both firms' profits as λ grows.

When PL's quality is high ($Q_P = 0.7, 1$), as shown in Table 1, if the retailer promotes PL (Case P), the NB manufacturer's profit increases more than the retailer's as λ increases; if the retailer promotes NB (Case N), the retailer's profit increases more than the NB manufacturer's as λ increases. When PL's quality is low ($Q_P = 0.4$), in Case P the trend remains the same. However, in Case N even though PL gets the spillover benefit, the retailer's profit increases less than the NB manufacturer's. This is because PL's marginal profit is low due to its low quality as compared to NB.

5.1.2 Impact on the profits when competition is moderate or high

When the competition is moderate or high, the two firms' profits may not monotonically increase with the spillover rate, because the horizontal competition caused by the spillover effect could significantly hurt both firms. Below we illustrate several representative cases.

Impact on the two firms' profits in Case N

Here we study the impact of the spillover rate from NB to PL, λ_P, on the retailer's profit. As λ_P increases, PL's demand increases but NB's demand decreases because of competition. When the competition is low (e.g., $\gamma = 0.1$), the demand decrease of NB is insignificant and can be compensated by the demand increase of PL. Thus the retailer's profit monotonically increases with λ_P. When the competition is moderate ($\gamma = 0.5$), the demand loss of NB is no longer ignorable. But, if PL's quality is sufficiently high ($Q_P = 0.95$) and the profit margin of selling PL is close to that of NB, the retailer' s profit still monotonically increases with λ_P, see Figure 7.

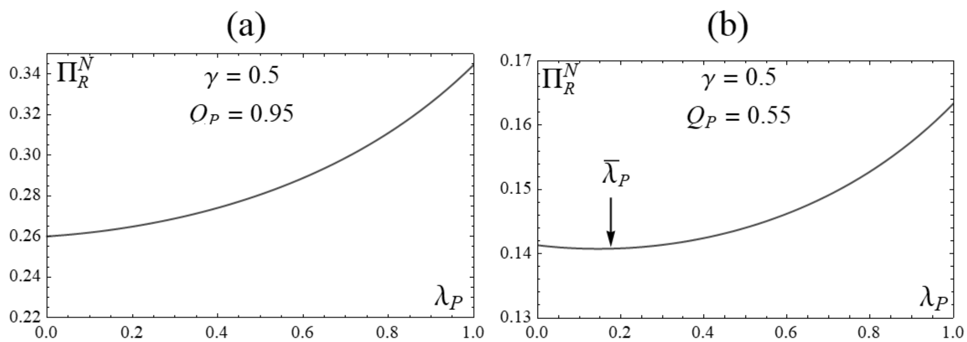

Figure 7: The retailer's profit in Case N

The nuance comes in Figure 7(b), where the competition is moderate and PL's quality is low ($\gamma = 0.5$ and $Q_P = 0.55$) in Case N. When λ_P increases, there are two effects on the retailer's profit. On the one hand, it incurs a negative effect where the high margin NB product's demand decreases even though the low margin PL product's demand increases. On the other hand, it incurs a positive effect on the retailer's profit, because the retailer keeps all the revenue gain caused by the PL's demand increase, though shares the revenue loss caused by the NB's demand decrease with the NB manufacturer. When λ_P is lower than a certain threshold ($\bar{\lambda}_P$ shown on the graph), the negative effect dominates and the retailer's profit decreases as λ_P increases. When λ_P is higher than the threshold, the positive effect dominates and the retailer's profit increases. Similar results can be observed for the manufacturer, for example, see Figure 10 which is to be further discussed in the next subsection.

Impact on the two firms' profits in Case P

In Case P, we find that both firms' profits monotonically increase with λ_N, and both firms prefer high spillover. Figure 8 shows two examples of the impacts of the spillover rate from PL to NB, λ_N, on the two firms. Intuitively, the manufacturer's profit increases more rapidly than the retailer's as the spillover rate from PL to NB, λ_N, grows. A comparison of the two graphs in Figure 8 indicates that the retailer's advantages decrease as PL's quality drops from 0.95 to 0.55.

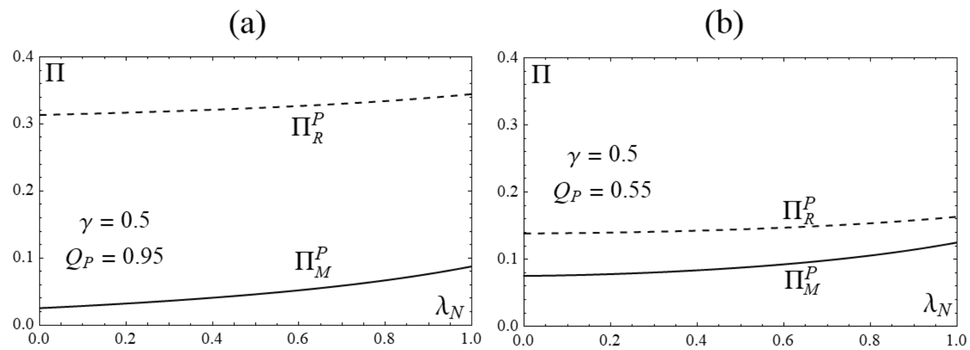

Figure 8: The two firms' profits in Case P

5.1.3 Impact on the supply chain's profit in Cases P and N

Figure 9 shows the impacts of the two spillover rates on the supply chain's profit under Case P (solid) and Case N (dashed). In many scenarios the supply chain's profit monotonically increases with the spillover rate, except in Case N when the competition is moderate and the PL's quality is low, or both the competition and PL's quality are high. Noticeably, however, the dashed line (Case N) in Figure 9(b) (i.e., $Q_p = 0.55$) shows a non-monotonic trend. This is because this curve (the supply chain's profit) is a combination of Figure 7(b) (the retailer's profit) and Figure 10(a) (the NB manufacturer's profit). This result suggests that the supply chain profit does not always increase as the spillover rate increases.

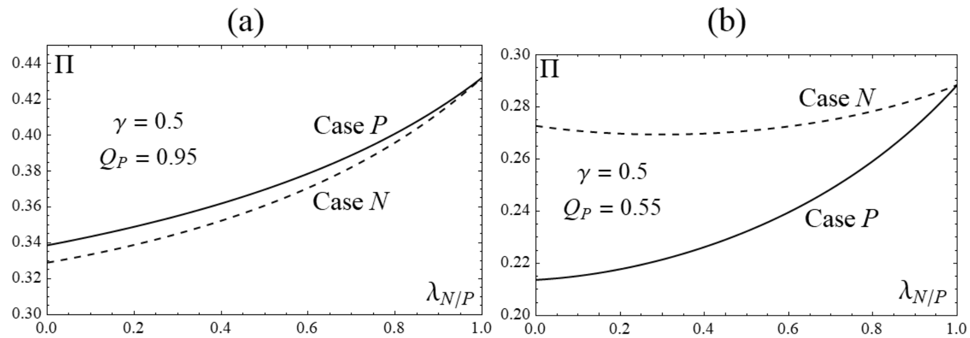

Figure 9: The supply chain's profit in Case P and N

5.2 The Firms' Preferences of Spillover Rates

As we discussed in the paper, the spillover rates can be asymmetric. In practice, spillover rates are controllable. For example, when promoting one brand, the retailer can place the other brand right beside, front, behind, above, below the shelf, or far away to create different spillover effects. One can argue that placing the two products side by side will create a higher spillover effect than placing them far away. Placing one product at the proper eye level and the other below the eye level will create asymmetric spillover rates (Valenzuela and Raghubir 2015). While for tractability the paper has so far explored the situations with exogenous spillover rates, this subsection investigates the firms' preference of spillover rates when they are controllable.

To showcase the firms' preference of spillover rates, let us examine the impact of the spillover rate from NB to PL, λ_P, on NB manufacturer's profit, as shown in Figure 10. As λ_P increases, there will be a trade-off affecting NB product's demand. On the one hand, the demand of PL product will increase due to the spillover effect, which in turn encroaches on NB product's demand (competition effect). On the other hand, the retailer has an incentive to step up the promotion level because of the spillover effect; consequently, the demand of NB increases (complementary effect). As illustrated in Figure 10(a), when γ is moderate and PL's quality is low ($\gamma = 0.5$ and $Q_P = 0.55$), the competition effect dominates such that the NB manufacturer's profit monotonically decreases with λ_P. Therefore, the manufacturer's optimal preference of λ_P will be zero to prevent the competition effect although the complementary effect is subdued accordingly.

When γ is moderate and PL's quality is high ($\gamma = 0.65$ and $Q_P = 0.95$, see Figure 10(b)), however, the complementary effect dominates when λ_P is low, such that the NB manufacturer's

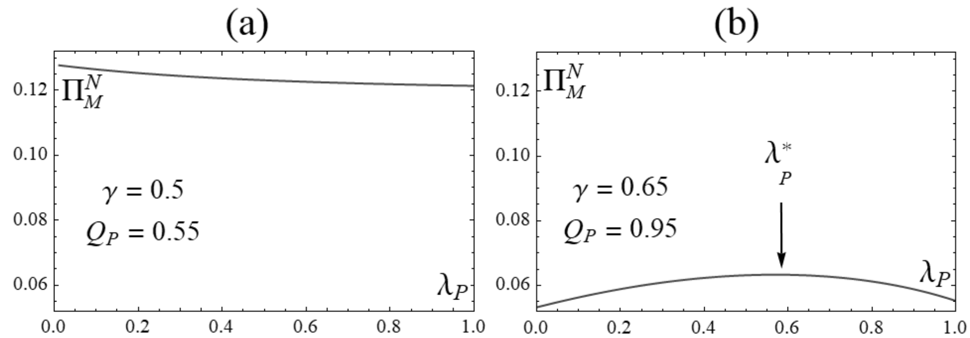

Figure 10: The NB manufacturer's profit in Case N

profit first increases then decreases, as the competition effect surpasses the complementary effect when λ_P is high. Therefore, there exists an optimal spillover rate λ_P^* for the NB manufacturer, $\lambda_P^* = \underset{\lambda_P \in [0,1]}{\text{argmax}} \Pi_M^N$. Although it is difficult to analytically provide the closed form of λ_P^*, we numerically observe the following property: the NB manufacturer's optimal spillover rate (λ_P^*) decreases with the product substitutability level (γ). We further illustrate this property in Figure 11.

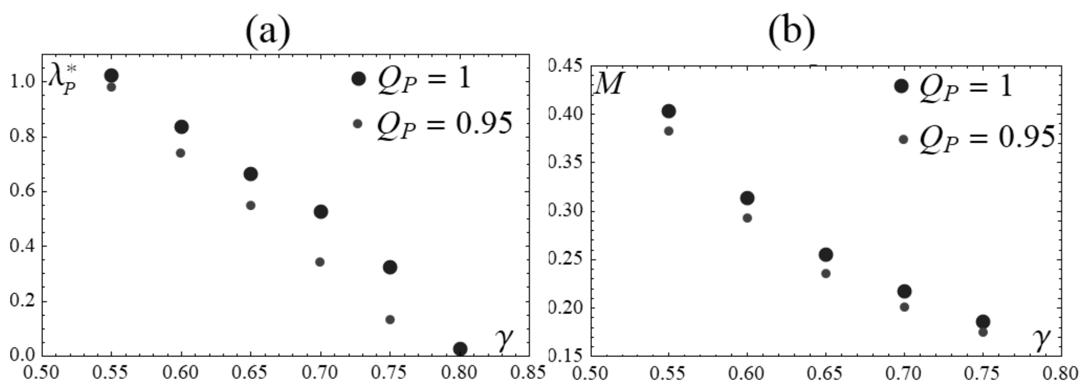

Figure 11: Optimal spillover rate (a) and promotional effort (b) when γ increases

Figure 11 also shows the optimal manufacturer-preferred spillover rate increases as Q_P increases. This is because when Q_P increases, the retailer has more incentives to exert more promotional effort to attract more consumers (see Figure 11(b)). Since NB product's demand directly benefits from the promotion (the complementary effect), the NB manufacturer will prefer a higher spillover rate for PL to stimulate a higher promotional effort from the retailer.

We now summarize the retailer's and manufacturer's preferences on spillover rate (λ_P^*) in Case N in Table 2. For example, if the product substitutability is moderate and PL product's quality is high, we obtain {High, Moderate}, which means that the retailer prefers high spillover, whereas the NB manufacturer prefers moderate spillover. In Case P, both players prefer high spillover in all scenarios.

Retailer **Manufacturer**	γ is low	γ is moderate	γ is high
Q_P is high	High High	High Moderate	High or Low Low
Q_P is low	High High	High or Low Low	Infeasible

Table 2: The {retailer's, manufacturer's} preferences on the spillover rate (λ_P^*) in Case N.

5.3 Impacts of Non-zero Production Cost

In this section we relax the assumption of zero production cost, and conduct numerical tests to show its impacts on the structure of the main results studied in Section 4. For this purpose, we introduce unit production cost functions $c_N = a_N + b_N Q_N$ and $c_P = a_P + b_P Q_P$, where $a_{N/P}$ is the quality-independent cost for basic material and labor and $b_{N/P}$ is the cost coefficient for quality improvement. The profit functions change from Equation (2) to:

$$\begin{cases} \Pi_M^P = (w_N - c_N)D_N; \\ \Pi_R^P = (p_N - w_N)D_N + (p_P - c_P)D_P - M^2. \end{cases}$$

It is complicated to obtain the analytical solutions for the three cases P/N/B as in Section 3. We solve the problem numerically and find the unique equilibrium solution for each case.

Notice that the production cost affects the intensity of competition and the firms' preferences. For example, when $\gamma = 0.1$ and $\lambda_{N/P} = 0.3$, the retailer will prefer not to introduce PL, i.e., Case B $\succ$ Case N and Case B $\succ$ Case P, if $a_{N/P} \geq 0.25$ and $b_{N/P} \geq 0.3$. Therefore for meaningful discussions, we exclude the cases where the production cost is so high that introducing PL is no longer beneficial for the retailer. In the following, we set $\{\gamma = 0.1, \lambda_{N/P} = 0.3\}$ and keep $a_{N/P} \leq 0.2$ and $b_{N/P} \leq 0.25$.

We conducted numerical tests with a matrix of values of cost functions, ranging from symmetric production cost functions, $c_N = 0.05 + 0.05Q_N$ and $c_P = 0.05 + 0.05Q_P$, to asymmetric cost functions where that the NB incurs only a quarter of PL's production cost, $c_N = 0.05 + 0.05Q_N$ and $c_P = 0.2 + 0.2Q_P$. The structure of the results are consistent. In the rest of this section, we only present the results with cost functions $c_N = 0.12 + 0.05Q_N$ and $c_P = 0.18 + 0.07Q_P$, as NB usually has some advantage in the production cost compared with PL.

For the scenario with symmetric spillover rates and asymmetric quality, we find the following preference sequences for the retailer:

- Case N $\succ$ Case B;
- Case P $\succ$ Case B if and only if $Q_P > \bar{Q}_P^{PB} = 0.423$;
- Case P $\succ$ Case N if and only if $Q_P > \bar{Q}_P^{PN} = 0.648 > \bar{Q}_P^{PB}$.

These results are in line with Proposition 3.

The preference sequences for the NB manufacture are:

- Case N $\succ$ Case P;
- Case N $\succ$ Case B if and only if $Q_P < \bar{Q}_P^{NB} = 0.358$;
- Case P $\succ$ Case B if and only if $Q_P < \bar{Q}_P^{PB} = 0.112 < \bar{Q}_P^{NB}$.

These results are in line with Proposition 4.

For the scenario with asymmetric spillover and symmetric quality, we find that

- For the retailer, Case P $\succ$ Case N if and only if $\lambda_N > \bar{\lambda}_N^R = 0.287$.
- For the NB manufacturer, Case P $\succ$ Case N if and only if $\lambda_N > \bar{\lambda}_N^M = 0.791 > \bar{\lambda}_N^R$;

These results are in line with Proposition 5.

The structure of the results are consistent throughout the numerical tests conducted. So the analytical properties obtained with zero production costs still hold with non-zero production costs.

5.4 Impacts of NB manufacturer's Out-store Marketing Effort

In addition to the retailer's in-store promotion, the NB manufacturer may also insert effort to promote its product outside of stores, through TV or the Internet. While the retailer's in-store promotion effort could spill over to other products, the NB manufacturer's out-store promotion is more likely to be predatory than cooperative. The manufacturer may try to promote its own product by revealing the limitations of the competitor's product. In this section we examine the NB manufacturer's out-store promotion efforts on the retailer's decisions.

We assume the out-store promotion increases the NB product's perceived quality by $M_N = k_N Q_N$, which means the better the quality of the product itself, the more effective the promotion. The NB manufacturer's cost for the promotion effort is $c_k = a_k + b_k k_N^2$, where a_k is a fixed cost for booking the channel and b_k is the variable effort cost. Let λ_k be the impact of NB manufacturer's promotion on PL product. The demand functions in Case P change to

$$\begin{cases} D_N = \dfrac{1}{1-\gamma^2}(Q_N - \gamma Q_P + (1-\gamma\lambda_k)k_N Q_N + (\lambda_N - \gamma)M - p_N + \gamma p_P); \\ D_P = \dfrac{1}{1-\gamma^2}(Q_P - \gamma Q_N + (\lambda_k - \gamma)k_N Q_N + (1-\gamma\lambda_N)M - p_P + \gamma p_N). \end{cases}$$

The demand functions in Case N change to

$$\begin{cases} D_N = \dfrac{1}{1-\gamma^2}(Q_N - \gamma Q_P + (1-\gamma\lambda_k)k_N Q_N + (1-\gamma\lambda_P)M - p_N + \gamma p_P); \\ D_P = \dfrac{1}{1-\gamma^2}(Q_P - \gamma Q_N + (\lambda_k - \gamma)k_N Q_N + (\lambda_P - \gamma)M - p_P + \gamma p_N). \end{cases}$$

The profit functions change from Equation (2) to:

$$\begin{cases} \Pi_M^P = w_N D_N - c_k; \\ \Pi_R^P = (p_N - w_N)D_N + p_P D_P - M^2. \end{cases}$$

The problem becomes more complicated after introducing the out-store promotion. We solve the three cases P/N/B numerically and find the unique equilibrium solution for each case.

To keep the out-store promotion a profitable option to the manufacturer, the promotion cost cannot be too high. In this section, we use $\{\gamma = 0.1, \lambda_{N/P} = 0.3, \theta = 0.02\}$ and keep $a_k \leq 0.1$, $b_k \leq 0.9$, $-0.5 \leq \lambda_k \leq 0.5$. Although the NB manufacturer's out-store promotion is more likely to be predatory than cooperative, here we allow λ_k to be either negative or positive.

Similar to the previous section, our numerical results show that the introducing the out-store promotion does not change the structure of the major results, i.e., Propositions 3, 4 and 5. In addition, we find that in both cases P and N, the NB manufacturer's profit decreases in λ_k, while the retailer's profit increases in λ_k. This result is easy to understand intuitively.

More importantly, our results show that the predatory effect the manufacturer's out-store promotion may make the retailer uninterested in introducing the PL product. For example, when $Q_P = 0.8$, $\gamma = 0.1$ and $\lambda_N = 0.2$, if $\lambda_k \leq -0.25$, the retailer's preference sequence changes from Case P $\succ$ Case N $\succ$ Case B to Case B $\succ$ Case P $\succ$ Case N. So the retailer will not introduce PL because a powerful "predatory" advertisement of NB product undermines the profitability of PL product.

Our results also show that the predatory effect of the out-store promotion can be mitigated by a high spillover rate of the in-store promotion. Following the scenario described above, if λ_N increases from 0.2 to 0.7 while other values remain unchanged, the retailer holds its preference sequence of Case P $\succ$ Case N $\succ$ Case B when $\lambda_k = -0.25$. To change the retailer's preference sequence to Case B $\succ$ Case P $\succ$ Case N, λ_k needs to be -0.3 or lower. So the retailer can still introduce the PL product unless the predatory effect of the out-store promotion is very strong, because a "cooperative" in-store promotion can reduce the predatory effect of the NB manufacturer's out-store promotion. For example, if the retailer strategically put the NB and PL products next to each other, consumers searching for the NB product may end up buying by the PL product. The key driver behind those different marketing strategies is, the retailer earns profit from both products sold in the store, while the NB manufacturer earns profit only from its own product.

6 Conclusion

This paper extends the extant literature on retailer-owned private label product by simultaneously considering the retailer's in-store promotional effort and the spillover effect. We compare three different market scenarios: no private label while promoting the national brand, introducing a private label and promoting it, and introducing a private label but promoting the national brand. We find that the introduction of private label is not preferable for the national brand if there is no spillover effect. However, if spillover effect exists, we find a national brand manufacturer may benefit from the introduction of a competing private label product if the spillover effect is high and product substitutability is low.

We also study impacts of the retailer's in-store promotion decision on firms. First, when the spillover rates between the two products are symmetric, the retailer prefers to promote the private label product and the national brand manufacturer prefer its own product to be promoted. Second, when the spillover from the national brand to the private label is significantly higher than the spillover in the opposite direction, both firms prefer to promote the national brand product. Third, when the spillover from private label to national brand is significantly higher than the opposite spillover, both firms prefer to promote the private label product.

When the spillover rates between the two firms are symmetric, the products' qualities play important roles in the two firms' preferences. On the one hand, the retailer always prefers the private label introduction. If the private label product's quality is high, the retailer also prefers to promote it; otherwise the retailer promotes the national brand product. On the other hand, the manufacturer prefers the private label introduction if and only if the product substitutability is low, or the substitution factor is high but the private label product's quality is low. This is because in both scenarios, the private label product does not substantially challenge the national brand product, which is consistent with the empirical finding by Pauwels and Srinivasan (2004) that premium-brand manufacturers can benefit from private label introduction.

Our numerical analysis reveals that a higher spillover rate does not always benefit the whole supply chain. When the retailer introduces a low quality private label and promotes the high quality national brand, a higher spillover rate from the national brand product to the private label product may intensify the channel conflict and the supply chain profit does not monotonically increase with the spillover rate. This finding helps explain why some retailers do not put the national brand product and the private label product close together. Our numerical study also suggests that the firms would opt for controlling the in-store promotion spillover rates to optimize their profits. Our numerical results also show that the structure of the major results does not change by introducing quality-related production costs or the NB manufacturer's out-store promotions. In addition, our results show that the predatory effect the manufacturer's out-store promotion may dampen the retailer's interest to introduce the PL product, but the predatory effect of the out-store promotion can be mitigated by the spillover effect of the in-store promotion.

This paper has its limitations. First, for tractability, we consider only one national brand product. In reality, a retailer may sell products from multiple national brand manufacturers.

Second, while some of our results are supported by existing empirical studies, the data on in-store media promotion and spillover effect is rare. Therefore, cooperating with retailers to design some field experiments can be a future research priority. Finally, given that the private label introduction is inevitable, how to help manufacturers improve their competitive edge will be the next challenging but intriguing subject.

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Appendix: Online Supplements

Proof of Lemma 1

The Stackelberg game proceeds in two stages. In stage two the retailer (follower) decides the retail prices and the promotional effort. In stage one the NB manufacturer (leader) decides the wholesale price.

1) The retailer determines the retail prices and promotion level:

The demand functions are:

$$\begin{cases} D_N = \dfrac{1}{1-\gamma^2}(Q_N - \gamma Q_P + (\lambda_N - \gamma)M - p_N + \gamma p_P), \\ D_P = \dfrac{1}{1-\gamma^2}(Q_P - \gamma Q_N + (1 - \gamma\lambda_N)M - p_P + \gamma p_N). \end{cases}$$

The revenue functions are:

$$\begin{cases} \Pi_M = w_N D_N, \\ \Pi_R = (p_N - w_N)D_N + p_P D_P - \theta M^2. \end{cases}$$

From this point we apply the assumption $Q_N = 1$ and $\theta = 1$. Notice Π_R is quadratic and concave on $\{p_P, p_N, M\}$, because $\dfrac{\partial^2 \Pi_R}{\partial p_P^2} = \dfrac{\partial^2 \Pi_R}{\partial p_N^2} = -\dfrac{2}{1-\gamma^2} < 0$, and $\dfrac{\partial^2 \Pi_R}{\partial M^2} = -2 < 0$. Therefore the unique optimal solution of retail prices and promotional effort exists.

Using the first order condition, we solve $\{\dfrac{\partial \Pi_R}{\partial p_P} = 0, \dfrac{\partial \Pi_R}{\partial p_N} = 0, \dfrac{\partial \Pi_R}{\partial M} = 0\}$ and get

$$\begin{cases} p_P = -\dfrac{\gamma\Gamma - 4Q_P - \gamma\Gamma w_N - \Gamma\lambda_N - \gamma\Gamma Q_P\lambda_N + \Gamma w_N\lambda_N + \Gamma Q_P\lambda_N^2}{2(4 - \Gamma + 2\gamma\Gamma\lambda_N - \Gamma\lambda_N^2)}, \\ p_N = -\dfrac{-4 + \Gamma - 4w_N + \Gamma w_N - \gamma\Gamma\lambda_N - \Gamma Q_P\lambda_N - 3\gamma\Gamma w_N\lambda_N + \gamma\Gamma Q_P\lambda_N^2 + 2\Gamma w_N\lambda_N^2}{2(4 - \Gamma + 2\gamma\Gamma\lambda_N - \Gamma\lambda_N^2)}, \\ M = -\dfrac{\Gamma(\gamma - Q_P - \gamma w_N - \lambda_N + \gamma Q_P\lambda_N + w_N\lambda_N)}{4 - \Gamma + 2\gamma\Gamma\lambda_N - \Gamma\lambda_N^2}. \end{cases}$$

where $\Gamma = \dfrac{1}{1-\gamma^2}$.

2) The NB manufacturer decides the wholesale price:

Substitute the above $\{p_P, p_N, M\}$ into Π_M. We verify $\dfrac{\partial^2 \Pi_M}{\partial w_N^2} < 0$ and the optimal wholesale price exists. Solve $\dfrac{\partial \Pi_M}{\partial w_N} = 0$ to get $w_N = \dfrac{1}{6}(3 + Q_P(-4\gamma + \lambda_N))$.

Substitute the above w_N into other variables and get the whole set of closed-form solution as follows.

$$\begin{cases} w_N = \dfrac{1}{6}(3 + Q_P(-4\gamma + \lambda_N)), \\ p_N = \dfrac{3(-9 + 12\gamma^2 - 5\gamma\lambda_N + 2\lambda_N^2) + Q_P(4\gamma(3 - 4\gamma^2) + (-9 + 16\gamma^2)\lambda_N - 5\gamma\lambda_N^2 + 2\lambda_N^3)}{12(-3 + 4\gamma^2 - 2\gamma\lambda_N + \lambda_N^2)}, \\ p_P = \dfrac{3(\gamma - \lambda_N) + Q_P(-24 + 28\gamma^2 - 11\gamma\lambda_N + 7\lambda_N^2)}{12(-3 + 4\gamma^2 - 2\gamma\lambda_N + \lambda_N^2)}, \\ M = \dfrac{3(\gamma - \lambda_N) + Q_P(-6 + 4\gamma^2 + \gamma\lambda_N + \lambda_N^2)}{6(-3 + 4\gamma^2 - 2\gamma\lambda_N + \lambda_N^2)}, \\ D_N = -\dfrac{3 + Q_P(-4\gamma + \lambda_N)}{4(-3 + 4\gamma^2 - 2\gamma\lambda_N + \lambda_N^2)}, \\ D_P = \dfrac{12\gamma - 3\lambda_N + Q_P(-24 + 16\gamma^2 - 8\gamma\lambda_N + 7\lambda_N^2)}{12(-3 + 4\gamma^2 - 2\gamma\lambda_N + \lambda_N^2)}. \end{cases}$$

The NB manufacturer's and the retailer's profits are

$$\begin{cases} \Pi_M^P = \dfrac{(-3 + Q_P(4\gamma - \lambda_N))^2}{24(3 - 4\gamma^2 + 2\gamma\lambda_N - \lambda_N^2)}, \\ \Pi_R^P = \dfrac{3 - Q_P(8\gamma - 2\lambda_N) + Q_P^2(16(1 - \gamma^2) + 8\gamma\lambda_N - 5\lambda_N^2)}{16(3 - 4\gamma^2 + 2\gamma\lambda_N - \lambda_N^2)}. \end{cases}$$

Notice if $\lambda_N = 0$, $D_N = \dfrac{3 - 4\gamma Q_P}{4(3 - 4\gamma^2)}$ and $w_N = 1/6(3 - 4\gamma Q_P)$. Given the basic non-negative condition $Q_P < 1/\gamma$ and $3 - 4\gamma Q_P > 0$, we have a non-negative condition as $3 - 4\gamma^2 > 0$, or $\gamma < \sqrt{3}/2$. Q.E.D.

Proof of Lemma 2

The process is similar to the proof of Lemma 1 as follows.

1) The retailer determines the retail prices and promotion level:

$$\begin{cases} p_P = -\dfrac{-4Q_P + \Gamma Q_P - \Gamma\lambda_P - \gamma\Gamma Q_P\lambda_P + \Gamma w_N\lambda_P + \gamma\Gamma\lambda_P^2 - \gamma\Gamma w_N\lambda_P^2}{2(4 - \Gamma + 2\gamma\Gamma\lambda_P - \Gamma\lambda_P^2)}, \\ p_N = -\dfrac{-4 + \gamma\Gamma Q_P - 4w_N + 2\Gamma w_N - \gamma\Gamma\lambda_P - \Gamma Q_P\lambda_P - 3\gamma\Gamma w_N\lambda_P + \Gamma\lambda_P^2 + \Gamma w_N\lambda_P^2}{2(4 - \Gamma + 2\gamma\Gamma\lambda_P - \Gamma\lambda_P^2)}, \\ M = \dfrac{\Gamma(1 - \gamma Q_P - w_N - \gamma\lambda_P + Q_P\lambda_P + \gamma w_N\lambda_P)}{4 - \Gamma + 2\gamma\Gamma\lambda_P - \Gamma\lambda_P^2}. \end{cases}$$

where $\Gamma = \dfrac{1}{1 - \gamma^2}$.

2) The NB manufacturer decides the wholesale price:

Substitute the above $\{p_P, p_N, M\}$ into Π_M. We verify $\dfrac{\partial^2\Pi_M}{\partial w_N^2} < 0$ and the optimal wholesale price exists. Solve $\dfrac{\partial\Pi_M}{\partial w_N} = 0$ to get $w_N = \dfrac{-4 + Q_P(4\gamma - \lambda_P) + \lambda_P^2}{2(-4 + \lambda_P^2)}$.

Substitute the above w_N into other variables and get the whole set of closed-form solution as follows.

$$\begin{cases} w_N = \dfrac{-4 + Q_P(4\gamma - \lambda_P) + \lambda_P^2}{2(-4 + \lambda_P^2)}, \\ p_N = \dfrac{(-4 + \lambda_P^2)(-10 + 12\gamma^2 - 5\gamma\lambda_P + 3\lambda_P^2) + Q_P(16\gamma(-1 + \gamma^2) - 2(-5 + 8\gamma^2)\lambda_P + 9\gamma\lambda_P^2 - 3\lambda_P^3)}{4(-4 + \lambda_P^2)(-3 + 4\gamma^2 - 2\gamma\lambda_P + \lambda_P^2)}, \\ p_P = \dfrac{\lambda_P(-1 + \gamma\lambda_P)(-4 + \lambda_P^2) + Q_P(24 - 32\gamma^2 + 12\gamma\lambda_P + (-7 + 4\gamma^2)\lambda_P^2 - \gamma\lambda_P^3)}{4(-4 + \lambda_P^2)(-3 + 4\gamma^2 - 2\gamma\lambda_P + \lambda_P^2)}, \\ M = \dfrac{(-1 + \gamma\lambda_P)(-4 + \lambda_P^2) + Q_P(-4\gamma + (7 - 4\gamma^2)\lambda_P + 3\gamma\lambda_P^2 - 2\lambda_P^3)}{2(-4 + \lambda_P^2)(-3 + 4\gamma^2 - 2\gamma\lambda_P + \lambda_P^2)}, \\ D_N = \dfrac{-4 + Q_P(4\gamma - \lambda_P) + \lambda_P^2}{4(-3 + 4\gamma^2 - 2\gamma\lambda_P + \lambda_P^2)}, \\ D_P = \dfrac{Q_P(24 - 16\gamma^2 + 8\gamma\lambda_P - 7\lambda_P^2) + (4\gamma - \lambda_P)(-4 + \lambda_P^2)}{4(-4 + \lambda_P^2)(-3 + 4\gamma^2 - 2\gamma\lambda_P + \lambda_P^2)}. \end{cases}$$

the NB manufacturer's and the retailer's profits are

$$\begin{cases} \Pi_M^N = \dfrac{(-4 + Q_P(4\gamma - \lambda_P) + \lambda_P^2)^2}{8(-4 + \lambda_P^2)(-3 + 4\gamma^2 - 2\gamma\lambda_P + \lambda_P^2)}, \\ \Pi_R^N = \dfrac{2Q_P(4\gamma - \lambda_P)(-4 + \lambda_P^2) + (-4 + \lambda_P^2)^2 - 3Q_P^2(16(-1 + \gamma^2) - 8\gamma\lambda_P + 5\lambda_P^2)}{16(-4 + \lambda_P^2)(-3 + 4\gamma^2 - 2\gamma\lambda_P + \lambda_P^2)}. \end{cases}$$

Q.E.D.

Proof of Lemma 3

The process is similar to the proof of Lemma 1 as follows. Note that only NB product is in the market.

1) The retailer determines the retail price and promotion level:

$$\begin{cases} p_N = \dfrac{2\theta Q_N - w_N + 2\theta w_N}{4\theta - 1} \\ M = \dfrac{Q_N - w_N}{4\theta - 1} \end{cases}$$

2) The NB manufacturer decides the wholesale price: Substitute the above $\{p_N, M\}$ into Π_M. Given $\theta = 1$, $\dfrac{\partial^2\Pi_M}{\partial w_N^2} = \dfrac{4\theta}{1 - 4\theta} < 0$ and the optimal wholesale price exists. Solve $\dfrac{\partial\Pi_M}{\partial w_N} = 0$

to get $w_N = Q_N/2$.

Substitute the above w_N into other variables and get the whole set of closed-form solution as follows.

$$\begin{cases} w_N = Q_N/2, \\ p_N = (6\theta - 1)Q_N/(8\theta - 2), \\ M = Q_N/(8\theta - 2), \\ D_N = \theta Q_N/(4\theta - 1). \end{cases}$$

the NB manufacturer's and the retailer's profits are

$$\begin{cases} \Pi_M = \theta Q_N^2/(8\theta - 2), \\ \Pi_R = \theta Q_N^2/(16\theta - 4). \end{cases}$$

After applying $Q_N = 1$ and $\theta = 1$, $\{w_N, p_N, M, D_N, \Pi_M, \Pi_R\} = \{1/2, 5/6, 1/6, 1/3, 1/6, 1/12\}$. Q.E.D.

Proof of Lemma 4

The models without promotion are simplified versions of Case B, Case P and N by removing the promotional effort M. By solving the simplified models, the NB manufacturer's and the retailer's profits without PL introduction are $\Pi_M^0 = Q_N^2/8$ and $\Pi_R^0 = Q_N^2/16$. The profits with PL introduction are

$$\begin{cases} \Pi_M^1 = \dfrac{(Q_N - \gamma Q_P)^2}{8(1-\gamma^2)}, \\ \Pi_R^1 = \dfrac{Q_N^2 - 2\gamma Q_N Q_P + (4 - 3\gamma^2)Q_P^2}{16(1-\gamma^2)}. \end{cases}$$

Since $\Pi_R^1 - \Pi_R^0 = \dfrac{3(1-\gamma^2)Q_P^2 + (Q_P - \gamma)^2}{16(1-\gamma^2)} > 0$, the retailer prefers to introduce PL product. Since $\Pi_M^1 - \Pi_M^0 = -\dfrac{(Q_P - \gamma) + Q_P(1 - \gamma Q_P)}{8(1-\gamma^2)} < 0$, the NB manufacturer prefers not. Q.E.D.

Proof of Lemma 5

In this proof we apply the following non-negative conditions: $\gamma < Q_P < 1/\gamma$ and $3 - 4\gamma^2 > 0$ (see the end of Lemma 1).

From Lemmas 1, 2 and 3 and by setting $\lambda_N = \lambda_P = 0$, we have the closed-form solutions of the

NB manufacturer's profits in Case B, Case P and N as follows.

$$\begin{cases} \Pi_M^B = 1/6, \\ \Pi_M^P = \dfrac{(4\gamma Q_P - 3)^2}{24(4(1-\gamma^2)-1)}, \\ \Pi_M^N = \dfrac{(1-\gamma Q_P)^2}{6-8\gamma^2}. \end{cases}$$

$\Pi_M^N - \Pi_M^B = \dfrac{(-1+\gamma Q_P)^2}{6-8\gamma^2} - \dfrac{1}{6} < 0 \Leftrightarrow 4\gamma - 6Q_P + 3\gamma Q_P^2 < 0$. Let $f = 4\gamma - 6Q_P + 3\gamma Q_P^2$. $\dfrac{\partial f}{\partial Q_P} = -6(1-\gamma Q_P) < 0$, that is, f decreases with Q_P. When $Q_P = 1/\gamma$ we have $f = (4\gamma^2 - 3)/\gamma < 0$. That is, for all $Q_P < 1/\gamma$, $f < 0$ holds. Then we can conclude $\Pi_M^B > \Pi_M^N$.

$\Pi_M^N - \Pi_M^P = \dfrac{3-4\gamma^2 Q_P^2}{24(3-4\gamma^2)}$. In Lemma 1 we have the non-negative conditions $3 - 4\gamma Q_P > 0$. Then $3 - 4\gamma^2 Q^2 > 3 - 4\gamma Q > 0$ holds. In summary, we have $\Pi_M^N > \Pi_M^P$. Q.E.D.

Proof of Lemma 6

In this proof we apply the following non-negative conditions: $\gamma < Q_P < 1/\gamma$ and $3 - 4\gamma^2 > 0$ (see the end of Lemma 1).

From Lemmas 1, 2 and 3 and by setting $\lambda_N = \lambda_P = 0$, we have the closed-form solutions of retailer's profit in Case B, Case P and N as follows.

$$\begin{cases} \Pi_R^B = 1/12, \\ \Pi_R^P = \dfrac{-3+8\gamma Q_P + 16(-1+\gamma^2)Q_P^2}{-48+64\gamma^2}. \\ \Pi_R^N = \dfrac{-1+2\gamma Q_P + 3(-1+\gamma^2)Q_P^2}{4(-3+4\gamma^2)}. \end{cases}$$

$\Pi_R^N - \Pi_R^B = \dfrac{-4\gamma^2 + 6\gamma Q_P + 9(-1+\gamma^2)Q_P^2}{-36+48\gamma^2} > 0 \Leftrightarrow 4\gamma^2 - 6\gamma Q_P + 9(1-\gamma^2)Q_P^2 > 0$. Let $f = 4\gamma^2 - 6\gamma Q_P + 9(1-\gamma^2)Q_P^2$. $\dfrac{\partial f}{\partial Q_P} = 6(3(1-\gamma^2)Q_P - \gamma) > 0$, that is, f increases with Q_P. When $Q_P = \gamma$ we have $f = \gamma^2(7-9\gamma^2) > 0$. That is, for all $Q_P > \gamma$, $f > 0$ holds. Then we conclude $\Pi_R^N > \Pi_R^B$.

Solve the quadratic equation $\Pi_R^P - \Pi_R^B = \dfrac{48(1-\gamma^2)Q_P^2 - 3 + 16\gamma^2 - 24\gamma Q_P}{48(3-4\gamma^2)} = 0$ on Q_P, we will have two roots:

$$Q_+ = \frac{3\gamma + \sqrt{3}\sqrt{16\gamma^4 - 16\gamma^2 + 3}}{12(1-\gamma^2)}, Q_- = \frac{3\gamma - \sqrt{3}\sqrt{16\gamma^4 - 16\gamma^2 + 3}}{12(1-\gamma^2)}.$$

It is easy to verify $Q_- < \gamma$ which contradicts the non-negative condition $Q_P > \gamma$, and then the larger root $Q_+ = \bar{Q}_P^{PB}$ is the only feasible threshold. Noticing that the quadratic equation is convex on Q_P, we can conclude that when $Q_P > \bar{Q}_P^{PB}$, $\Pi_R^P > \Pi_R^B$.

Consider $\Pi_R^P - \Pi_R^N = \dfrac{4(1-\gamma^2)Q_P^2 - 1}{16(3-4\gamma^2)} = 0$. We find that when $Q_P > \bar{Q}_P^{PN} = \dfrac{1}{2\sqrt{1-\gamma^2}}$, $\Pi_R^P > \Pi_R^N$.

Comparing between the two thresholds, we find that $\bar{Q}_P^{PB} < \bar{Q}_P^{PN}$. Q.E.D.

Proof of Proposition 1

From Lemmas 1, 2 and 3 we have the closed-form solutions of Case P, Case N and Case B. When $\lambda_P = \lambda_N = \lambda$ and $Q_P = Q_N = 1$, the retailer's profit gap between Case P and Case N is

$$\Delta_{PN} = \Pi_R^P - \Pi_R^N = \frac{16\gamma^2 - 8\gamma(\lambda-1) + 5\lambda^2 - 2\lambda - 19}{16(4\gamma^2 - 2\gamma\lambda + \lambda^2 - 3)} - \frac{64 - 48\gamma^2 + 8\gamma(\lambda^2 + 3\lambda - 4) + \lambda^4 - 2\lambda^3 - 23\lambda^2 + 8\lambda}{16(\lambda^2 - 4)(4\gamma^2 - 2\gamma\lambda + \lambda^2 - 3)}.$$

To find the minimal and maximal values of Δ_{PN}, we solve two constrained nonlinear programming problems given $0 \le \lambda \le 1$ and $0 \le \gamma \le 4/5$. We find $0 \le \Delta_{PN} \le 0.0625$, that is Case P $\succ$ Case N.

Similarly, let

$$\Delta_{NB} = \Pi_R^N - \Pi_R^B = \frac{64 - 48\gamma^2 + 8\gamma(\lambda^2 + 3\lambda - 4) + \lambda^4 - 2\lambda^3 - 23\lambda^2 + 8\lambda}{16(\lambda^2 - 4)(4\gamma^2 - 2\gamma\lambda + \lambda^2 - 3)} - \frac{1}{12}.$$

To find the minimal and maximal values of Δ_{NB}, we solve two constrained nonlinear programming problems given $0 \le \lambda \le 1$ and $0 \le \gamma \le 4/5$. We find $0.1875 \le \Delta_{NB} \le 0.4167$, that is Case N $\succ$ Case B. Q.E.D.

Proof of Proposition 2

From Lemmas 1, 2 and 3, we have the closed-form solutions of Case P, Case N and Case B. When $\lambda_P = \lambda_N = \lambda$ and $Q_P = Q_N = 1$, the NB manufacturer's profits are

$$\begin{cases} \Pi_M^P = \dfrac{((\lambda - 4\gamma) + 3)^2}{24(-4\gamma^2 + 2\gamma\lambda - \lambda^2 + 3)}, \\ \Pi_M^N = \dfrac{(\lambda^2 + (4\gamma - \lambda) - 4)^2}{8(4-\lambda^2)(-4\gamma^2 + 2\gamma\lambda - \lambda^2 + 3)}, \\ \Pi_M^B = 1/6. \end{cases}$$

Firstly, $\Pi_M^N - \Pi_M^P = \dfrac{1-\lambda^2}{6(4-\lambda^2)} > 0$, that is Case N $\succ$ Case P. Secondly, $\Pi_M^N - \Pi_M^B = 0$ can be transformed into a quadratic equation of γ:

$$\gamma^2(16(4-\lambda^2)+48)+\gamma(24\lambda^2-8(4-\lambda^2)\lambda-24\lambda-96)$$
$$+3\lambda^4-6\lambda^3+4(4-\lambda^2)\lambda^2-21\lambda^2-12(4-\lambda^2)+24\lambda+48=0.$$

The above function is convex because $16(4-\lambda^2)+48>0$. The equation has two roots as $\gamma_-=\lambda/4$ and $\gamma_+=\dfrac{\lambda^3+6\lambda^2-7\lambda-24}{4(\lambda^2-7)}$. By solving a constrained nonlinear programming problem given $0\leq\lambda\leq 1$, we find $\gamma_+\geq 0.857$ (infeasible). Such that γ_- is the only feasible threshold. Then we can conclude that Case N $\succ$ Case B when $\lambda>4\gamma=\bar{\lambda}^M_{NB}$. This threshold implies that when $\gamma>1/4$, Case B $\succ$ Case N because $\lambda\leq 1$.

Thirdly, $\Pi^P_M-\Pi^B_M=0$ can be transformed into a quadratic equation of λ:

$$30\lambda^2+(36-96\gamma)\lambda+192\gamma^2-144\gamma-18=0.$$

The above function is convex. The equation has two roots as $\lambda_-=\frac{1}{5}(-2\sqrt{6}\sqrt{-4\gamma^2+3\gamma+1}+8\gamma-3)$ and $\lambda_+=\frac{1}{5}(2\sqrt{6}\sqrt{-4\gamma^2+3\gamma+1}+8\gamma-3)$. By solving a constrained nonlinear programming problem given $0\leq\gamma\leq 0.8$, we find $\lambda_-\leq -0.218$ (infeasible). Such that λ_+ is the only feasible threshold. Then we can conclude that Case P $\succ$ Case B when $\lambda>\lambda_+=\bar{\lambda}^M_{PB}$.

Lastly, to compare between the two thresholds we only need to consider the situation of $\gamma\leq 1/4$, because when $\gamma>1/4$, Case B $\succ$ Case N $\succ$ Case P. We find $\bar{\lambda}^M_{PB}-\bar{\lambda}^M_{NB}\geq 0$ given $\gamma\leq 1/4$.

To express the thresholds as functions of λ, firstly we notice $\bar{\gamma}^M_{NB}=\lambda/4$ can be inferred from Proposition 2 directly. From Proposition 2, the threshold between Case P and Case B in the space $\{\gamma,\lambda\}$ is $(2\sqrt{6}\sqrt{-4\gamma^2+3\gamma+1}+8\gamma-3)/5=\lambda$. Solving γ from this equation yields two roots as $\gamma_-=(2\lambda+3-\sqrt{3}\sqrt{5-2\lambda^2})/8$ and $\gamma_+=(2\lambda+3+\sqrt{3}\sqrt{5-2\lambda^2})/8$. By solving a constrained nonlinear programming problem given $0\leq\lambda\leq 1$, we find $\gamma_+\geq 0.86$ (infeasible). Such that γ_- is the only feasible threshold. To compare between the two thresholds, we find $\bar{\gamma}^M_{PB}-\bar{\gamma}^M_{NB}\leq 0$ given $0\leq\lambda\leq 1$. Q.E.D.

Proof of Proposition 3

From Lemmas 1, 2 and 3, we have the closed-form solutions of Case P, Case N and Case B.

When $\lambda_P = \lambda_N = \lambda$ and $Q_N = 1$, the retailer's profits are

$$\begin{cases} \Pi_R^P = \dfrac{(-16\gamma^2 + 8\gamma\lambda - 5\lambda^2 + 16)Q_P^2 + (2\lambda - 8\gamma)Q_P + 3}{16(-4\gamma^2 + 2\gamma\lambda - \lambda^2 + 3)}, \\ \Pi_R^N = \dfrac{(\lambda^2 - 4)^2 - 3(16\gamma^2 - 8\gamma\lambda + 5\lambda^2 - 16)Q_P^2 + 2(\lambda^2 - 4)(4\gamma - \lambda)Q_P}{16(\lambda^2 - 4)(4\gamma^2 - 2\gamma\lambda + \lambda^2 - 3)}, \\ \Pi_R^B = 1/12. \end{cases}$$

Firstly, by solving a constrained nonlinear programming problem given $0 \le \lambda \le 1$, $0 \le \gamma \le 4/5$ and $\gamma < Q_P < min(1/\gamma, 2)$, we find $\Pi_R^N - \Pi_R^B \ge 0$.

Secondly, $\Pi_R^P - \Pi_R^B = 0$ can be transformed into a quadratic equation of Q_P:

$$12(-16\gamma^2 + 8\gamma\lambda - 5\lambda^2 + 16)Q_P^2 + 12(2\lambda - 8\gamma)Q_P - 16(-4\gamma^2 + 2\gamma\lambda - \lambda^2 + 3) + 36 = 0.$$

The above function is convex, because $-16\gamma^2 + 8\gamma\lambda - 5\lambda^2 + 16 \ge 5.76$ given $0 \le \lambda \le 1$ and $0 \le \gamma \le 4/5$. The equation has two roots:

$$\begin{cases} Q_- = \dfrac{-2\sqrt{3}\sqrt{64\gamma^4 - 64\gamma^3\lambda + 52\gamma^2\lambda^2 - 64\gamma^2 - 18\gamma\lambda^3 + 32\gamma\lambda + 5\lambda^4 - 19\lambda^2 + 12} + 12\gamma - 3\lambda}{3(-16\gamma^2 + 8\gamma\lambda - 5\lambda^2 + 16)}, \\ Q_+ = \dfrac{2\sqrt{3}\sqrt{64\gamma^4 - 64\gamma^3\lambda + 52\gamma^2\lambda^2 - 64\gamma^2 - 18\gamma\lambda^3 + 32\gamma\lambda + 5\lambda^4 - 19\lambda^2 + 12} + 12\gamma - 3\lambda}{3(-16\gamma^2 + 8\gamma\lambda - 5\lambda^2 + 16)}. \end{cases}$$

By solving a constrained nonlinear programming problem given $0 \le \lambda \le 1$ and $0 \le \gamma \le 4/5$, we find $Q_- \le -0.111$ (infeasible). Such that $Q_+ = \bar{Q}_P^{PB}$ is the only feasible threshold. Notice $\bar{Q}_P^{PB}$ is the larger root, and we conclude that Case P $\succ$ Case B when $Q_P > \bar{Q}_P^{PB}$.

Thirdly, $\Pi_R^P - \Pi_R^N = 0$ can be transformed into a quadratic equation of Q_P:

$$\begin{aligned} &(3(16\gamma^2 - 8\gamma\lambda + 5\lambda^2 - 16) + (4 - \lambda^2)(-16\gamma^2 + 8\gamma\lambda - 5\lambda^2 + 16))Q_P^2 \\ &+ ((4 - \lambda^2)(2\lambda - 8\gamma) - 2(\lambda^2 - 4)(4\gamma - \lambda))Q_P - (\lambda^2 - 4)^2 + 3(4 - \lambda^2) = 0. \end{aligned}$$

The above function is convex, because $3(16\gamma^2 - 8\gamma\lambda + 5\lambda^2 - 16) + (4 - \lambda^2)(-16\gamma^2 + 8\gamma\lambda - 5\lambda^2 + 16) \ge$

0.143 given $0 \le \lambda \le 1$ and $0 \le \gamma \le 4/5$. The equation has two roots:

$$\begin{cases} Q_- = -\dfrac{\sqrt{\lambda^4 - 5\lambda^2 + 4}}{\sqrt{16\gamma^2\lambda^2 - 16\gamma^2 - 8\gamma\lambda^3 + 8\gamma\lambda + 5\lambda^4 - 21\lambda^2 + 16}}, \\ Q_+ = \dfrac{\sqrt{\lambda^4 - 5\lambda^2 + 4}}{\sqrt{16\gamma^2\lambda^2 - 16\gamma^2 - 8\gamma\lambda^3 + 8\gamma\lambda + 5\lambda^4 - 21\lambda^2 + 16}}. \end{cases}$$

It is easy to see the larger root $Q_+ = \bar{Q}_P^{PN}$ is the only feasible threshold. Then we can conclude that Case P $\succ$ Case N when $Q_P > \bar{Q}_P^{PN}$.

Lastly, to compare between the two thresholds, we find $\bar{Q}_P^{PN} - \bar{Q}_P^{PB} \ge 0$ given $0 \le \lambda \le 1$ and $0 \le \gamma \le 4/5$. Q.E.D.

Proof of Proposition 4

From Lemmas 1, 2 and 3, we have the closed-form solutions of Case P, Case N and Case B. When $\lambda_P = \lambda_N = \lambda$ and $Q_N = 1$, the NB manufacturer's profits are

$$\begin{cases} \Pi_M^P = \dfrac{((\lambda - 4\gamma)Q_P + 3)^2}{24(-4\gamma^2 + 2\gamma\lambda - \lambda^2 + 3)}, \\ \Pi_M^N = \dfrac{(\lambda^2 + (4\gamma - \lambda)Q_P - 4)^2}{8(4 - \lambda^2)(-4\gamma^2 + 2\gamma\lambda - \lambda^2 + 3)}, \\ \Pi_M^B = 1/6. \end{cases}$$

Firstly, by solving a constrained nonlinear programming problem given $0 \le \lambda \le 1$, $0 \le \gamma \le 4/5$ and $\gamma < Q_P < min(1/\gamma, 2)$, we find $\Pi_M^N - \Pi_M^P \ge 0$.

Secondly, $\Pi_M^N - \Pi_M^B = 0$ can be transformed into a quadratic equation of Q_P:

$$3(4\gamma-\lambda)^2Q_P^2 + (6\lambda^2(4\gamma-\lambda) - 24(4\gamma-\lambda))Q_P - 4(4-\lambda^2)(-4\gamma^2+2\gamma\lambda-\lambda^2+3) + 3\lambda^4 - 24\lambda^2 + 48 = 0.$$

The above function is convex. The equation has two roots:

$$\begin{cases} Q_- = \dfrac{-2\sqrt{3}\sqrt{4\gamma^2\lambda^2 - 16\gamma^2 - 2\gamma\lambda^3 + 8\gamma\lambda + \lambda^4 - 7\lambda^2 + 12} - 3\lambda^2 + 12}{3(4\gamma - \lambda)}, \\ Q_+ = \dfrac{2\sqrt{3}\sqrt{4\gamma^2\lambda^2 - 16\gamma^2 - 2\gamma\lambda^3 + 8\gamma\lambda + \lambda^4 - 7\lambda^2 + 12} - 3\lambda^2 + 12}{3(4\gamma - \lambda)}. \end{cases}$$

By solving a constrained nonlinear programming problem given $0 \le \lambda \le 1$ and $0 \le \gamma \le 4/5$, we find $Q_+ \ge 2.294$ (infeasible). Such that $Q_- = \bar{Q}_P^{NB}$ is the only feasible threshold.

Whether $\bar{Q}_P^{NB}$ is the smaller root or larger root depends on the sign of $4\gamma - \lambda$. If $4\gamma - \lambda > 0$, $\bar{Q}_P^{NB}$ is the smaller root and Case N $\succ$ Case B when $Q_P < \bar{Q}_P^{NB}$. If $4\gamma - \lambda < 0$, $\bar{Q}_P^{NB}$ is the larger root and Case N $\succ$ Case B when $Q_P > \bar{Q}_P^{NB}$. However, when $4\gamma - \lambda < 0$, we find $\bar{Q}_P^{NB} < 0$ by solving this constrained nonlinear programming problem, which means Case N $\succ$ Case B for all $Q_P \geq 0$, and thus the threshold $\bar{Q}_P^{NB}$ can be dropped in this scenario.

Thirdly, $\Pi_M^P - \Pi_M^B = 0$ can be transformed into a quadratic equation of Q_P:

$$(\lambda - 4\gamma)^2 Q_P^2 + 6(\lambda - 4\gamma)Q_P - 4(-4\gamma^2 + 2\gamma\lambda - \lambda^2 + 3) + 9 = 0.$$

The above function is convex. The equation has two roots:

$$\begin{cases} Q_- = \dfrac{-\sqrt{(6\lambda - 24\gamma)^2 - 4(16\gamma^2 - 8\gamma\lambda + \lambda^2)(16\gamma^2 - 8\gamma\lambda + 4\lambda^2 - 3)} + 24\gamma - 6\lambda}{2(16\gamma^2 - 8\gamma\lambda + \lambda^2)}, \\ Q_+ = \dfrac{\sqrt{(6\lambda - 24\gamma)^2 - 4(16\gamma^2 - 8\gamma\lambda + \lambda^2)(16\gamma^2 - 8\gamma\lambda + 4\lambda^2 - 3)} + 24\gamma - 6\lambda}{2(16\gamma^2 - 8\gamma\lambda + \lambda^2)}. \end{cases}$$

By solving a constrained nonlinear programming problem given $0 \leq \lambda \leq 1$ and $0 \leq \gamma \leq 4/5$, we find $Q_+ \geq 2.36$ (infeasible). Such that $Q_- = \bar{Q}_P^{PB}$ is the only feasible threshold. Then we can conclude that Case P $\succ$ Case B when $Q_P < \bar{Q}_P^{PB}$.

Lastly, to compare between the two thresholds, we find $\bar{Q}_P^{NB} - \bar{Q}_P^{PB} \geq 0$ given $0 \leq \lambda \leq 1$ and $0 \leq \gamma \leq 4/5$. Q.E.D.

Proof of Proposition 5

From Lemmas 1 and 2 we have the closed-form solutions of Case P and Case N, and we keep $Q_P = Q_N = 1$. It is easy to verify that $\Pi_M^P < \Pi_M^N$ and $\Pi_R^P < \Pi_R^N$ when $\{\lambda_P, \lambda_N\} = \{1, 0\}$. Similarly, $\Pi_M^P > \Pi_M^N$ and $\Pi_R^P > \Pi_R^N$ when $\{\lambda_P, \lambda_N\} = \{0, 1\}$. Such that there exist two thresholds of $\{\lambda_P, \lambda_N\}$ for each firm to prefer Case P or Case N. In the following we show that $\Pi_R^N - \Pi_R^P$ and $\Pi_M^N - \Pi_M^P$ are quadratic functions of λ_N and analyze their monotonic properties around the thresholds.

The threshold for the retailer

For the retailer, the closed-form of the threshold in space $\{\gamma, \lambda_P, \lambda_N\}$ is determined by

$$\Pi_R^N - \Pi_R^P = \frac{1}{16}(\frac{16(-3\gamma^2 - 2\gamma + 4) - (23 - 8\gamma)\lambda_P^2 + 8(3\gamma + 1)\lambda_P + \lambda_P^4 - 2\lambda_P^3}{(4 - \lambda_P^2)(-4\gamma^2 + 2\gamma\lambda_P - \lambda_P^2 + 3)}$$
$$- \frac{-16\gamma^2 - 8\gamma + 2(4\gamma + 1)\lambda_N - 5\lambda_N^2 + 19}{-4\gamma^2 + 2\gamma\lambda_N - \lambda_N^2 + 3}) = 0.$$

The above condition can be transformed into a quadratic equation of λ_N as $a_2\lambda_N^2 + a_1\lambda_N + a_0 = 0$, where

$$\begin{cases} a_2 = -16(-3\gamma^2 - 2\gamma + 4) + 5(4 - \lambda_P^2)(-4\gamma^2 + 2\gamma\lambda_P - \lambda_P^2 + 3) + (23 - 8\gamma)\lambda_P^2 - 8(3\gamma + 1)\lambda_P - \lambda_P^4 + 2\lambda_P^3, \\ a_1 = 2\gamma(16(-3\gamma^2 - 2\gamma + 4) - (23 - 8\gamma)\lambda_P^2 + 8(3\gamma + 1)\lambda_P + \lambda_P^4 - 2\lambda_P^3) \\ - 2(4\gamma + 1)(4 - \lambda_P^2)(-4\gamma^2 + 2\gamma\lambda_P - \lambda_P^2 + 3), \\ a_0 = 16\gamma^2(4 - \lambda_P^2)(-4\gamma^2 + 2\gamma\lambda_P - \lambda_P^2 + 3) - 4\gamma^2(16(-3\gamma^2 - 2\gamma + 4) \\ - (23 - 8\gamma)\lambda_P^2 + 8(3\gamma + 1)\lambda_P + \lambda_P^4 - 2\lambda_P^3) + 8\gamma(4 - \lambda_P^2)(-4\gamma^2 + 2\gamma\lambda_P - \lambda_P^2 + 3) \\ - 19(4 - \lambda_P^2)(-4\gamma^2 + 2\gamma\lambda_P - \lambda_P^2 + 3) + 3(16(-3\gamma^2 - 2\gamma + 4) - (23 - 8\gamma)\lambda_P^2 + 8(3\gamma + 1)\lambda_P + \lambda_P^4 - 2\lambda_P^3). \end{cases}$$

The equation has two roots as $\lambda_{N-} = \dfrac{-a_1 - \sqrt{a_1^2 - 4a_0a_2}}{2a_2}$ and $\lambda_{N+} = \dfrac{-a_1 + \sqrt{a_1^2 - 4a_0a_2}}{2a_2}$. Firstly, to find the minimal and maximal values of λ_{N+}, we solve two constrained nonlinear programming problems given $0 \leq \lambda_P \leq 1$, $0 \leq \gamma \leq 4/5$ and $a_2 \geq 0$, and find $2.49 \leq \lambda_{N+} \leq +\infty$; similarly we find $-1.71 \leq \lambda_{N-} \leq -0.28$. That means when $a_2 \geq 0$ there is no feasible threshold for $\lambda_N \in [0, 1]$. Secondly, under the constraints of $a_1^2 - 4a_0a_2 \geq 0$ and $a_2 \leq 0$ we find $-2.39 \leq \lambda_{N+} \leq -1.34$ (infeasible), which means λ_{N-} is the only feasible threshold for $\lambda_N \in [0, 1]$. Since we have $a_2 \leq 0$, $a_2\lambda_N^2 + a_1\lambda_N + a_0 = 0$ is a concave function and λ_{N-} is the larger root. Finally we can conclude that $\Pi_M^N - \Pi_M^P < 0$ (Case P $\succ$ Case N) when $\lambda_N > \lambda_{N-} = \bar{\lambda}_N^R$ in the feasible region.

The threshold for the NB manufacturer

For the NB manufacturer, the closed-form of the threshold in space $\{\gamma, \lambda_P, \lambda_N\}$ is determined by

$$\Pi_M^N - \Pi_M^P = \frac{1}{8}(\frac{(-4\gamma - \lambda_P^2 + \lambda_P + 4)^2}{(4 - \lambda_P^2)(-4\gamma^2 + 2\gamma\lambda_P - \lambda_P^2 + 3)} - \frac{(-4\gamma + \lambda_N + 3)^2}{3(-4\gamma^2 + 2\gamma\lambda_N - \lambda_N^2 + 3)}) = 0$$

The above condition can be transformed into a quadratic equation of λ_N as $b_2\lambda_N^2 + b_1\lambda_N + b_0 = 0$, where

$$\begin{cases} b_2 = (\lambda_P^2 - 4)(-4\gamma^2 + 2\gamma\lambda_P - \lambda_P^2 + 3) - 3(-4\gamma - \lambda_P^2 + \lambda_P + 4)^2, \\ b_1 = 8\gamma(4 - \lambda_P^2)(-4\gamma^2 + 2\gamma\lambda_P - \lambda_P^2 + 3) - 6(4 - \lambda_P^2)(-4\gamma^2 + 2\gamma\lambda_P - \lambda_P^2 + 3) + 6\gamma(-4\gamma - \lambda_P^2 + \lambda_P + 4)^2 \\ b_0 = -12\gamma^2(-4\gamma - \lambda_P^2 + \lambda_P + 4)^2 - 16\gamma^2(4 - \lambda_P^2)(-4\gamma^2 + 2\gamma\lambda_P - \lambda_P^2 + 3) \\ \quad + 24\gamma(4 - \lambda_P^2)(-4\gamma^2 + 2\gamma\lambda_P - \lambda_P^2 + 3) - 9(4 - \lambda_P^2)(-4\gamma^2 + 2\gamma\lambda_P - \lambda_P^2 + 3) + 9(-4\gamma - \lambda_P^2 + \lambda_P + 4)^2. \end{cases}$$

The equation has two roots as $\lambda_{N-} = \dfrac{-b_1 - \sqrt{b_1^2 - 4b_0b_2}}{2b_2}$ and $\lambda_{N+} = \dfrac{-b_1 + \sqrt{b_1^2 - 4b_0b_2}}{2b_2}$. By solving two constrained nonlinear programming problems given $0 \le \lambda_P \le 1$ and $0 \le \gamma \le 4/5$, we find $-1.67 \le \lambda_{N+} \le -0.14$ (infeasible), which means λ_{N-} is the only feasible threshold for $\lambda_N \in [0, 1]$. We also find $-64.74 \le b_2 \le -3.69$, which means $b_2\lambda_N^2 + b_1\lambda_N + b_0$ is a concave function. Notice λ_{N-} is the larger root. Then we can conclude that $\Pi_M^N - \Pi_M^P < 0$ (Case P $\succ$ Case N) when $\lambda_N > \lambda_{N-} = \bar{\lambda}_N^M$ in the feasible region.

Comparison between the two thresholds

By solving two constrained nonlinear programming problems given $a_2 \le 0$, we find $-3.73 \le \bar{\lambda}_N^R - \bar{\lambda}_N^M \le -1.29$, which means $\bar{\lambda}_N^R < \bar{\lambda}_N^M$ in the feasible region. Q.E.D.

Impact of In-store Promotion and Spillover Effect on Private Label Introduction

Abstract

This paper investigates the impact of in-store promotion and its spillover effect on private label introductions. We study different retail supply chain scenarios where the retailer carrying a national brand may introduce its own private label product and promote either the national brand or the private label inside the store. The in-store promotion on one product has a positive spillover effect on the other product. Without in-store promotion and spillover effect, the conventional wisdom indicates that, in a retail supply chain, the national brand manufacturer will be negatively impacted by the introduction of a private label product. With in-store promotion and spillover effect, however, the national brand manufacturer can actually benefit from the private label introduction. When the spillover from national brand to private label is high, the retailer prefers to promote the national brand product. When the spillover from private label to national brand is high, promoting the private label product can also benefit the national brand manufacturer. With symmetric spillover rate, the national brand manufacturer can still benefit from the private label introduction, as long as the retailer promotes the national brand product, the horizontal competition is not intense or the private label product quality is sufficiently low.

Key words: private label; in-store promotion; spillover effect; game theory

1 Introduction

The penetration of private labels has been a notable trend in the retail industry. For example, Wal-Mart has introduced a wide variety of products under its own private label *Great Value* to challenge the dominating position of national brands. According to the Private Label Manufacturer's Association, in 2014 one of every four products sold was a private label, $1 of every $5 of sales was generated by private labels, and the annual revenue of private labels was $112 billion. Thus, private labels have attracted growing attentions from both retailers and manufacturers.

Private label introduction benefits the retailer in many ways. The direct benefit is that private label products bring higher gross margin by diminishing double marginalization (Narasimhan and Wilcox 1998, Sachon and Martinez 2009). Private labels also benefit retailers by increasing their bargain power with manufacturers (Pauwels and Srinivasan 2004, Groznik and Heese 2010), increasing customer loyalty to the retailer (Vahie and Paswan 2006), and enhancing a unique store image for the retailer (Ailawadi and Keller 2004).

However, national brand manufacturers are at odds with private label introductions. On one hand, it is considered conventional wisdom that the introduction of a private label encroaches on the national brand's existing market share. A number of theoretical studies show that the introduction of a private label forces the manufacturer to lower the wholesale price, and thus hurts the manufacturer's profit (please refer to Choi and Coughlan (2006), Mills (1995; 1999), Raju et al. (1995), Sayman et al. (2002), Sayman and Raju (2004) and the references therein). On the other hand, a few empirical studies show that the wholesale price of a national brand product might increase and the manufacturer might benefit when a competing private label debuts (Ailawadi and Harlam (2004), Chintagunta et al. (2002), Pauwels and Srinivasan (2004)). However, the latter argument is rarely backed by any theoretical literature, except Ru et al. (2015), who show that in a retailer-led Stackelberg game with markup pricing, the retailer may lower the retail markup of the national brand when a competing private label is introduced, and the wholesale price and the demand of the national brand may both increase.

In this paper, we show that presence of in-store promotion and its spillover may alter a national brand manufacturer's preference of private label introductions. In-store promotion is a norm nowadays for almost all major retailers, see, e.g., Shaffer and Zettelmeyer (2009), Dukes and Liu

(2010), Schultz and Block (2011), and Nordfalt and Lange (2013), and it is typically accompanied with the spillover effect. In a market with multiple brands of similar products, when one product gets promoted, demand for other products will likely be impacted. The impact could be positive if the promotion is "cooperative," or be negative if the promotion is "predatory" (Piga 2000). Inside the same store, in-store promotional efforts exerted by the retailer to increase the sales for a particular product, such as prominent locations, eye-level shelf space, special decoration or lighting, and in-store media (Dukes and Liu 2010), are typically cooperative, such that all products in its store can have enhanced demand from the promotion.

The spillover between two products can be either symmetric or asymmetric, according to Cellini et al. (2008), Giannakas et al. (2012), Lei et al. (2008) and Norman et al. (2008). For example, the spillover rates could be symmetric if both products are placed in the same shelf level, adjacent to each other. Consumers will have the same chance of finding both products after viewing the promotion advertisement through the in-store media. On the other hand, the spillover rates could be asymmetric, for example, if one product is located at the eye-level shelf space while the other at the bottom level (Valenzuela and Raghubir 2015). If the distance between two products is small, the spillover rate is high; otherwise, the spillover rate is low.

To explore the impact of in-store promotion and its spillover rate on the national brand manufacturer's preference of the private label introduction, we establish a Stackelberg game framework to examine the competition between a private label product and a national brand product carried by a common retailer. The national brand manufacturer decides the wholesale price of its product. The retailer decides the retail prices of the two products, the in-store promotional effort level, and which product to promote.

First, if there is no spillover effect, we confirm the conventional wisdom that the introduction of a private label is not preferable for the national brand manufacturer, even though there is an in-store promotion. However, if the spillover effect is symmetric and the private label has the same quality as the national brand, the national brand manufacturer can actually benefit from the introduction of a private label product if the spillover effect is high and the product substitutability is low. This result occurs because, while the competition from the private label product is not significant, the retailer will exert a high promotional effort to increase the sales. Nevertheless, with the symmetric spillover effect, the national brand (NB) manufacturer and the retailer always conflict on the in-store promotion type: the retailer prefers to promote the private label (PL) product other than the national brand product, whereas the manufacturer prefers otherwise.

Second, conditional on symmetric quality, we find that the spillover effect asymmetry has significant impact on both firms' preferences. When the spillover from the national brand to the private label is significantly higher than from the private label to the national brand, both firms prefer to promote the national brand because the promotion is more effective. Nevertheless, when the spillover from the national brand to the private label is significantly lower than from the private label to the national brand, both firms prefer to promote the private label. This demonstrates that, if the retailer can maneuver the spillover effect asymmetrically, the national brand manufacturer would actually prefer to let the retailer introduce and promote the private label.

Third, assuming symmetric spillover rate, we find that, under asymmetric quality, the national brand manufacturer can still benefit from the private label introduction. The manufacturer prefers the private label introduction, as long as the retailer promotes the national brand when the horizontal competition is not intense or the private label's quality is sufficiently low. This is because the disadvantage of the private label introduction to the manufacturer is smaller than the advantage of reduced double marginalization. This finding is consistent with the empirical result shown by Pauwels and Srinivasan (2004) that premium-brand manufacturers, but not second-tier brand manufacturers, can benefit from private label introductions.

In addition, we observe that a high spillover rate may not always benefit the whole supply chain. When the retailer introduces a low quality private label product but promotes a high quality the national brand product, a high spillover rate from the national brand product to the private label product may intensify the channel conflict, and thus the whole supply chain profit does not monotonically increase with the spillover rate. This finding may explain why some retailers do not put their private label products immediately next to their national brand counterparts.

Finally, our results show that the predatory effect the manufacturer's out-store promotion may dampen the retailer's interest to introduce the private label product, but the predatory effect of the out-store promotion can be mitigated by the spillover effect of the in-store promotion.

This paper contributes to the extant literature on private labels by investigating the interaction between a retailer and a national brand manufacturer in the presence of the retailer's in-store promotion and the associated spillover effect. Our findings suggest that a national brand manufacturer may actually benefit from a private label introduction if either the spillover effect is substantially asymmetric or the quality level of the private label product is sufficiently lower than that of the national brand product.

The remaining of the paper is organized as follows. We review the related literature in Section 2 and establish the model in Section 3. The main analysis and numerical studies are provided in Section 4 and Section 5, respectively. We conclude in Section 6 and list all proofs in the Appendix.

2 Literature Review

Private labels have drawn a lot of attention from both academia and practice. Steiner (2004) provides a retrospective of the history of competition between national brands and private labels, and analyzes the advantage of using private labels to balance the market power between retailers and national brand manufacturers. Consumer welfare is usually improved when the competition becomes intense. Kumar and Steenkamp (2007) offer a comprehensive analysis on the private label topic. They describe the common strategies that retailers use to introduce private labels, and propose strategies for national brand manufacturers to compete against or collaborate with private labels. To bridge between academic research and business practices, Sethuraman (2009) assesses the external validity of 44 analytical results that appeared in literature and their applicabilities in practice.

There is a stream of literature studying the impacts of private labels on retailers and supply chains. Narasimhan and Wilcox (1998) show that when retailers introduce private labels, they not only profit directly but also use them as a strategic tool to gain market power against national brand manufacturers. Sachon and Martinez (2009) point out that a supply chain's total profit increases from a private label introduction only when the competition between the private label and the national brand is not intense. Groznik and Heese (2010) analyze how private labels cause channel conflicts in both single-retailer and multi-retailer channels. Chen et al. (2011) study the role of private label introduction in supply chain coordination. They characterize the conditions under which the retailer will introduce the private label, and the conditions under which the introduction is beneficial or detrimental to the overall supply chain. The above papers focus on the decisions of the retailer, but the reactions of the national brand manufacturer are not considered.

Another stream of literature study the impacts of private labels on national brand manufacturers and their reactions. Wedel and Zhang (2004) study the competition between national brands and private labels across the subcategories, and they show asymmetrical price competition exists both within and across subcategories: the cross-subcategory impact of national brands on store brands is greater than that of store brands on national brands. Pauwels and Srinivasan (2004) empirically show that private label penetration benefits the retailer, the consumers, and premium-brand manufacturers, but it hurts second-tier brand manufacturers. Geyskens et al. (2010) examine the impact of economy and premium private labels on mainstream-quality and premium-quality national brands and existing private labels. They show that both economy and premium private labels cannibalize incumbent private labels, and economy private label introductions benefit mainstream-quality national brands because the latter become a middle option in the retailer's assortment. Gielens (2012) investigates how new product introduction helps national brand manufacturers boot their market shares. They suggest that, to fight economy private labels successfully, national brands should maintain a smaller price gap, while offering products that focus less on intrinsic and usage benefits. The above papers focus on the reactions of national brand manufacturers. The interactions between the manufacturer and the retailer are still rarely examined in the literature. Differently, our paper considers the national brand manufacturer and the retailer in an interactive scenario where they contemplate each other's strategy and take actions accordingly.

The in-store promotional efforts have attracted interest from a large group of researchers, who examine the issues from a variety of aspects. Shaffer and Zettelmeyer (2009) study the problem of comparative advertising and in-store displays. They explain why manufacturers may or may not want to engage in comparative advertising, especially in regard to in-store displays, in the channel perspective. Dukes and Liu (2010) study the effects of in-store media, which allows manufacturers to advertise their products. They show that in-store media plays an important role in coordinating a distribution channel and the competition between suppliers. Schultz and Block (2011) study many types of in-store promotion to find which promotion techniques influence consumers' purchase decisions the most. They develop models to predict consumers' response to different combinations of promotional efforts. Nordfalt and Lange (2013) perform two large field experiments to show that in-store promotions are powerful tools to increase sales. They find the effectiveness of in-store promotions varies widely depending on when and how the promotions are executed. Those papers study the promotional effort without considering the spillover effect. We establish scenarios to examine the impacts of in-store promotion along with its spillover effect to find new insights.

The perceived quality of products may affect retailers' decisions on promotional efforts. In this paper, perceived quality is used as a measure of the product's attractiveness by itself, exclusive of price and promotion effect. Besides the product's physical quality, perceived quality also includes the brand's reputation. Many private label products have lower perceived quality compared to their national brand counterparts because of the lack of reputation, which takes time to accumulate (Heese 2010).

The spillover effect of promotion is analyzed separately by many scholars. Cellini and Lambertini (2003) illustrate a Cournot oligopoly game where firms sell similar goods and invest in promotion activities with spillover effects. They find the social welfare of a centralized firm will be larger than that of two oligopoly firms. Norman et al. (2008) investigate the promotion activities in homogeneous goods markets where one firm's promotional effort tends to spill over to rival firms. Since such a phenomenon discourages the promotion investment, they suggest collecting mandatory fees for all firms to support a joint advertising effort. Dharmasena et al. (2010) study the spillover effects of promotions in the U.S. non-alcoholic beverage market. They find asymmetric spillover effects where the promotional effort on one product can positively affect one group of products but negatively affect another group. Therefore, one firm needs to pay attention to the promotional efforts of other firms even if they do not produce the same type of products. Giannakas et al. (2012) develop a theoretical framework to analyze the effect of advertising spillover on firms' productivity. They use the data of meat processing firms in Greece during 1983-2008 and find the spillover effect is one of the important drivers to improve firms' productivity. Those papers do not investigate the spillover effect in a private label context as our research.

It is noticeable that the competition between private label and national brand with both in-store promotion and spillover effect has not been fully studied in the literature. Our work will contribute to fill this void.

3 The Model

We investigate a two-echelon supply chain where a national brand (NB) manufacturer sells its product through a retailer. The retailer has an option to introduce a private label (PL) product, which will inevitably compete with NB product. For the sake of brevity, we use PL and NB to represent the private label product and the national brand product, respectively, in the rest of the paper. Inside its own store, the retailer can utilize its in-store media to promote either product. We use subscripts N and P to denote NB and PL products, respectively. As illustrated in Figure 1, there are three possible scenarios:

1. Case P: The retailer introduces PL and promotes it;
2. Case N: The retailer introduces PL, but promotes NB;
3. Case B: This is a baseline case. The retailer does not introduce PL while promoting NB.

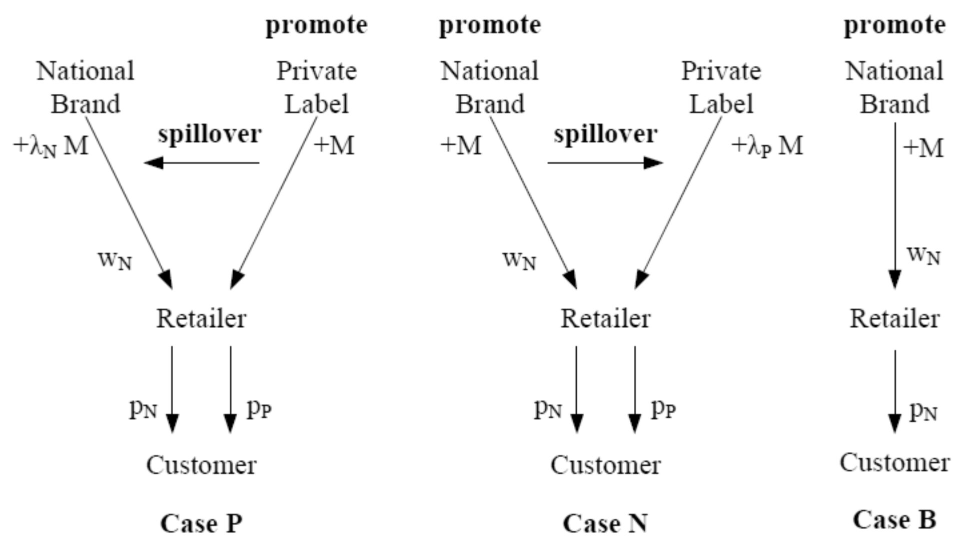

Figure 1: Channel structures

As shown in Figure 1, w_N represents the wholesale price of NB. The retail prices of NB and PL are denoted by p_N and p_P, respectively. The retailer's in-store promotional effort is M, which incurs a cost of θM^2. We normalize θ to 1 without affecting our qualitative results. Similar simplification has also been adopted by Choi and Coughlan (2006), Chen et al. (2009), and Liu et al. (2014).

The quality of NB is denoted by Q_N and normalized to 1. PL's quality is Q_P, which can be either lower or higher than 1. For tractability, both products' quality are assumed to be exogenous. We assume the production costs to be zero for the purpose of analytical tractability. Our numerical analysis later shows that including quality-related production costs will not change the structure of the major results.

Given that both NB and PL are located inside the same store, the promotion of one product will have a spillover effect on the other product. We let λ_N, $0 \leq \lambda_N \leq 1$, denote the spillover rate to the NB when PL is promoted, and λ_P, $0 \leq \lambda_P \leq 1$, denote the spillover rate vice versa.

The sequence of events in this Stackelberg game is as follows. In Stage 1, the manufacturer decides the wholesale price w_N. In Stage 2, the retailer decides both products' retail prices $p_{N/P}$, and the promotion level M on PL or NB. We discuss three scenarios as follows.

3.1 Case P: The retailer introduces PL and promotes it

When a product is promoted, its demand will increase, the demand of the other product will also increase if the spillover effect is high and the substitutability is low. In line with Sayman et al. (2002) and Choi and Coughlan (2006) on modeling the competition between PL and NB, we adopt the following quadratic and strictly concave function to describe the utility of a representative customer group who purchase a certain mixture of substitutable products, which is widely used in similar research (Cai et al. 2012, Singh and Vives 1984, Hackner 2003, Ingene 2004).

$$U(D_N, D_P) = D_N(Q_N + \lambda_N M) + D_P(Q_P + M) - (D_N^2 + 2\gamma D_N D_P + D_P^2)/2 - p_P D_P - p_N D_N, \tag{1}$$

where $D_{N/P}$ is the demand of NB/PL and $Q_{N/P}$ is the physical quality of NB/PL. The retailer's promotion effort on PL increases the perceived quality from the physical quality by M and $\lambda_N M$ for PL and NB, respectively. The parameter γ is the product substitutability between the two products. The third term represents the fact that the value of using both substitutable products is less than the sum of the separate values of using each product by itself (Samuelson 1974). The consumer utility decreases as products become more substitutable, i.e., as γ increases, everything else held constant. A more complex function of the initial base demand based on the quality level will not change our main results qualitatively, but quickly leads to intractability.

Maximizing the above utility function yields the following demand functions.

$$\begin{cases} D_N = \dfrac{1}{1-\gamma^2}(Q_N - \gamma Q_P + (\lambda_N - \gamma)M - p_N + \gamma p_P); \\ D_P = \dfrac{1}{1-\gamma^2}(Q_P - \gamma Q_N + (1 - \gamma\lambda_N)M - p_P + \gamma p_N). \end{cases} \tag{2}$$

A nice feature of the chosen utility function is that the quality of the product, $Q_{N/P}$, naturally becomes the initial base demand of NB/PL when consumers maximize their utility. Intuitively, given other factors unchanged, the higher the quality of the product, the larger the demand for it.

Let $\Pi^P_{M/R}$ denote the profit of the manufacturer/retailer in Case P. They are calculated as follows.

$$\begin{cases} \Pi^P_M = w_N D_N; \\ \Pi^P_R = (p_N - w_N)D_N + p_P D_P - M^2. \end{cases} \tag{3}$$

Both the manufacturer and the retailer attempt to maximize their own profits, which leads to the following result (please refer to the Appendix for all the solutions and proofs in this paper).

Lemma 1 *There exists a unique equilibrium solution of* (w_N, p_N, p_P, M) *in Case P.*

Note the above model is built in a Bertrand setting in which the retailer decides the prices and then the utility-maximizing demands are derived. Alternatively, we can build the model in a Cournot setting in which the retailer decides both products' ordering quantities $D_{N/P}$ and then the customers pay the utility-maximizing prices as follows.

$$\begin{cases} p_N = Q_N + M\lambda_N - D_N - \gamma D_P; \\ p_P = Q_P + M - \gamma D_N - D_P. \end{cases}$$

The above price functions are inverse functions of Equation (2). After solving the Cournot model, we find the solutions of $\{w_N, p_N, p_P, M\}$ are the same as those in the Bertrand model because the common retailer determines both retail prices or both order quantities. Therefore, this paper focuses on the Bertrand setting, and the same results also apply for the Cournot setting.

3.2 Case N: The retailer introduces PL but promotes NB

In Case N, the promotion increases the initial base demand of NB, Q_N, and the spillover effect of promotion enhances the initial base demand of PL, Q_P, by $M\lambda_P$. The representative customer' utility function is correspondingly described as follows.

$$U(D_N, D_P) = D_N(Q_N + M) + D_P(Q_P + \lambda_P M) - (D_N^2 + 2\gamma D_N D_P + D_P^2)/2 - p_P D_P - p_N D_N. \tag{4}$$

Maximizing the above utility function results in the demand functions as follows:

$$\begin{cases} D_N = \dfrac{1}{1-\gamma^2}(Q_N - \gamma Q_P + (1-\gamma\lambda_P)M - p_N + \gamma p_P); \\ D_P = \dfrac{1}{1-\gamma^2}(Q_P - \gamma Q_N + (\lambda_P - \gamma)M - p_P + \gamma p_N). \end{cases}$$

The profit functions $\{\Pi_M^N, \Pi_R^N\}$ take the same forms as in Equation (3). Similarly, we have

Lemma 2 *There exists a unique equilibrium solution of (w_N, p_N, p_P, M) in Case N.*

3.3 Case B: The retailer does not introduce PL and promotes NB

Case B serves as a baseline case to compare with Cases P and N. The utility function of Case B is similar to that in Case N. Because there is no PL, the demand of PL is zero, that is, $D_P = 0$. Plugging this constraint into Equation (4) and maximizing the utility function results in

$$D_N = Q_N + M - p_N.$$

The profit functions are the same as in Equation (3) (with $D_P = 0$). Similarly, there exists a unique equilibrium solution.

Lemma 3 *There exists a unique equilibrium solution of (w_N, p_N, M) in Case B.*

To ensure the products' demands are non-negative in all scenarios for meaningful discussion, we make two more assumptions, $\gamma < Q_P < min(1/\gamma, 2)$ and $0 < \gamma \leq 0.8$, for the following reasons. First, without the pricing and promotion issues, the basic demands for both products should be positive, so the non-negative conditions are $Q_N - \gamma Q_P > 0$ and $Q_P - \gamma Q_N > 0$. Considering the normalization of $Q_N = 1$, the above non-negative condition can be rewritten as $\gamma < Q_P < 1/\gamma$. However, the above upper bound $1/\gamma$ can be very large if γ is small. Since PL is generally designed to imitate NB and Q_P will not be significantly different from Q_N, we limit $\gamma < Q_P < min(1/\gamma, 2)$, which means PL's quality will not be twice as good as NB. Second, for γ, as PL is a substitutable product for NB, we have the lower bound as $\gamma > 0$. It is easy to see γ should be less than a certain value to ensure non-negative demand in Equation (2). We find that $\sqrt{3}/2 = 0.866$ is the largest allowed value of γ in the no-spillover model (see the proof of Lemma 1). We tighten the upper bound to its first digit as $\gamma <= 0.8$ for simplicity.

4 Analytical Results

This section serves two major purposes. The first is to study the retailer's two strategic decisions: (1) whether or not to introduce PL; and (2) if PL is introduced, whether to promote PL or NB. The second purpose is to investigate whether or not the manufacturer would benefit from the retailer's introduction of PL and in-store promotion. In the following, we first introduce the benchmark case without spillover, then we proceed to analyze the impact of spillover effect.

4.1 No-spillover: Preliminaries

Many of the studies on the introduction of PL do not consider the in-store promotion as well as the spillover effect of promotion. Normally, channel conflicts arise as the retailer introduces PL, which hurts the manufacturer (see, e.g., Groznik and Heese (2010), Heese (2010), Chen et al. (2011)). The following lemma confirms the same message.

Lemma 4 *Without promotion, the retailer prefers to introduce PL, which always hurts the NB manufacturer.*

Conventional wisdom tells us that the retailer can benefit from selling PL, which encroaches into NB's market. The retailer gains a higher marginal profit in selling PL than selling NB. In contrast, the manufacturer suffers from losing its monopoly of NB in the market and has to reduce its wholesale price because of the horizontal competition from PL.

Without the spillover effect of promotion, would in-store promotion upon either PL or NB change the manufacturer's preference regarding the introduction of PL? The following lemma suggests the answer is no.

Lemma 5 *With promotion but no spillover, the manufacturer's preference on the three cases are Case B $\succ$ Case N $\succ$ Case P.*

This result is not surprising, because the manufacturer's best scenario is to maintain its monopoly (i.e., Case B). If PL is launched, the manufacturer certainly prefers its own product to be promoted as compared with PL being promoted (i.e., Case N $\succ$ Case P). The retailer's preference is different from the manufacturer's, as demonstrated below.

Lemma 6 *With promotion but no spillover, the retailer's preference on the three cases are as follows:*

- *Case N $\succ$ Case B;*
- *Case P $\succ$ Case B if and only if $Q_P > \bar{Q}_P^{PB}(\gamma) = \dfrac{3\gamma + \sqrt{3}\sqrt{16\gamma^4 - 16\gamma^2 + 3}}{12(1-\gamma^2)}$;*
- *Case P $\succ$ Case N if and only if $Q_P > \bar{Q}_P^{PN}(\gamma) = \dfrac{1}{2\sqrt{1-\gamma^2}}$, where $\bar{Q}_P^{PB}(\gamma) < \bar{Q}_P^{PN}(\gamma)$.*

Lemma 6 confirms that the retailer can be better off by introducing PL (i.e., Case N $\succ$ Case B). Provided that NB is promoted in both scenarios, introducing PL reduces the double marginalization and hence increases the total demand of both products for the retailer. Therefore, if the manufacturer demands the retailer to promote NB, the retailer will choose to introduce PL.

The nuance comes when the retailer is determined to promote PL in Case P. When PL's quality is low, Case B outperforms Case P for the retailer, because promoting a low quality PL leads to lower profit margin than promoting the higher quality NB in a monopoly market (i.e., Case B). This result indicates that introducing and promoting PL does not always benefit the retailer. As PL's quality improves, the benefit of having more demand from selling both products outweighs the relatively higher profit margin of selling only NB in a monopoly market; as a result, Case P outperforms Case B for the retailer.

We can also infer from Lemma 6 that both thresholds $\bar{Q}_P^{PN}(\gamma)$ and $\bar{Q}_P^{PB}(\gamma)$ increase with γ. It means that, as the competition between the two products becomes more intense, for the retailer, PL must have a sufficiently high quality to make Case P more preferable than the other two cases.

As illustrated in Figure 2, if PL's quality is low, i.e., $Q_P < \bar{Q}_P^{PN}$, the retailer will promote NB to enlarge the market. If PL's quality is high, i.e., $Q_P > \bar{Q}_P^{PN}$, the retailer will instead promote PL to capture a higher profit margin. Therefore, the retailer has the incentive to introduce PL, although whether to promote either NB or PL depends on whether PL's quality is high or low. The "N/A" areas in Figure 2 do not satisfy the non-negativity conditions stipulated after Lemma 3.

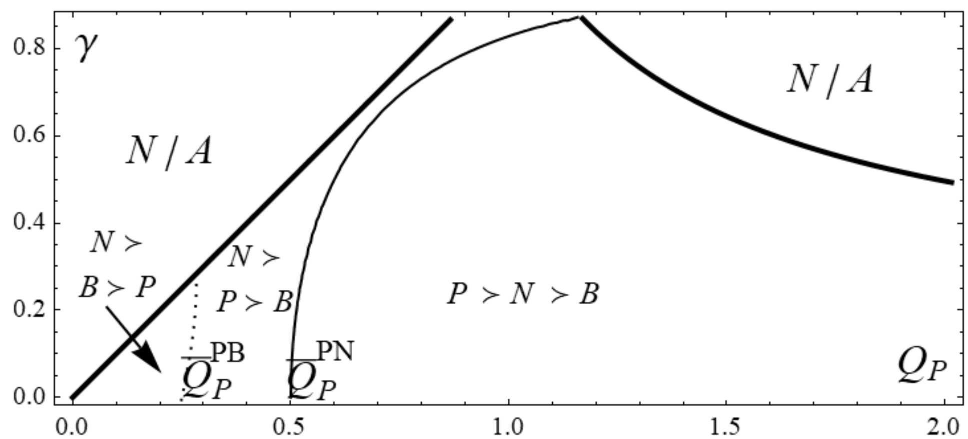

Figure 2: The retailer's preference without spillover.

Comparing Lemma 6 with Lemma 5, we can conclude that the manufacturer will be at odds with the retailer when PL is introduced, especially when PL's quality is high and Case P is chosen over Case N in in-store promotion. Note that this result is obtained under no spillover effect. With spillover considered, however, will the conflict between the manufacturer and the retailer over the introduction of PL and promotion be lessened? To answer this question, in the next sections, we explore the cases of symmetric spillover and asymmetric spillover.

4.2 Impact of Symmetric Spillover Rate

To single out the impact of spillover on the two firms' preferences, we start with a simple case with symmetric spillover ($\lambda_N = \lambda_P = \lambda \in [0, 1]$) between the two products and keep both products' qualities equal ($Q_P = Q_N = 1$).

Impact on the retailer

When PL's quality is equal to NB's, we find that the retailer's preference is the same as that in the no-spillover scenario (Lemma 6) with a sufficiently high quality PL.

Proposition 1 *For the retailer, when the spillover rates between the two products are symmetric and the product qualities are equal, Case P $\succ$ Case N $\succ$ Case B.*

This result reveals that for the retailer, if PL's quality is as high as NB's, the magnitude of a symmetric spillover will not change its preference. In other words, the impact of PL's quality dominates the impact of spillover on the retailer's preference.

Impact on the NB manufacturer

In the no-spillover scenario, we find the NB manufacturer never prefers introducing PL. With spillover effect, however, the NB manufacturer can benefit from the introduction of PL when the spillover rate is high, which deviates from the conventional wisdom.

Proposition 2 *For the NB manufacturer, when the spillover rates between the two products are symmetric and the product qualities are equal, its preference on the three cases, N, P, and B, are as follows.*

- *Case N $\succ$ Case B if and only if $\lambda > \bar{\lambda}^M_{NB}(\gamma) = 4\gamma$;*
- *Case P $\succ$ Case B if and only if $\lambda > \bar{\lambda}^M_{PB}(\gamma) = (2\sqrt{3}\sqrt{-8\gamma^2 + 6\gamma + 2} + 8\gamma - 3)/5$ where $\bar{\lambda}^M_{NB}(\gamma) < \bar{\lambda}^M_{PB}(\gamma)$;*
- *Case N $\succ$ Case P.*

To interpret these results, we use Figure 3 to more vividly demonstrate how the NB manufacturer's preference changes in term of the spillover rate λ. First, given any product substitutability level, when the spillover rate λ is low, the NB manufacturer does not favor the introduction of PL. However, when λ is sufficiently high, the NB manufacturer can actually benefit from the introduction of PL as long as the retailer promotes NB. There are two drivers behind this phenomenon. First, the NB manufacturer benefits from the in-store promotion of NB. Second, the spillover effect boosts the horizontal competition between the two products and hence reduces double marginalization to generate higher demand for the manufacturer. Although the introduction of PL encroaches into NB's market share, the benefit of a greater market size to the NB manufacturer outweighs its loss, such that Case N is more preferable to Case B for the manufacturer.

As λ continues to grow higher, Case P can be even better than Case B for the NB manufacturer, as long as γ is sufficiently low. In comparison to Case N, the retailer will exert more promotional

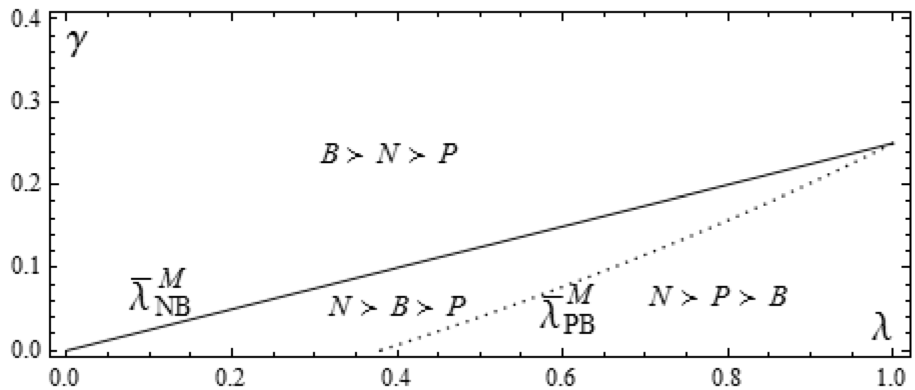

Figure 3: The NB manufacturer's preference with the same spillover/quality.

effort in Case B due to the higher profit margin of PL. Provided that the spillover effect is sufficiently high (i.e., λ is high), the NB manufacturer can also significantly benefit from a larger market size and reduced double marginalization. Overall, a high spillover rate alleviates the negative impact of intense competition on the manufacturer. Nevertheless, conditional on the symmetric spillover rates, if NB manufacturer can determine the promotion type, it always prefers its own product, instead of PL, to be promoted.

From Proposition 2, one can also infer that both thresholds $\bar{\lambda}^M_{NB}(\gamma)$ and $\bar{\lambda}^M_{PB}(\gamma)$ rise as γ increases. It means when the competition between the two products becomes more intense, the manufacturer will more likely prefer the introduction of PL, regardless of the promotion type, if and only if the spillover rate becomes sufficiently higher.

Equivalently, the above results can be described from the perspective of product substitutability level γ. For the NB manufacturer, when the spillover rates between the two products are symmetric and the product qualities are equal, its preference on the three cases are:

- Case N $\succ$ Case B if and only if $\gamma < \bar{\gamma}^M_{NB}(\lambda) = \lambda/4$;
- Case P $\succ$ Case B if and only if $\gamma < \bar{\gamma}^M_{PB}(\lambda) = (-\sqrt{3}\sqrt{5-2\lambda^2} + 2\lambda + 3)/8$ where $\bar{\gamma}^M_{PB}(\lambda) < \bar{\gamma}^M_{NB}(\lambda)$;
- Case N $\succ$ Case P.

Given the same spillover rate, when the substitutability level is sufficiently low, the introduction of PL can be beneficial to the NB manufacturer. As γ increases, Case N may still be better than

Case B, but Case P will be worse than Case B, because the NB manufacturer will suffer from overly intense horizontal competition. When γ is even higher, the introduction of PL will hurt the NB manufacturer.

Combining Proposition 1 for the retailer and Proposition 2 for the NB manufacturer, we can conclude that the spillover effect makes the introduction of PL more preferable for both firms. However, with the symmetric spillover rates, the NB manufacturer and the retailer are always at odds with the in-store promotion type: the retailer prefers Case P to Case N, whereas the NB manufacturer prefers Case N to Case P.

4.3 Impact of Asymmetric Product Quality

To single out the impact of PL's quality, we hereby assume symmetric spillover rates, that is $\lambda_N = \lambda_P = \lambda \in [0, 1]$.

Impact on the retailer

We first extend the result from the no-spillover case in Lemma 6 by explicitly including symmetric spillover rates for both firms and obtain the following result.

Proposition 3 *For the retailer, when the spillover rates between the two products are symmetric, its preference on the three cases is as follows:*

- *Case N $\succ$ Case B;*
- *Case P $\succ$ Case B if and only if*

$$Q_P > \bar{Q}_P^{PB}(\lambda, \gamma) = \frac{2\sqrt{3}\sqrt{64\gamma^4 - 64\gamma^3\lambda + 52\gamma^2\lambda^2 - 64\gamma^2 - 18\gamma\lambda^3 + 32\gamma\lambda + 5\lambda^4 - 19\lambda^2 + 12} + 12\gamma - 3\lambda}{3(-16\gamma^2 + 8\gamma\lambda - 5\lambda^2 + 16)};$$

- *Case P $\succ$ Case N if and only if*

$$Q_P > \bar{Q}_P^{PN}(\lambda, \gamma) = \frac{\sqrt{\lambda^4 - 5\lambda^2 + 4}}{\sqrt{16\gamma^2\lambda^2 - 16\gamma^2 - 8\gamma\lambda^3 + 8\gamma\lambda + 5\lambda^4 - 21\lambda^2 + 16}};$$

- $\bar{Q}_P^{PB}(\lambda, \gamma) < \bar{Q}_P^{PN}(\lambda, \gamma)$.

Similar to the result in Lemma 6 without spillover, the retailer is always better off by introducing PL (Case N $\succ$ Case B). With the symmetric spillover rate, the threshold values of Q_P in Lemma

6 are altered accordingly to include $\{\lambda, \gamma\}$. Similar to Lemma 6, the retailer's preference can be categorized as below:

1. When Q_P is low, i.e., $Q_P \leq \bar{Q}_P^{PB}$, Case N $\succ$ Case B $\succ$ Case P;
2. When Q_P is medium, i.e., $\bar{Q}_P^{PB} < Q_P \leq \bar{Q}_P^{PN}$, Case N $\succ$ Case P $\succ$ Case B;
3. When Q_P is high, i.e., $Q_P > \bar{Q}_P^{PN}$, Case P $\succ$ Case N $\succ$ Case B.

Proposition 3 confirms that the retailer does not always prefer to promote PL. Instead, when PL's quality is low and medium, the retailer can benefit from promoting NB rather than PL. This is different from Proposition 1 with symmetric quality levels. Given that the retailer has stakes in both products, promoting the higher quality product can lead to a higher profit margin.

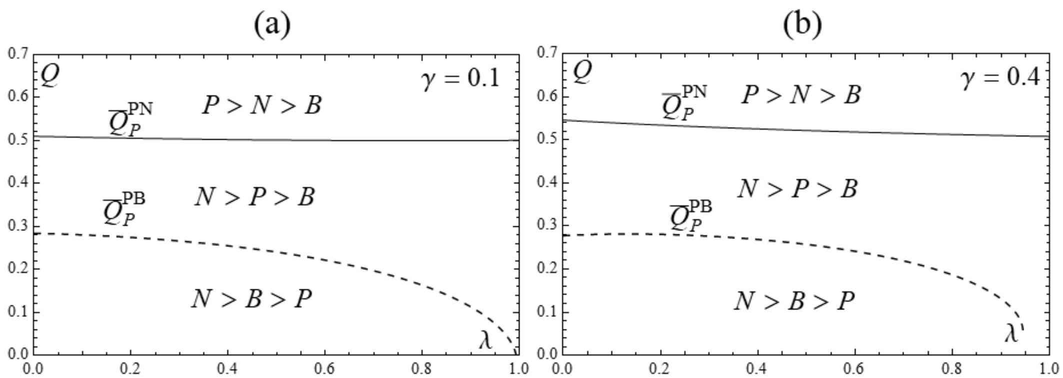

Figure 4: The trends of $\bar{Q}_P^{PN}$ and $\bar{Q}_P^{PB}$ for the retailer when λ increases.

Note that both thresholds, $\bar{Q}_P^{PB}(\lambda, \gamma)$ and $\bar{Q}_P^{PN}(\lambda, \gamma)$, decrease with λ. This suggests, as λ grows, it is more likely for the retailer to introduce and promote PL, which is illustrated in Figure 4. Figure 4 also shows that the lines of $\bar{Q}_P^{PB}(\lambda, \gamma)$ and $\bar{Q}_P^{PN}(\lambda, \gamma)$ shift up as γ increases from 0.1 to 0.4, which indicates that it is more likely for the retailer to introduce PL and promote NB as product substitutability increases.

Impacts on the NB manufacturer

We now extend the result from Proposition 2 with symmetric quality to asymmetric quality as follows.

Proposition 4 *For the NB manufacturer, when the spillover rates between the two products are symmetric, its preference on the three cases is as follows:*

- *Case N $\succ$ Case P;*
- *Case N $\succ$ Case B if and only if*
  - $\gamma < \lambda/4$ *(low competition); or*
  - $\gamma > \lambda/4$ *(high competition), and*

$$Q_P < \bar{Q}_P^{NB}(\lambda,\gamma) = \frac{-2\sqrt{3}\sqrt{4\gamma^2\lambda^2 - 16\gamma^2 - 2\gamma\lambda^3 + 8\gamma\lambda + \lambda^4 - 7\lambda^2 + 12} - 3\lambda^2 + 12}{3(4\gamma - \lambda)};$$

- *Case P $\succ$ Case B if and only if*

$$Q_P < \bar{Q}_P^{PB}(\lambda,\gamma) = \frac{-\sqrt{(6\lambda - 24\gamma)^2 - 4(16\gamma^2 - 8\gamma\lambda + \lambda^2)(16\gamma^2 - 8\gamma\lambda + 4\lambda^2 - 3)} + 24\gamma - 6\lambda}{2(16\gamma^2 - 8\gamma\lambda + \lambda^2)};$$

- $\bar{Q}_P^{PB}(\lambda,\gamma) < \bar{Q}_P^{NB}(\lambda,\gamma)$.

Similar to Proposition 2, we find the same result that the NB manufacturer always prefers its own product to be promoted regardless of PL quality level. Because of the asymmetric quality, however, it is more likely for the manufacturer to accept the introduction of PL. Compared to Proposition 2, in addition to the original condition (i.e., $\gamma < \lambda/4$), the manufacturer might prefer Case N to Case B if $\gamma > \lambda/4$ and $Q_P < \bar{Q}_P^{NB}(\lambda,\gamma)$. In other words, the manufacturer is tolerant of the introduction of PL, as long as the retailer promotes NB when the horizontal competition is not intense or PL's quality is sufficiently low. In this situation, the disadvantage of the introduction of PL to the manufacturer is smaller than the advantage of reduced double marginalization. If PL's quality is even lower, the benefit of reduced double marginalization can be higher, such that the manufacturer might even prefer Case P to Case B.

Combining Proposition 3 and Proposition 4, we illustrate the two firms' preferences in Figure 5. On the one hand, the retailer prefers Case P when Q_P is high (to the right of the dashed line), or Case N when Q_P is low (to the left of the dashed line). On the other hand, the NB manufacturer prefers Case N if the competition is low (below the lower solid line), or the competition is high but Q_P is low (to the left of the upper solid line); otherwise the manufacturer prefers Case B. As a result, both the retailer and the NB manufacturer can prefer the same Case N when Q_P is sufficiently low. This is a deviation from the scenario with symmetric spillover rate and symmetric quality levels where the retailer and NB manufacturer always conflict over the in-store promotion type. In summary, the introduction of PL with in-store promotion of NB can be a Pareto choice for both the retailer and the manufacturer.

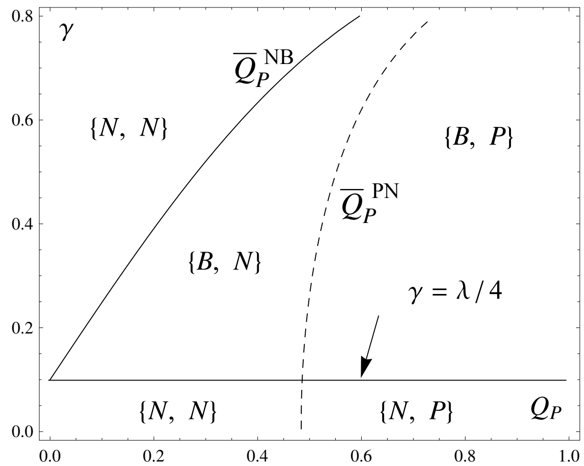

Figure 5: The thresholds of the {Manufacturer's, Retailer's} preference as PL's quality varies with fixed $\lambda = 0.4$.

4.4 Impact of Asymmetric Spillover Rates

We now focus on the two firms' preference for Cases N or P under a general asymmetric spillover setting ($\{\lambda_N, \lambda_P\} \in [0, 1]$). For tractability, we again assume the products' qualities are equal. Case B is not needed in this analysis for the following two reasons. First, for the NB manufacturer, the thresholds of Case P and Case N over Case B have been described in Proposition 2, which are functions of the substitutability level γ. Case B is inferior to Cases N or P as long as γ is not substantially large. Second, for the retailer, Case B is always dominated by Case P or Case N as Proposition 1 shows. As a result, it is reasonable to assume PL has been introduced and, thus, we preclude Case B in this analysis and focus on the comparison between Cases N and P.

Proposition 5 *When the two products have asymmetric spillover effects and the product qualities are equal,*

- *for the retailer, Case P $\succ$ Case N if and only if* $\lambda_N > \bar{\lambda}_N^R(\lambda_P, \gamma)$;
- *for the NB manufacturer, Case P $\succ$ Case N if and only if* $\lambda_N > \bar{\lambda}_N^M(\lambda_P, \gamma)$;
- $\bar{\lambda}_N^R(\lambda_P, \gamma) < \bar{\lambda}_N^M(\lambda_P, \gamma)$.

Proposition 5 is described in terms of λ_N. In terms of the spillover rate from NB to PL, λ_P, Proposition 5 can be rewritten as follows[1]:

- For the retailer, Case N $\succ$ Case P if and only if $\lambda_P > \bar{\lambda}_P^R(\lambda_N, \gamma)$;
- For the NB manufacturer, Case N $\succ$ Case P if and only if $\lambda_P > \bar{\lambda}_P^M(\lambda_N, \gamma)$;
- $\bar{\lambda}_P^M(\lambda_N, \gamma) < \bar{\lambda}_P^R(\lambda_N, \gamma)$.

We use Figure 6 to graphically illustrate Proposition 5. The two thresholds, $\bar{\lambda}_N^R(\lambda_P, \gamma)$ and $\bar{\lambda}_N^M(\lambda_P, \gamma)$, segment the feasible region $\{\lambda_N, \lambda_P\} \in [0, 1]$ based on the two firms' preferences. First, when λ_N and λ_P are not sufficiently different, the NB manufacturer prefers Case N and the retailer prefers Case P, which is similar to the result with the symmetric spillover rates.

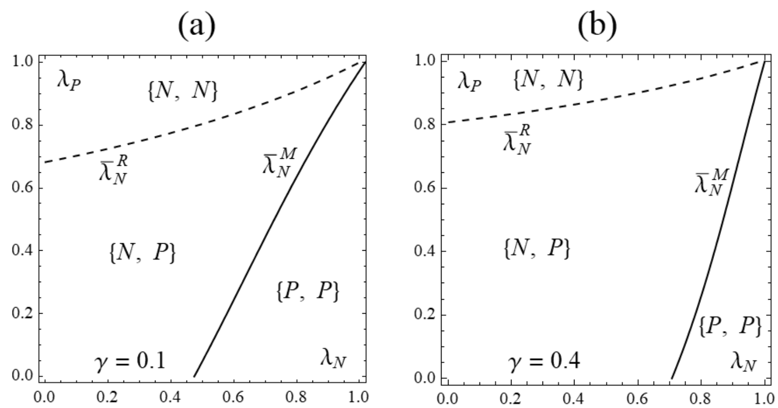

Figure 6: The {Manufacturer's, Retailer's} preferences of Case P or Case N with asymmetric spillover rates.

When λ_P is significantly greater than λ_N (the spillover effect from NB to PL is much larger than from PL to NB), both firms prefer Case N (upper left corner). This occurs because the same amount of promotional effort in Case N leads to a larger overall market size for both firms than in Case P. It is intuitive that the promotion decision has a higher impact on the manufacturer than the retailer, because the retailer still keeps a portion of the sales revenue of NB, but the manufacturer earns nothing from the sales of PL. Although the retailer has to sacrifice some profit for promoting NB instead of PL, it benefits from the significant reduction of double marginalization caused by

[1] From the proof of Proposition 5, we can see that the λ_P-based thresholds $(\bar{\lambda}_P^R(\lambda_N, \gamma), \bar{\lambda}_P^M(\lambda_N, \gamma))$ can be expressed as the inverse functions of the λ_N-based ones $(\bar{\lambda}_N^R(\lambda_P, \gamma), \bar{\lambda}_N^M(\lambda_P, \gamma))$. We hereby skip the complex functions for parsimony.

more intense horizontal competition due to higher spillover rates. Similarly, when λ_N is significantly larger than λ_P (the spillover effect from PL to NB is much larger than from NB to PL), both firms prefer Case P to Case N (lower right corner). Comparing Propositions 4 and 5, it shows that the manufacturer's preference sequence is not affected by asymmetric quality levels, but it changes when asymmetric promotion spillover effects are considered. In contrast, the retailer's preference sequence changes when either the quality levels or the spillover effects become asymmetric.

As the product substitutability (γ) grows, the area of {N,P} enlarges, whereas those of {N,N} and {P,P} shrink. These results indicate that the manufacturer is more likely to prefer Case N while the retailer is more likely to prefer Case P, when products become more substitutable. The more intense horizontal competition reduces the benefit of lessened double marginalization, therefore, the benefit of direct in-store promotion becomes more critical to both firms.

In summary, Proposition 5 delivers an unconventional message that both the retailer and the NB manufacturer can actually prefer the introduction of PL and the same in-store promotion type, that is {N,N} and {P,P}, conditional on the asymmetric spillover effects. In other words, the retailer can actually benefit from promoting NB rather than promoting its own product, whereas the NB manufacturer can be better off from the introduction of PL with a positive spillover effect in either type of in-store promotions.

5 Extended Numerical Analysis

For analytical tractability, our previous analysis is limited to either only asymmetric spillover rates or only asymmetric quality. For simplicity, we assumed normalize the production cost to zero, and we did not include the NB manufacturer's out-store promotions in the model. In this section, we conduct numerical tests to examine the impact of asymmetric spillover and asymmetric quality simultaneously. Since the spillover rates are controllable in practice, we also study the firms' preferences of spillover rates. We also conduct numerical analysis to examine the impacts of quality-related production costs and the NB manufacturer's out-store promotions.

5.1 Impacts with Asymmetric Spillover and Asymmetric Quality

5.1.1 Improvement of the profits when competition is low

We start with the case of low competition assuming $\gamma = 0.1$. We examine the firms' profits under spillover rates $\lambda = 0.1, 0.5, 0.9$, respectively. We find that both firms' profits increase monotonically when λ increases, because the firms benefit more from the market expansion effect of spillover when the horizontal competition is less of a concern. Due to different game settings, the magnitudes of those profit increases are different in Cases P and N. In Table 1, we underscore each firm's profit increase in each scenario when λ increases from 0.1 to 0.9 (e.g., in Case P when $Q_P = 1$, for the manufacturer, the profit increase is $0.219 - 0.102 = 0.117$).

		γ = 0.1			
		Case P (λ_N)		Case N (λ_P)	
		Π_M	Π_R	Π_M	Π_R
Q_P = 1	λ = 0.1	0.102	0.384	0.144	0.322
	λ = 0.5	0.142	0.405	0.176	0.355
	λ = 0.9	0.219	0.443	0.229	0.428
	Increase	**0.117**	**0.058**	**0.085**	**0.106**
Q_P = 0.7	λ = 0.1	0.109	0.218	0.151	0.198
	λ = 0.5	0.140	0.233	0.173	0.217
	λ = 0.9	0.201	0.264	0.211	0.259
		0.091	**0.046**	**0.060**	**0.061**
Q_P = 0.4	λ = 0.1	0.116	0.112	0.158	0.119
	λ = 0.5	0.137	0.122	0.170	0.128
	λ = 0.9	0.183	0.145	0.193	0.147
		0.067	**0.033**	**0.035**	**0.028**

Table 1: The increase of both firms' profits as λ grows.

When PL's quality is high ($Q_P = 0.7, 1$), as shown in Table 1, if the retailer promotes PL (Case P), the manufacturer's profit increases more than the retailer's as λ increases; if the retailer promotes NB (Case N), the retailer's profit increases more than the manufacturer's as λ increases. When PL's quality is low ($Q_P = 0.4$), in Case P the trend remains the same. However, in Case N even though PL gets the spillover benefit, the retailer's profit increases less than the manufacturer's. This is because PL's marginal profit is low due to its low quality as compared to NB.

5.1.2 Impact on the profits when competition is moderate or high

When the competition is moderate or high, the two firms' profits may not monotonically increase with the spillover rate, because the horizontal competition caused by the spillover effect could significantly hurt both firms. Below we illustrate several representative cases.

Impact on the two firms' profits in Case N

Here we study the impact of the spillover rate from NB to PL, λ_P, on the retailer's profit. As λ_P increases, PL's demand increases but NB's demand decreases because of competition. When the competition is low (e.g., $\gamma = 0.1$), the demand decrease of NB is insignificant and can be compensated by the demand increase of PL. Thus the retailer's profit monotonically increases with λ_P. When the competition is moderate ($\gamma = 0.5$), the demand loss of NB is no longer ignorable. But, if PL's quality is sufficiently high ($Q_P = 0.95$) and the profit margin of selling PL is close to that of NB, the retailer' s profit still monotonically increases with λ_P, see Figure 7.

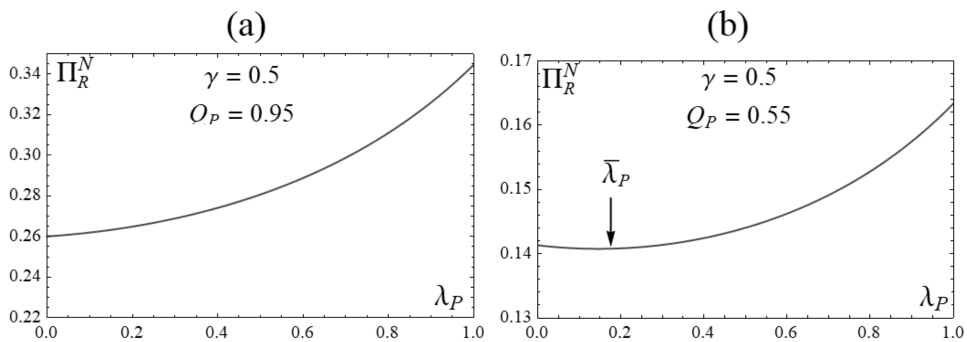

Figure 7: The retailer's profit in Case N

The nuance comes in Figure 7(b), where the competition is moderate and PL's quality is low ($\gamma = 0.5$ and $Q_P = 0.55$) in Case N. When λ_P increases, there are two effects on the retailer's profit. On the one hand, it incurs a negative effect where the high margin NB's demand decreases even though the low margin PL's demand increases. On the other hand, it incurs a positive effect on the retailer's profit, because the retailer keeps all the revenue gain caused by PL's demand increase, though shares the revenue loss caused by NB's demand decrease with the NB manufacturer. When λ_P is lower than a certain threshold ($\bar{\lambda}_P$ shown on the graph), the negative effect dominates and the retailer's profit decreases as λ_P increases. When λ_P is higher than the threshold, the positive effect dominates and the retailer's profit increases. Similar results can be observed for the manufacturer, for example, see Figure 10 which is to be further discussed in the next subsection.

Impact on the two firms' profits in Case P

In Case P, we find that both firms' profits monotonically increase with λ_N, and both firms prefer high spillover. Figure 8 shows two examples of the impacts of the spillover rate from PL to NB, λ_N, on the two firms. Intuitively, the manufacturer's profit increases more rapidly than the retailer's as the spillover rate from PL to NB, λ_N, grows. A comparison of the two graphs in Figure 8 indicates that the retailer's advantages decrease as PL's quality drops from 0.95 to 0.55.

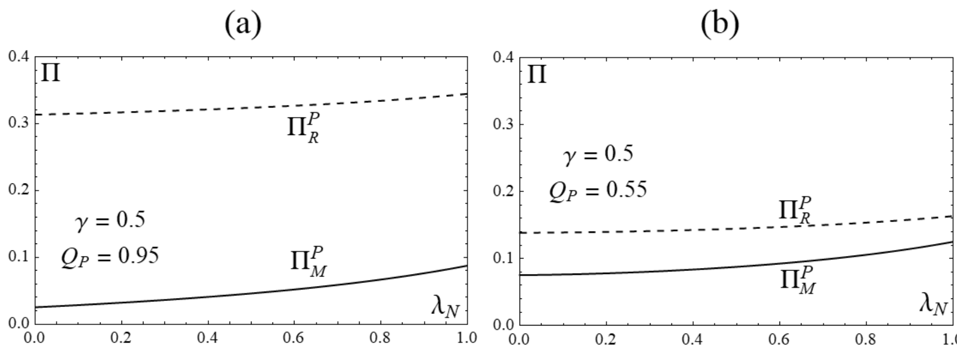

Figure 8: The two firms' profits in Case P

5.1.3 Impact on the supply chain's profit in Cases P and N

Figure 9 shows the impacts of the two spillover rates on the supply chain's profit under Case P (solid) and Case N (dashed). In many scenarios the supply chain's profit monotonically increases with the spillover rate, except in Case N when the competition is moderate and the PL's quality is low, or both the competition and PL's quality are high. Noticeably, however, the dashed line (Case N) in Figure 9(b) (i.e., $Q_p = 0.55$) shows a non-monotonic trend. This is because this curve (the supply chain's profit) is a combination of Figure 7(b) (the retailer's profit) and Figure 10(a) (the NB manufacturer's profit). This result suggests that the supply chain profit does not always increase as the spillover rate increases.

5.2 The Firms' Preferences of Spillover Rates

As we discussed in the paper, the spillover rates can be asymmetric. In practice, spillover rates are controllable. For example, when promoting one brand, the retailer can place the other brand right

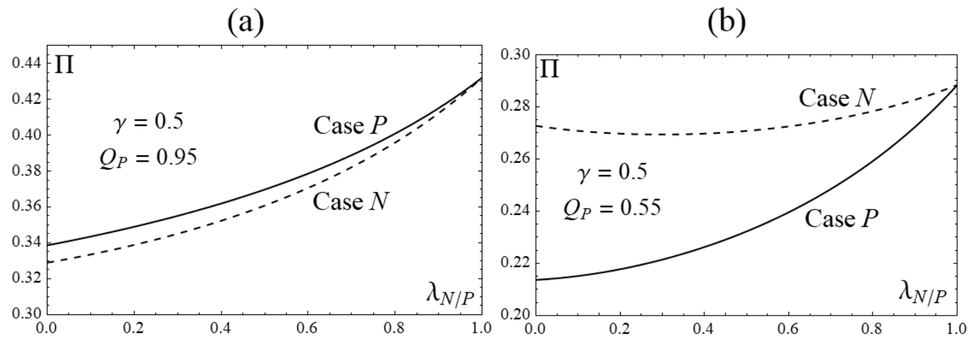

Figure 9: The supply chain's profit in Case P and N

beside, front, behind, above, below the shelf, or far away to create different spillover effects. One can argue that placing the two products side by side will create a higher spillover effect than placing them far away. Placing one product at the proper eye level and the other below the eye level will create asymmetric spillover rates (Valenzuela and Raghubir 2015). While for tractability the paper has so far explored the situations with exogenous spillover rates, this subsection investigates the firms' preference of spillover rates when they are controllable.

To showcase the firms' preference of spillover rates, let us examine the impact of the spillover rate from NB to PL, λ_P, on NB manufacturer's profit, as shown in Figure 10. As λ_P increases, there will be a trade-off affecting NB's demand. On the one hand, the demand of PL will increase due to the spillover effect, which in turn encroaches on NB's demand (competition effect). On the other hand, the retailer has an incentive to step up the promotion level because of the spillover effect; consequently, the demand of NB increases (complementary effect). As illustrated in Figure 10(a), when γ is moderate and PL's quality is low ($\gamma = 0.5$ and $Q_P = 0.55$), the competition effect dominates such that the NB manufacturer's profit monotonically decreases with λ_P. Therefore, the manufacturer's optimal preference of λ_P will be zero to prevent the competition effect although the complementary effect is subdued accordingly.

When γ is moderate and PL's quality is high ($\gamma = 0.65$ and $Q_P = 0.95$, see Figure 10(b)), however, the complementary effect dominates when λ_P is low, such that the NB manufacturer's profit first increases then decreases, as the competition effect surpasses the complementary effect when λ_P is high. Therefore, there exists an optimal spillover rate λ_P^* for the NB manufacturer, $\lambda_P^* = \underset{\lambda_P \in [0,1]}{\text{argmax}} \Pi_M^N$. Although it is difficult to analytically provide the closed form of λ_P^*, we numerically observe the following property: the NB manufacturer's optimal spillover rate (λ_P^*) decreases with

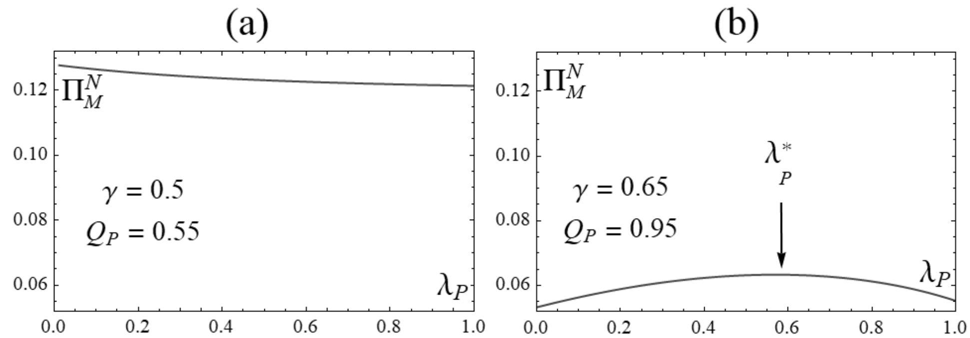

Figure 10: The NB manufacturer's profit in Case N

the product substitutability level (γ). We further illustrate this property in Figure 11.

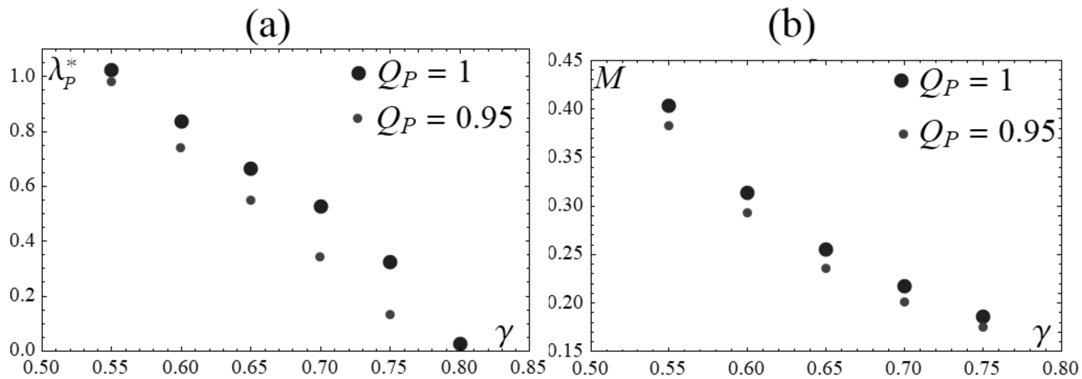

Figure 11: Optimal spillover rate (a) and promotional effort (b) when γ increases

Figure 11 also shows the manufacturer's preferred spillover rate increases as Q_P increases. This is because when Q_P increases, the retailer has more incentives to exert more promotional effort to attract more consumers (see Figure 11(b)). Since NB's demand directly benefits from the promotion (the complementary effect), the NB manufacturer will prefer a higher spillover rate from NB to PL to stimulate a higher promotional effort from the retailer.

We now summarize the retailer's and manufacturer's preferences on the spillover rate (λ_P^*) in Case N in Table 2. For example, if the product substitutability is moderate and PL's quality is high, we obtain {High, Moderate}, which means that the retailer prefers high spillover, whereas the manufacturer prefers moderate spillover. In Case P, both players prefer high spillover in all scenarios.

Retailer Manufacturer	γ is low	γ is moderate	γ is high
Q_P is high	High High	High Moderate	High or Low Low
Q_P is low	High High	High or Low Low	Infeasible

Table 2: The {retailer's, manufacturer's} preferences on the spillover rate (λ_P^*) in Case N.

5.3 Impacts of Non-zero Production Cost

In this section we relax the assumption of zero production costs, and conduct numerical tests to show its impacts on the structure of the main results studied in Section 4. For this purpose, we introduce unit production cost functions $c_N = a_N + b_N Q_N$ and $c_P = a_P + b_P Q_P$, where $a_{N/P}$ is the quality-independent cost for basic material and labor and $b_{N/P}$ is the cost coefficient for quality improvement for NB/PL. The profit functions change from Equation (3) to:

$$\begin{cases} \Pi_M^P = (w_N - c_N)D_N; \\ \Pi_R^P = (p_N - w_N)D_N + (p_P - c_P)D_P - M^2. \end{cases}$$

It is complicated to obtain the analytical solutions for the three cases P/N/B as in Section 3. We solve the problem numerically and find the unique equilibrium solution for each case.

Notice that the production cost affects the intensity of competition and the firms' preferences. For example, when $\gamma = 0.1$ and $\lambda_{N/P} = 0.3$, the retailer will prefer not to introduce PL, i.e., Case B $\succ$ Case N and Case B $\succ$ Case P, if $a_{N/P} \geq 0.25$ and $b_{N/P} \geq 0.3$. Therefore for meaningful discussions, we exclude the cases where the production cost is so high that introducing PL is no longer beneficial for the retailer. In the following, we set $\{\gamma = 0.1, \lambda_{N/P} = 0.3\}$ and keep $a_{N/P} \leq 0.2$ and $b_{N/P} \leq 0.25$.

We conducted numerical tests with a matrix of values of cost functions, ranging from symmetric production cost functions, $c_N = 0.05{+}0.05Q_N$ and $c_P = 0.05{+}0.05Q_P$, to asymmetric cost functions where NB incurs only a quarter of PL's production cost, $c_N = 0.05 + 0.05Q_N$ and $c_P = 0.2{+}0.2Q_P$. The structure of the results are consistent. In the rest of this section, we only present the results with cost functions $c_N = 0.12{+}0.05Q_N$ and $c_P = 0.18{+}0.07Q_P$, as NB usually has some advantage in the production cost compared with PL.

For the scenario with symmetric spillover rates and asymmetric quality, we find the following preference sequences for the retailer:

- Case N $\succ$ Case B;
- Case P $\succ$ Case B if and only if $Q_P > \bar{Q}_P^{PB} = 0.423$;
- Case P $\succ$ Case N if and only if $Q_P > \bar{Q}_P^{PN} = 0.648 > \bar{Q}_P^{PB}$.

These results are in line with Proposition 3.

The preference sequences for the NB manufacturer are:

- Case N $\succ$ Case P;
- Case N $\succ$ Case B if and only if $Q_P < \bar{Q}_P^{NB} = 0.358$;
- Case P $\succ$ Case B if and only if $Q_P < \bar{Q}_P^{PB} = 0.112 < \bar{Q}_P^{NB}$.

These results are in line with Proposition 4.

For the scenario with asymmetric spillover and symmetric quality, we find that

- For the retailer, Case P $\succ$ Case N if and only if $\lambda_N > \bar{\lambda}_N^R = 0.287$.
- For the NB manufacturer, Case P $\succ$ Case N if and only if $\lambda_N > \bar{\lambda}_N^M = 0.791 > \bar{\lambda}_N^R$;

These results are in line with Proposition 5. The structure of the results are consistent throughout the numerical tests conducted. So the analytical properties obtained with zero production costs still hold with non-zero production costs.

5.4 Impacts of NB manufacturer's Out-store Marketing Effort

In addition to the retailer's in-store promotion, the NB manufacturer may also insert effort to promote its product outside of stores, through TV or the Internet. While the retailer's in-store promotion effort could spill over to other products, the NB manufacturer's out-store promotion is more likely to be predatory than cooperative. The manufacturer may try to promote its own product by revealing the limitations of the competitor's product. In this section we examine the NB manufacturer's out-store promotion efforts on the retailer's decisions.

We assume the out-store promotion increases NB's perceived quality by $M_N = k_N Q_N$, which means the better the quality of the product itself, the more effective the promotion. The NB manufacturer's cost for the promotion effort is $c_k = a_k + b_k k_N^2$, where a_k is a fixed cost for booking the channel and b_k is the variable effort cost. Let λ_k be the impact of the NB manufacturer's promotion on PL. The demand functions in Case P change to

$$\begin{cases} D_N = \dfrac{1}{1-\gamma^2}(Q_N - \gamma Q_P + (1-\gamma\lambda_k)k_N Q_N + (\lambda_N - \gamma)M - p_N + \gamma p_P); \\ D_P = \dfrac{1}{1-\gamma^2}(Q_P - \gamma Q_N + (\lambda_k - \gamma)k_N Q_N + (1-\gamma\lambda_N)M - p_P + \gamma p_N). \end{cases}$$

The demand functions in Case N change to

$$\begin{cases} D_N = \dfrac{1}{1-\gamma^2}(Q_N - \gamma Q_P + (1-\gamma\lambda_k)k_N Q_N + (1-\gamma\lambda_P)M - p_N + \gamma p_P); \\ D_P = \dfrac{1}{1-\gamma^2}(Q_P - \gamma Q_N + (\lambda_k - \gamma)k_N Q_N + (\lambda_P - \gamma)M - p_P + \gamma p_N). \end{cases}$$

The profit functions change from Equation (3) to:

$$\begin{cases} \Pi_M^P = w_N D_N - c_k; \\ \Pi_R^P = (p_N - w_N)D_N + p_P D_P - M^2. \end{cases}$$

The problem becomes more complicated after introducing the out-store promotion. We solve the three cases P/N/B numerically and find the unique equilibrium solution for each case.

To keep the out-store promotion a profitable option to the manufacturer, the promotion cost cannot be too high. In this section, we use $\{\gamma = 0.1, \lambda_{N/P} = 0.3, \theta = 0.02\}$ and keep $a_k \leq 0.1$, $b_k \leq 0.9$, $-0.5 \leq \lambda_k \leq 0.5$. Although the NB manufacturer's out-store promotion is more likely to be predatory than cooperative, here we allow λ_k to be either negative or positive.

Similar to the previous section, our numerical results show that the introducing the out-store promotion does not change the structure of the major results, i.e., Propositions 3, 4 and 5. In addition, we find that in both cases P and N, the NB manufacturer's profit decreases in λ_k, while the retailer's profit increases in λ_k. This result is easy to understand intuitively.

More importantly, our results show that the predatory effect the manufacturer's out-store promotion may make the retailer uninterested in introducing PL. For example, when $Q_P = 0.8$, $\gamma = 0.1$ and $\lambda_N = 0.2$, if $\lambda_k \leq -0.25$, the retailer's preference sequence changes from Case P $\succ$ Case N $\succ$ Case B to Case B $\succ$ Case P $\succ$ Case N. So the retailer will not introduce PL because a powerful "predatory" advertisement of NB undermines the profitability of PL.

Our results also show that the predatory effect of the out-store promotion can be mitigated by a high spillover rate of the in-store promotion. Following the scenario described above, if λ_N increases from 0.2 to 0.7 while other values remain unchanged, the retailer holds its preference sequence of Case P $\succ$ Case N $\succ$ Case B when $\lambda_k = -0.25$. To change the retailer's preference sequence to Case B $\succ$ Case P $\succ$ Case N, λ_k needs to be -0.3 or lower. So the retailer can still introduce PL unless the predatory effect of the out-store promotion is very strong, because a "cooperative" in-store promotion can reduce the predatory effect of the NB manufacturer's out-store promotion. For example, if the retailer strategically put NB and PL products next to each other, consumers searching for NB may end up buying PL. The key driver behind those different marketing strategies is that the retailer earns profit from both products sold in the store, while the NB manufacturer earns profit only from its own product.

6 Conclusion

This paper extends the extant literature on retailer-owned private label product by simultaneously considering the retailer's in-store promotional effort and the spillover effect. We compare three different market scenarios: no private label while promoting the national brand, introducing a private label and promoting it, and introducing a private label but promoting the national brand. We find that the introduction of private label is not preferable for the national brand if there is no spillover effect. However, if spillover effect exists, we find a national brand manufacturer may benefit from the introduction of a competing private label product if the spillover effect is high and product substitutability is low.

We also study impacts of the retailer's in-store promotion decision on firms. First, when the spillover rates between the two products are symmetric, the retailer prefers to promote the private label product and the national brand manufacturer prefers its own product to be promoted. Second, when the spillover from the national brand to the private label is significantly higher than the spillover in the opposite direction, both firms prefer to promote the national brand product. Third, when the spillover from the private label to the national brand is significantly higher than the opposite spillover, both firms prefer to promote the private label product.

When the spillover rates between the two firms are symmetric, the products' qualities play important roles in the two firms' preferences. On one hand, the retailer always prefers the private label introduction. If the private label product's quality is high, the retailer also prefers to promote it; otherwise the retailer promotes the national brand product. On the other hand, the manufacturer prefers the private label introduction if and only if the product substitutability is low, or the substitution factor is high but the private label product's quality is low. This is because in both scenarios, the private label product does not substantially challenge the national brand product, which is consistent with the empirical finding by Pauwels and Srinivasan (2004) that premium-brand manufacturers can benefit from private label introductions.

Our numerical analysis reveals that a higher spillover rate does not always benefit the whole supply chain. When the retailer introduces a low quality private label product but promotes the high quality national brand, a higher spillover rate from the national brand to the private label product may intensify channel conflicts and the supply chain profit does not monotonically increase with the spillover rate. This finding helps explain why some retailers do not put the national brand and the private label products close to each other. Our numerical results also show that the structure of the major results do not change by introducing quality-related production costs or the manufacturer's out-store promotions. In addition, our results show that the predatory effect the manufacturer's out-store promotion may dampen the retailer's interest to introduce the private label, but the predatory effect of the out-store promotion can be mitigated by the spillover effect of the in-store promotion.

This paper has its limitations. First, for tractability, we consider only one national brand product. In reality, a retailer may sell products from multiple national brand manufacturers. Second, while some of our results are supported by existing empirical studies, the data on in-store media promotion and spillover effect is rare. Therefore, collaborating with retailers to design field experiments can be a future research priority. Finally, given that private label introductions are inevitable, how to help manufacturers improve their competitive edge will be the next challenging but intriguing subject.

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Appendix: Online Supplements

Proof of Lemma 1

The Stackelberg game proceeds in two stages. In stage two the retailer (follower) decides the retail prices and the promotional effort. In stage one the NB manufacturer (leader) decides the wholesale price.

1) The retailer determines the retail prices and promotion level:

The demand functions are:

$$\begin{cases} D_N = \dfrac{1}{1-\gamma^2}(Q_N - \gamma Q_P + (\lambda_N - \gamma)M - p_N + \gamma p_P), \\ D_P = \dfrac{1}{1-\gamma^2}(Q_P - \gamma Q_N + (1 - \gamma\lambda_N)M - p_P + \gamma p_N). \end{cases}$$

The revenue functions are:

$$\begin{cases} \Pi_M = w_N D_N, \\ \Pi_R = (p_N - w_N)D_N + p_P D_P - \theta M^2. \end{cases}$$

From this point we apply the assumption $Q_N = 1$ and $\theta = 1$. Notice Π_R is quadratic and concave on $\{p_P, p_N, M\}$, because $\dfrac{\partial^2 \Pi_R}{\partial p_P^2} = \dfrac{\partial^2 \Pi_R}{\partial p_N^2} = -\dfrac{2}{1-\gamma^2} < 0$, and $\dfrac{\partial^2 \Pi_R}{\partial M^2} = -2 < 0$. Therefore the unique optimal solution of retail prices and promotional effort exists.

Using the first order condition, we solve $\{\dfrac{\partial \Pi_R}{\partial p_P} = 0, \dfrac{\partial \Pi_R}{\partial p_N} = 0, \dfrac{\partial \Pi_R}{\partial M} = 0\}$ and get

$$\begin{cases} p_P = -\dfrac{\gamma\Gamma - 4Q_P - \gamma\Gamma w_N - \Gamma\lambda_N - \gamma\Gamma Q_P\lambda_N + \Gamma w_N\lambda_N + \Gamma Q_P\lambda_N^2}{2(4 - \Gamma + 2\gamma\Gamma\lambda_N - \Gamma\lambda_N^2)}, \\ p_N = -\dfrac{-4 + \Gamma - 4w_N + \Gamma w_N - \gamma\Gamma\lambda_N - \Gamma Q_P\lambda_N - 3\gamma\Gamma w_N\lambda_N + \gamma\Gamma Q_P\lambda_N^2 + 2\Gamma w_N\lambda_N^2}{2(4 - \Gamma + 2\gamma\Gamma\lambda_N - \Gamma\lambda_N^2)}, \\ M = -\dfrac{\Gamma(\gamma - Q_P - \gamma w_N - \lambda_N + \gamma Q_P\lambda_N + w_N\lambda_N)}{4 - \Gamma + 2\gamma\Gamma\lambda_N - \Gamma\lambda_N^2}. \end{cases}$$

where $\Gamma = \dfrac{1}{1-\gamma^2}$.

2) The NB manufacturer decides the wholesale price:

Substitute the above $\{p_P, p_N, M\}$ into Π_M. We verify $\dfrac{\partial^2 \Pi_M}{\partial w_N^2} < 0$ and the optimal wholesale price exists. Solve $\dfrac{\partial \Pi_M}{\partial w_N} = 0$ to get $w_N = \dfrac{1}{6}(3 + Q_P(-4\gamma + \lambda_N))$.

Substitute the above w_N into other variables and get the whole set of closed-form solution as follows.

$$\begin{cases} w_N = \dfrac{1}{6}(3 + Q_P(-4\gamma + \lambda_N)), \\ p_N = \dfrac{3(-9 + 12\gamma^2 - 5\gamma\lambda_N + 2\lambda_N^2) + Q_P(4\gamma(3 - 4\gamma^2) + (-9 + 16\gamma^2)\lambda_N - 5\gamma\lambda_N^2 + 2\lambda_N^3)}{12(-3 + 4\gamma^2 - 2\gamma\lambda_N + \lambda_N^2)}, \\ p_P = \dfrac{3(\gamma - \lambda_N) + Q_P(-24 + 28\gamma^2 - 11\gamma\lambda_N + 7\lambda_N^2)}{12(-3 + 4\gamma^2 - 2\gamma\lambda_N + \lambda_N^2)}, \\ M = \dfrac{3(\gamma - \lambda_N) + Q_P(-6 + 4\gamma^2 + \gamma\lambda_N + \lambda_N^2)}{6(-3 + 4\gamma^2 - 2\gamma\lambda_N + \lambda_N^2)}, \\ D_N = -\dfrac{3 + Q_P(-4\gamma + \lambda_N)}{4(-3 + 4\gamma^2 - 2\gamma\lambda_N + \lambda_N^2)}, \\ D_P = \dfrac{12\gamma - 3\lambda_N + Q_P(-24 + 16\gamma^2 - 8\gamma\lambda_N + 7\lambda_N^2)}{12(-3 + 4\gamma^2 - 2\gamma\lambda_N + \lambda_N^2)}. \end{cases}$$

The NB manufacturer's and the retailer's profits are

$$\begin{cases} \Pi_M^P = \dfrac{(-3 + Q_P(4\gamma - \lambda_N))^2}{24(3 - 4\gamma^2 + 2\gamma\lambda_N - \lambda_N^2)}, \\ \Pi_R^P = \dfrac{3 - Q_P(8\gamma - 2\lambda_N) + Q_P^2(16(1 - \gamma^2) + 8\gamma\lambda_N - 5\lambda_N^2)}{16(3 - 4\gamma^2 + 2\gamma\lambda_N - \lambda_N^2)}. \end{cases}$$

Notice if $\lambda_N = 0$, $D_N = \dfrac{3 - 4\gamma Q_P}{4(3 - 4\gamma^2)}$ and $w_N = 1/6(3 - 4\gamma Q_P)$. Given the basic non-negative condition $Q_P < 1/\gamma$ and $3 - 4\gamma Q_P > 0$, we have a non-negative condition as $3 - 4\gamma^2 > 0$, or $\gamma < \sqrt{3}/2$. Q.E.D.

Proof of Lemma 2

The process is similar to the proof of Lemma 1 as follows.

1) The retailer determines the retail prices and promotion level:

$$\begin{cases} p_P = -\dfrac{-4Q_P + \Gamma Q_P - \Gamma\lambda_P - \gamma\Gamma Q_P\lambda_P + \Gamma w_N\lambda_P + \gamma\Gamma\lambda_P^2 - \gamma\Gamma w_N\lambda_P^2}{2(4 - \Gamma + 2\gamma\Gamma\lambda_P - \Gamma\lambda_P^2)}, \\ p_N = -\dfrac{-4 + \gamma\Gamma Q_P - 4w_N + 2\Gamma w_N - \gamma\Gamma\lambda_P - \Gamma Q_P\lambda_P - 3\gamma\Gamma w_N\lambda_P + \Gamma\lambda_P^2 + \Gamma w_N\lambda_P^2}{2(4 - \Gamma + 2\gamma\Gamma\lambda_P - \Gamma\lambda_P^2)}, \\ M = \dfrac{\Gamma(1 - \gamma Q_P - w_N - \gamma\lambda_P + Q_P\lambda_P + \gamma w_N\lambda_P)}{4 - \Gamma + 2\gamma\Gamma\lambda_P - \Gamma\lambda_P^2}. \end{cases}$$

where $\Gamma = \dfrac{1}{1 - \gamma^2}$.

2) The NB manufacturer decides the wholesale price:

Substitute the above $\{p_P, p_N, M\}$ into Π_M. We verify $\dfrac{\partial^2 \Pi_M}{\partial w_N^2} < 0$ and the optimal wholesale price exists. Solve $\dfrac{\partial \Pi_M}{\partial w_N} = 0$ to get $w_N = \dfrac{-4 + Q_P(4\gamma - \lambda_P) + \lambda_P^2}{2(-4 + \lambda_P^2)}$.

Substitute the above w_N into other variables and get the whole set of closed-form solution as follows.

$$\begin{cases} w_N = \dfrac{-4 + Q_P(4\gamma - \lambda_P) + \lambda_P^2}{2(-4 + \lambda_P^2)}, \\ p_N = \dfrac{(-4 + \lambda_P^2)(-10 + 12\gamma^2 - 5\gamma\lambda_P + 3\lambda_P^2) + Q_P(16\gamma(-1 + \gamma^2) - 2(-5 + 8\gamma^2)\lambda_P + 9\gamma\lambda_P^2 - 3\lambda_P^3)}{4(-4 + \lambda_P^2)(-3 + 4\gamma^2 - 2\gamma\lambda_P + \lambda_P^2)}, \\ p_P = \dfrac{\lambda_P(-1 + \gamma\lambda_P)(-4 + \lambda_P^2) + Q_P(24 - 32\gamma^2 + 12\gamma\lambda_P + (-7 + 4\gamma^2)\lambda_P^2 - \gamma\lambda_P^3)}{4(-4 + \lambda_P^2)(-3 + 4\gamma^2 - 2\gamma\lambda_P + \lambda_P^2)}, \\ M = \dfrac{(-1 + \gamma\lambda_P)(-4 + \lambda_P^2) + Q_P(-4\gamma + (7 - 4\gamma^2)\lambda_P + 3\gamma\lambda_P^2 - 2\lambda_P^3)}{2(-4 + \lambda_P^2)(-3 + 4\gamma^2 - 2\gamma\lambda_P + \lambda_P^2)}, \\ D_N = \dfrac{-4 + Q_P(4\gamma - \lambda_P) + \lambda_P^2}{4(-3 + 4\gamma^2 - 2\gamma\lambda_P + \lambda_P^2)}, \\ D_P = \dfrac{Q_P(24 - 16\gamma^2 + 8\gamma\lambda_P - 7\lambda_P^2) + (4\gamma - \lambda_P)(-4 + \lambda_P^2)}{4(-4 + \lambda_P^2)(-3 + 4\gamma^2 - 2\gamma\lambda_P + \lambda_P^2)}. \end{cases}$$

the NB manufacturer's and the retailer's profits are

$$\begin{cases} \Pi_M^N = \dfrac{(-4 + Q_P(4\gamma - \lambda_P) + \lambda_P^2)^2}{8(-4 + \lambda_P^2)(-3 + 4\gamma^2 - 2\gamma\lambda_P + \lambda_P^2)}, \\ \Pi_R^N = \dfrac{2Q_P(4\gamma - \lambda_P)(-4 + \lambda_P^2) + (-4 + \lambda_P^2)^2 - 3Q_P^2(16(-1 + \gamma^2) - 8\gamma\lambda_P + 5\lambda_P^2)}{16(-4 + \lambda_P^2)(-3 + 4\gamma^2 - 2\gamma\lambda_P + \lambda_P^2)}. \end{cases}$$

Q.E.D.

Proof of Lemma 3

The process is similar to the proof of Lemma 1 as follows. Note that only NB is in the market.

1) The retailer determines the retail price and promotion level:

$$\begin{cases} p_N = \dfrac{2\theta Q_N - w_N + 2\theta w_N}{4\theta - 1} \\ M = \dfrac{Q_N - w_N}{4\theta - 1} \end{cases}$$

2) The NB manufacturer decides the wholesale price: Substitute the above $\{p_N, M\}$ into Π_M. Given $\theta = 1$, $\dfrac{\partial^2 \Pi_M}{\partial w_N^2} = \dfrac{4\theta}{1 - 4\theta} < 0$ and the optimal wholesale price exists. Solve $\dfrac{\partial \Pi_M}{\partial w_N} = 0$ to get $w_N = Q_N/2$.

Substitute the above w_N into other variables and get the whole set of closed-form solution as follows.

$$\begin{cases} w_N = Q_N/2, \\ p_N = (6\theta - 1)Q_N/(8\theta - 2), \\ M = Q_N/(8\theta - 2), \\ D_N = \theta Q_N/(4\theta - 1). \end{cases}$$

the NB manufacturer's and the retailer's profits are

$$\begin{cases} \Pi_M = \theta Q_N^2/(8\theta - 2), \\ \Pi_R = \theta Q_N^2/(16\theta - 4). \end{cases}$$

After applying $Q_N = 1$ and $\theta = 1$, $\{w_N, p_N, M, D_N, \Pi_M, \Pi_R\} = \{1/2, 5/6, 1/6, 1/3, 1/6, 1/12\}$. Q.E.D.

Proof of Lemma 4

The models without promotion are simplified versions of Case B, Case P and N by removing the promotional effort M. By solving the simplified models, the NB manufacturer's and the retailer's profits without the introduction of PL are $\Pi_M^0 = Q_N^2/8$ and $\Pi_R^0 = Q_N^2/16$. The profits with the introduction of PL are

$$\begin{cases} \Pi_M^1 = \dfrac{(Q_N - \gamma Q_P)^2}{8(1-\gamma^2)}, \\ \Pi_R^1 = \dfrac{Q_N^2 - 2\gamma Q_N Q_P + (4 - 3\gamma^2)Q_P^2}{16(1-\gamma^2)}. \end{cases}$$

Since $\Pi_R^1 - \Pi_R^0 = \dfrac{3(1-\gamma^2)Q_P^2 + (Q_P - \gamma)^2}{16(1-\gamma^2)} > 0$, the retailer prefers to introduce PL. Since $\Pi_M^1 - \Pi_M^0 = -\dfrac{(Q_P -}{}$
the NB manufacturer prefers not. Q.E.D.

Proof of Lemma 5

In this proof we apply the following non-negative conditions: $\gamma < Q_P < 1/\gamma$ and $3 - 4\gamma^2 > 0$ (see the end of Lemma 1).

From Lemmas 1, 2 and 3 and by setting $\lambda_N = \lambda_P = 0$, we have the closed-form solutions of the

NB manufacturer's profits in Case B, Case P and N as follows.

$$\begin{cases} \Pi_M^B = 1/6, \\ \Pi_M^P = \dfrac{(4\gamma Q_P - 3)^2}{24(4(1-\gamma^2)-1)}, \\ \Pi_M^N = \dfrac{(1-\gamma Q_P)^2}{6-8\gamma^2}. \end{cases}$$

$\Pi_M^N - \Pi_M^B = \dfrac{(-1+\gamma Q_P)^2}{6-8\gamma^2} - \dfrac{1}{6} < 0 \Leftrightarrow 4\gamma - 6Q_P + 3\gamma Q_P^2 < 0$. Let $f = 4\gamma - 6Q_P + 3\gamma Q_P^2$. $\dfrac{\partial f}{\partial Q_P} = -6(1-\gamma Q_P) < 0$, that is, f decreases with Q_P. When $Q_P = 1/\gamma$ we have $f = (4\gamma^2 - 3)/\gamma < 0$. That is, for all $Q_P < 1/\gamma$, $f < 0$ holds. Then we can conclude $\Pi_M^B > \Pi_M^N$.

$\Pi_M^N - \Pi_M^P = \dfrac{3-4\gamma^2 Q_P^2}{24(3-4\gamma^2)}$. In Lemma 1 we have the non-negative conditions $3 - 4\gamma Q_P > 0$. Then $3 - 4\gamma^2 Q^2 > 3 - 4\gamma Q > 0$ holds. In summary, we have $\Pi_M^N > \Pi_M^P$. Q.E.D.

Proof of Lemma 6

In this proof we apply the following non-negative conditions: $\gamma < Q_P < 1/\gamma$ and $3 - 4\gamma^2 > 0$ (see the end of Lemma 1).

From Lemmas 1, 2 and 3 and by setting $\lambda_N = \lambda_P = 0$, we have the closed-form solutions of retailer's profit in Case B, Case P and N as follows.

$$\begin{cases} \Pi_R^B = 1/12, \\ \Pi_R^P = \dfrac{-3+8\gamma Q_P + 16(-1+\gamma^2)Q_P^2}{-48+64\gamma^2}. \\ \Pi_R^N = \dfrac{-1+2\gamma Q_P + 3(-1+\gamma^2)Q_P^2}{4(-3+4\gamma^2)}. \end{cases}$$

$\Pi_R^N - \Pi_R^B = \dfrac{-4\gamma^2 + 6\gamma Q_P + 9(-1+\gamma^2)Q_P^2}{-36+48\gamma^2} > 0 \Leftrightarrow 4\gamma^2 - 6\gamma Q_P + 9(1-\gamma^2)Q_P^2 > 0$. Let $f = 4\gamma^2 - 6\gamma Q_P + 9(1-\gamma^2)Q_P^2$. $\dfrac{\partial f}{\partial Q_P} = 6(3(1-\gamma^2)Q_P - \gamma) > 0$, that is, f increases with Q_P. When $Q_P = \gamma$ we have $f = \gamma^2(7 - 9\gamma^2) > 0$. That is, for all $Q_P > \gamma$, $f > 0$ holds. Then we conclude $\Pi_R^N > \Pi_R^B$.

Solve the quadratic equation $\Pi_R^P - \Pi_R^B = \dfrac{48(1-\gamma^2)Q_P^2 - 3 + 16\gamma^2 - 24\gamma Q_P}{48(3-4\gamma^2)} = 0$ on Q_P, we will have two roots:

$$Q_+ = \frac{3\gamma + \sqrt{3}\sqrt{16\gamma^4 - 16\gamma^2 + 3}}{12(1-\gamma^2)}, Q_- = \frac{3\gamma - \sqrt{3}\sqrt{16\gamma^4 - 16\gamma^2 + 3}}{12(1-\gamma^2)}.$$

It is easy to verify $Q_- < \gamma$ which contradicts the non-negative condition $Q_P > \gamma$, and then the larger root $Q_+ = \bar{Q}_P^{PB}$ is the only feasible threshold. Noticing that the quadratic equation is convex on Q_P, we can conclude that when $Q_P > \bar{Q}_P^{PB}$, $\Pi_R^P > \Pi_R^B$.

Consider $\Pi_R^P - \Pi_R^N = \dfrac{4(1-\gamma^2)Q_P^2 - 1}{16(3-4\gamma^2)} = 0$. We find that when $Q_P > \bar{Q}_P^{PN} = \dfrac{1}{2\sqrt{1-\gamma^2}}$, $\Pi_R^P > \Pi_R^N$.

Comparing between the two thresholds, we find that $\bar{Q}_P^{PB} < \bar{Q}_P^{PN}$. Q.E.D.

Proof of Proposition 1

From Lemmas 1, 2 and 3 we have the closed-form solutions of Case P, Case N and Case B. When $\lambda_P = \lambda_N = \lambda$ and $Q_P = Q_N = 1$, the retailer's profit gap between Case P and Case N is

$$\Delta_{PN} = \Pi_R^P - \Pi_R^N = \frac{16\gamma^2 - 8\gamma(\lambda-1) + 5\lambda^2 - 2\lambda - 19}{16(4\gamma^2 - 2\gamma\lambda + \lambda^2 - 3)} - \frac{64 - 48\gamma^2 + 8\gamma(\lambda^2 + 3\lambda - 4) + \lambda^4 - 2\lambda^3 - 23\lambda^2 + 8\lambda}{16(\lambda^2-4)(4\gamma^2 - 2\gamma\lambda + \lambda^2 - 3)}.$$

To find the minimal and maximal values of Δ_{PN}, we solve two constrained nonlinear programming problems given $0 \le \lambda \le 1$ and $0 \le \gamma \le 4/5$. We find $0 \le \Delta_{PN} \le 0.0625$, that is Case P $\succ$ Case N.

Similarly, let

$$\Delta_{NB} = \Pi_R^N - \Pi_R^B = \frac{64 - 48\gamma^2 + 8\gamma(\lambda^2 + 3\lambda - 4) + \lambda^4 - 2\lambda^3 - 23\lambda^2 + 8\lambda}{16(\lambda^2-4)(4\gamma^2 - 2\gamma\lambda + \lambda^2 - 3)} - \frac{1}{12}.$$

To find the minimal and maximal values of Δ_{NB}, we solve two constrained nonlinear programming problems given $0 \le \lambda \le 1$ and $0 \le \gamma \le 4/5$. We find $0.1875 \le \Delta_{NB} \le 0.4167$, that is Case N $\succ$ Case B. Q.E.D.

Proof of Proposition 2

From Lemmas 1, 2 and 3, we have the closed-form solutions of Case P, Case N and Case B. When $\lambda_P = \lambda_N = \lambda$ and $Q_P = Q_N = 1$, the NB manufacturer's profits are

$$\begin{cases} \Pi_M^P = \dfrac{((\lambda - 4\gamma) + 3)^2}{24(-4\gamma^2 + 2\gamma\lambda - \lambda^2 + 3)}, \\ \Pi_M^N = \dfrac{(\lambda^2 + (4\gamma - \lambda) - 4)^2}{8(4-\lambda^2)(-4\gamma^2 + 2\gamma\lambda - \lambda^2 + 3)}, \\ \Pi_M^B = 1/6. \end{cases}$$

Firstly, $\Pi_M^N - \Pi_M^P = \dfrac{1-\lambda^2}{6(4-\lambda^2)} > 0$, that is Case N $\succ$ Case P. Secondly, $\Pi_M^N - \Pi_M^B = 0$ can be transformed into a quadratic equation of γ:

$$\gamma^2(16(4-\lambda^2)+48)+\gamma(24\lambda^2-8(4-\lambda^2)\lambda-24\lambda-96)$$
$$+3\lambda^4-6\lambda^3+4(4-\lambda^2)\lambda^2-21\lambda^2-12(4-\lambda^2)+24\lambda+48=0.$$

The above function is convex because $16(4-\lambda^2)+48>0$. The equation has two roots as $\gamma_-=\lambda/4$ and $\gamma_+=\dfrac{\lambda^3+6\lambda^2-7\lambda-24}{4(\lambda^2-7)}$. By solving a constrained nonlinear programming problem given $0\leq\lambda\leq 1$, we find $\gamma_+\geq 0.857$ (infeasible). Such that γ_- is the only feasible threshold. Then we can conclude that Case N $\succ$ Case B when $\lambda>4\gamma=\bar{\lambda}^M_{NB}$. This threshold implies that when $\gamma>1/4$, Case B $\succ$ Case N because $\lambda\leq 1$.

Thirdly, $\Pi^P_M-\Pi^B_M=0$ can be transformed into a quadratic equation of λ:

$$30\lambda^2+(36-96\gamma)\lambda+192\gamma^2-144\gamma-18=0.$$

The above function is convex. The equation has two roots as $\lambda_-=\frac{1}{5}(-2\sqrt{6}\sqrt{-4\gamma^2+3\gamma+1}+8\gamma-3)$ and $\lambda_+=\frac{1}{5}(2\sqrt{6}\sqrt{-4\gamma^2+3\gamma+1}+8\gamma-3)$. By solving a constrained nonlinear programming problem given $0\leq\gamma\leq 0.8$, we find $\lambda_-\leq -0.218$ (infeasible). Such that λ_+ is the only feasible threshold. Then we can conclude that Case P $\succ$ Case B when $\lambda>\lambda_+=\bar{\lambda}^M_{PB}$.

Lastly, to compare between the two thresholds we only need to consider the situation of $\gamma\leq 1/4$, because when $\gamma>1/4$, Case B $\succ$ Case N $\succ$ Case P. We find $\bar{\lambda}^M_{PB}-\bar{\lambda}^M_{NB}\geq 0$ given $\gamma\leq 1/4$.

To express the thresholds as functions of λ, firstly we notice $\bar{\gamma}^M_{NB}=\lambda/4$ can be inferred from Proposition 2 directly. From Proposition 2, the threshold between Case P and Case B in the space $\{\gamma,\lambda\}$ is $(2\sqrt{6}\sqrt{-4\gamma^2+3\gamma+1}+8\gamma-3)/5=\lambda$. Solving γ from this equation yields two roots as $\gamma_-=(2\lambda+3-\sqrt{3}\sqrt{5-2\lambda^2})/8$ and $\gamma_+=(2\lambda+3+\sqrt{3}\sqrt{5-2\lambda^2})/8$. By solving a constrained nonlinear programming problem given $0\leq\lambda\leq 1$, we find $\gamma_+\geq 0.86$ (infeasible). Such that γ_- is the only feasible threshold. To compare between the two thresholds, we find $\bar{\gamma}^M_{PB}-\bar{\gamma}^M_{NB}\leq 0$ given $0\leq\lambda\leq 1$. Q.E.D.

Proof of Proposition 3

From Lemmas 1, 2 and 3, we have the closed-form solutions of Case P, Case N and Case B.

When $\lambda_P = \lambda_N = \lambda$ and $Q_N = 1$, the retailer's profits are

$$\begin{cases} \Pi_R^P = \dfrac{(-16\gamma^2 + 8\gamma\lambda - 5\lambda^2 + 16)Q_P^2 + (2\lambda - 8\gamma)Q_P + 3}{16(-4\gamma^2 + 2\gamma\lambda - \lambda^2 + 3)}, \\ \Pi_R^N = \dfrac{(\lambda^2-4)^2 - 3(16\gamma^2 - 8\gamma\lambda + 5\lambda^2 - 16)Q_P^2 + 2(\lambda^2-4)(4\gamma-\lambda)Q_P}{16(\lambda^2-4)(4\gamma^2 - 2\gamma\lambda + \lambda^2 - 3)}, \\ \Pi_R^B = 1/12. \end{cases}$$

Firstly, by solving a constrained nonlinear programming problem given $0 \le \lambda \le 1$, $0 \le \gamma \le 4/5$ and $\gamma < Q_P < min(1/\gamma, 2)$, we find $\Pi_R^N - \Pi_R^B \ge 0$.

Secondly, $\Pi_R^P - \Pi_R^B = 0$ can be transformed into a quadratic equation of Q_P:

$$12(-16\gamma^2 + 8\gamma\lambda - 5\lambda^2 + 16)Q_P^2 + 12(2\lambda - 8\gamma)Q_P - 16(-4\gamma^2 + 2\gamma\lambda - \lambda^2 + 3) + 36 = 0.$$

The above function is convex, because $-16\gamma^2 + 8\gamma\lambda - 5\lambda^2 + 16 \ge 5.76$ given $0 \le \lambda \le 1$ and $0 \le \gamma \le 4/5$. The equation has two roots:

$$\begin{cases} Q_- = \dfrac{-2\sqrt{3}\sqrt{64\gamma^4 - 64\gamma^3\lambda + 52\gamma^2\lambda^2 - 64\gamma^2 - 18\gamma\lambda^3 + 32\gamma\lambda + 5\lambda^4 - 19\lambda^2 + 12} + 12\gamma - 3\lambda}{3(-16\gamma^2 + 8\gamma\lambda - 5\lambda^2 + 16)}, \\ Q_+ = \dfrac{2\sqrt{3}\sqrt{64\gamma^4 - 64\gamma^3\lambda + 52\gamma^2\lambda^2 - 64\gamma^2 - 18\gamma\lambda^3 + 32\gamma\lambda + 5\lambda^4 - 19\lambda^2 + 12} + 12\gamma - 3\lambda}{3(-16\gamma^2 + 8\gamma\lambda - 5\lambda^2 + 16)}. \end{cases}$$

By solving a constrained nonlinear programming problem given $0 \le \lambda \le 1$ and $0 \le \gamma \le 4/5$, we find $Q_- \le -0.111$ (infeasible). Such that $Q_+ = \bar{Q}_P^{PB}$ is the only feasible threshold. Notice $\bar{Q}_P^{PB}$ is the larger root, and we conclude that Case P $\succ$ Case B when $Q_P > \bar{Q}_P^{PB}$.

Thirdly, $\Pi_R^P - \Pi_R^N = 0$ can be transformed into a quadratic equation of Q_P:

$$\begin{aligned} &(3(16\gamma^2 - 8\gamma\lambda + 5\lambda^2 - 16) + (4 - \lambda^2)(-16\gamma^2 + 8\gamma\lambda - 5\lambda^2 + 16))Q_P^2 \\ &+ ((4-\lambda^2)(2\lambda - 8\gamma) - 2(\lambda^2-4)(4\gamma - \lambda))Q_P - (\lambda^2-4)^2 + 3(4-\lambda^2) = 0. \end{aligned}$$

The above function is convex, because $3(16\gamma^2 - 8\gamma\lambda + 5\lambda^2 - 16) + (4-\lambda^2)(-16\gamma^2 + 8\gamma\lambda - 5\lambda^2 + 16) \ge 0.143$ given $0 \le \lambda \le 1$ and $0 \le \gamma \le 4/5$. The equation has two roots:

$$\begin{cases} Q_- = -\dfrac{\sqrt{\lambda^4 - 5\lambda^2 + 4}}{\sqrt{16\gamma^2\lambda^2 - 16\gamma^2 - 8\gamma\lambda^3 + 8\gamma\lambda + 5\lambda^4 - 21\lambda^2 + 16}}, \\ Q_+ = \dfrac{\sqrt{\lambda^4 - 5\lambda^2 + 4}}{\sqrt{16\gamma^2\lambda^2 - 16\gamma^2 - 8\gamma\lambda^3 + 8\gamma\lambda + 5\lambda^4 - 21\lambda^2 + 16}}. \end{cases}$$

It is easy to see the larger root $Q_+ = \bar{Q}_P^{PN}$ is the only feasible threshold. Then we can conclude that Case P $\succ$ Case N when $Q_P > \bar{Q}_P^{PN}$.

Lastly, to compare between the two thresholds, we find $\bar{Q}_P^{PN} - \bar{Q}_P^{PB} \geq 0$ given $0 \leq \lambda \leq 1$ and $0 \leq \gamma \leq 4/5$. Q.E.D.

Proof of Proposition 4

From Lemmas 1, 2 and 3, we have the closed-form solutions of Case P, Case N and Case B. When $\lambda_P = \lambda_N = \lambda$ and $Q_N = 1$, the NB manufacturer's profits are

$$\begin{cases} \Pi_M^P = \dfrac{((\lambda - 4\gamma)Q_P + 3)^2}{24(-4\gamma^2 + 2\gamma\lambda - \lambda^2 + 3)}, \\ \Pi_M^N = \dfrac{(\lambda^2 + (4\gamma - \lambda)Q_P - 4)^2}{8(4 - \lambda^2)(-4\gamma^2 + 2\gamma\lambda - \lambda^2 + 3)}, \\ \Pi_M^B = 1/6. \end{cases}$$

Firstly, by solving a constrained nonlinear programming problem given $0 \leq \lambda \leq 1$, $0 \leq \gamma \leq 4/5$ and $\gamma < Q_P < min(1/\gamma, 2)$, we find $\Pi_M^N - \Pi_M^P \geq 0$.

Secondly, $\Pi_M^N - \Pi_M^B = 0$ can be transformed into a quadratic equation of Q_P:

$$3(4\gamma-\lambda)^2 Q_P^2 + (6\lambda^2(4\gamma-\lambda) - 24(4\gamma-\lambda))Q_P - 4(4-\lambda^2)(-4\gamma^2+2\gamma\lambda-\lambda^2+3) + 3\lambda^4 - 24\lambda^2 + 48 = 0.$$

The above function is convex. The equation has two roots:

$$\begin{cases} Q_- = \dfrac{-2\sqrt{3}\sqrt{4\gamma^2\lambda^2 - 16\gamma^2 - 2\gamma\lambda^3 + 8\gamma\lambda + \lambda^4 - 7\lambda^2 + 12} - 3\lambda^2 + 12}{3(4\gamma - \lambda)}, \\ Q_+ = \dfrac{2\sqrt{3}\sqrt{4\gamma^2\lambda^2 - 16\gamma^2 - 2\gamma\lambda^3 + 8\gamma\lambda + \lambda^4 - 7\lambda^2 + 12} - 3\lambda^2 + 12}{3(4\gamma - \lambda)}. \end{cases}$$

By solving a constrained nonlinear programming problem given $0 \leq \lambda \leq 1$ and $0 \leq \gamma \leq 4/5$, we find $Q_+ \geq 2.294$ (infeasible). Such that $Q_- = \bar{Q}_P^{NB}$ is the only feasible threshold.

Whether $\bar{Q}_P^{NB}$ is the smaller root or larger root depends on the sign of $4\gamma - \lambda$. If $4\gamma - \lambda > 0$, $\bar{Q}_P^{NB}$ is the smaller root and Case N $\succ$ Case B when $Q_P < \bar{Q}_P^{NB}$. If $4\gamma - \lambda < 0$, $\bar{Q}_P^{NB}$ is the larger root and Case N $\succ$ Case B when $Q_P > \bar{Q}_P^{NB}$. However, when $4\gamma - \lambda < 0$, we find $\bar{Q}_P^{NB} < 0$ by solving this constrained nonlinear programming problem, which means Case N $\succ$ Case B for all $Q_P \geq 0$, and thus the threshold $\bar{Q}_P^{NB}$ can be dropped in this scenario.

Thirdly, $\Pi_M^P - \Pi_M^B = 0$ can be transformed into a quadratic equation of Q_P:

$$(\lambda - 4\gamma)^2 Q_P^2 + 6(\lambda - 4\gamma)Q_P - 4(-4\gamma^2 + 2\gamma\lambda - \lambda^2 + 3) + 9 = 0.$$

The above function is convex. The equation has two roots:

$$\begin{cases} Q_- = \dfrac{-\sqrt{(6\lambda - 24\gamma)^2 - 4(16\gamma^2 - 8\gamma\lambda + \lambda^2)(16\gamma^2 - 8\gamma\lambda + 4\lambda^2 - 3)} + 24\gamma - 6\lambda}{2(16\gamma^2 - 8\gamma\lambda + \lambda^2)}, \\ Q_+ = \dfrac{\sqrt{(6\lambda - 24\gamma)^2 - 4(16\gamma^2 - 8\gamma\lambda + \lambda^2)(16\gamma^2 - 8\gamma\lambda + 4\lambda^2 - 3)} + 24\gamma - 6\lambda}{2(16\gamma^2 - 8\gamma\lambda + \lambda^2)}. \end{cases}$$

By solving a constrained nonlinear programming problem given $0 \le \lambda \le 1$ and $0 \le \gamma \le 4/5$, we find $Q_+ \ge 2.36$ (infeasible). Such that $Q_- = \bar{Q}_P^{PB}$ is the only feasible threshold. Then we can conclude that Case P $\succ$ Case B when $Q_P < \bar{Q}_P^{PB}$.

Lastly, to compare between the two thresholds, we find $\bar{Q}_P^{NB} - \bar{Q}_P^{PB} \ge 0$ given $0 \le \lambda \le 1$ and $0 \le \gamma \le 4/5$. Q.E.D.

Proof of Proposition 5

From Lemmas 1 and 2 we have the closed-form solutions of Case P and Case N, and we keep $Q_P = Q_N = 1$. It is easy to verify that $\Pi_M^P < \Pi_M^N$ and $\Pi_R^P < \Pi_R^N$ when $\{\lambda_P, \lambda_N\} = \{1, 0\}$. Similarly, $\Pi_M^P > \Pi_M^N$ and $\Pi_R^P > \Pi_R^N$ when $\{\lambda_P, \lambda_N\} = \{0, 1\}$. Such that there exist two thresholds of $\{\lambda_P, \lambda_N\}$ for each firm to prefer Case P or Case N. In the following we show that $\Pi_R^N - \Pi_R^P$ and $\Pi_M^N - \Pi_M^P$ are quadratic functions of λ_N and analyze their monotonic properties around the thresholds.

The threshold for the retailer

For the retailer, the closed-form of the threshold in space $\{\gamma, \lambda_P, \lambda_N\}$ is determined by

$$\Pi_R^N - \Pi_R^P = \frac{1}{16}\Big(\frac{16(-3\gamma^2 - 2\gamma + 4) - (23 - 8\gamma)\lambda_P^2 + 8(3\gamma + 1)\lambda_P + \lambda_P^4 - 2\lambda_P^3}{(4 - \lambda_P^2)(-4\gamma^2 + 2\gamma\lambda_P - \lambda_P^2 + 3)} \\ - \frac{-16\gamma^2 - 8\gamma + 2(4\gamma + 1)\lambda_N - 5\lambda_N^2 + 19}{-4\gamma^2 + 2\gamma\lambda_N - \lambda_N^2 + 3}\Big) = 0.$$

The above condition can be transformed into a quadratic equation of λ_N as $a_2\lambda_N^2 + a_1\lambda_N + a_0 = 0$, where

$$\begin{cases} a_2 = -16(-3\gamma^2 - 2\gamma + 4) + 5(4 - \lambda_P^2)(-4\gamma^2 + 2\gamma\lambda_P - \lambda_P^2 + 3) + (23 - 8\gamma)\lambda_P^2 - 8(3\gamma + 1)\lambda_P - \lambda_P^4 + 2\lambda_P^3, \\ a_1 = 2\gamma(16(-3\gamma^2 - 2\gamma + 4) - (23 - 8\gamma)\lambda_P^2 + 8(3\gamma + 1)\lambda_P + \lambda_P^4 - 2\lambda_P^3) \\ - 2(4\gamma + 1)(4 - \lambda_P^2)(-4\gamma^2 + 2\gamma\lambda_P - \lambda_P^2 + 3), \\ a_0 = 16\gamma^2(4 - \lambda_P^2)(-4\gamma^2 + 2\gamma\lambda_P - \lambda_P^2 + 3) - 4\gamma^2(16(-3\gamma^2 - 2\gamma + 4) \\ - (23 - 8\gamma)\lambda_P^2 + 8(3\gamma + 1)\lambda_P + \lambda_P^4 - 2\lambda_P^3) + 8\gamma(4 - \lambda_P^2)(-4\gamma^2 + 2\gamma\lambda_P - \lambda_P^2 + 3) \\ - 19(4 - \lambda_P^2)(-4\gamma^2 + 2\gamma\lambda_P - \lambda_P^2 + 3) + 3(16(-3\gamma^2 - 2\gamma + 4) - (23 - 8\gamma)\lambda_P^2 + 8(3\gamma + 1)\lambda_P + \lambda_P^4 - 2\lambda_P^3). \end{cases}$$

The equation has two roots as $\lambda_{N-} = \dfrac{-a_1 - \sqrt{a_1^2 - 4a_0a_2}}{2a_2}$ and $\lambda_{N+} = \dfrac{-a_1 + \sqrt{a_1^2 - 4a_0a_2}}{2a_2}$. Firstly, to find the minimal and maximal values of λ_{N+}, we solve two constrained nonlinear programming problems given $0 \le \lambda_P \le 1$, $0 \le \gamma \le 4/5$ and $a_2 \ge 0$, and find $2.49 \le \lambda_{N+} \le +\infty$; similarly we find $-1.71 \le \lambda_{N-} \le -0.28$. That means when $a_2 \ge 0$ there is no feasible threshold for $\lambda_N \in [0, 1]$. Secondly, under the constraints of $a_1^2 - 4a_0a_2 \ge 0$ and $a_2 \le 0$ we find $-2.39 \le \lambda_{N+} \le -1.34$ (infeasible), which means λ_{N-} is the only feasible threshold for $\lambda_N \in [0, 1]$. Since we have $a_2 \le 0$, $a_2\lambda_N^2 + a_1\lambda_N + a_0 = 0$ is a concave function and λ_{N-} is the larger root. Finally we can conclude that $\Pi_M^N - \Pi_M^P < 0$ (Case P $\succ$ Case N) when $\lambda_N > \lambda_{N-} = \bar{\lambda}_N^R$ in the feasible region.

The threshold for the NB manufacturer

For the NB manufacturer, the closed-form of the threshold in space $\{\gamma, \lambda_P, \lambda_N\}$ is determined by

$$\Pi_M^N - \Pi_M^P = \frac{1}{8}\left(\frac{(-4\gamma - \lambda_P^2 + \lambda_P + 4)^2}{(4 - \lambda_P^2)(-4\gamma^2 + 2\gamma\lambda_P - \lambda_P^2 + 3)} - \frac{(-4\gamma + \lambda_N + 3)^2}{3(-4\gamma^2 + 2\gamma\lambda_N - \lambda_N^2 + 3)}\right) = 0$$

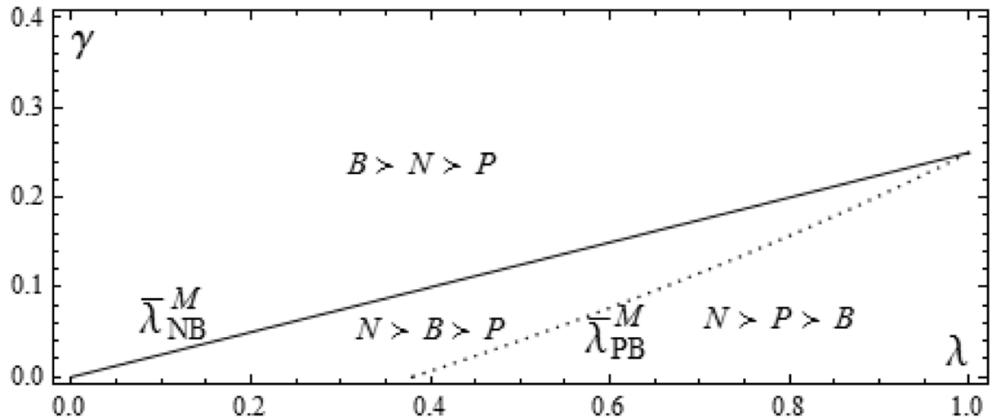
The above condition can be transformed into a quadratic equation of λ_N as $b_2\lambda_N^2 + b_1\lambda_N + b_0 = 0$, where

$$\begin{cases} b_2 = (\lambda_P^2 - 4)(-4\gamma^2 + 2\gamma\lambda_P - \lambda_P^2 + 3) - 3(-4\gamma - \lambda_P^2 + \lambda_P + 4)^2, \\ b_1 = 8\gamma(4 - \lambda_P^2)(-4\gamma^2 + 2\gamma\lambda_P - \lambda_P^2 + 3) - 6(4 - \lambda_P^2)(-4\gamma^2 + 2\gamma\lambda_P - \lambda_P^2 + 3) + 6\gamma(-4\gamma - \lambda_P^2 + \lambda_P + 4)^2 \\ b_0 = -12\gamma^2(-4\gamma - \lambda_P^2 + \lambda_P + 4)^2 - 16\gamma^2(4 - \lambda_P^2)(-4\gamma^2 + 2\gamma\lambda_P - \lambda_P^2 + 3) \\ + 24\gamma(4 - \lambda_P^2)(-4\gamma^2 + 2\gamma\lambda_P - \lambda_P^2 + 3) - 9(4 - \lambda_P^2)(-4\gamma^2 + 2\gamma\lambda_P - \lambda_P^2 + 3) + 9(-4\gamma - \lambda_P^2 + \lambda_P + 4)^2. \end{cases}$$

The equation has two roots as $\lambda_{N-} = \dfrac{-b_1 - \sqrt{b_1^2 - 4b_0b_2}}{2b_2}$ and $\lambda_{N+} = \dfrac{-b_1 + \sqrt{b_1^2 - 4b_0b_2}}{2b_2}$. By solving two constrained nonlinear programming problems given $0 \leq \lambda_P \leq 1$ and $0 \leq \gamma \leq 4/5$, we find $-1.67 \leq \lambda_{N+} \leq -0.14$ (infeasible), which means λ_{N-} is the only feasible threshold for $\lambda_N \in [0, 1]$. We also find $-64.74 \leq b_2 \leq -3.69$, which means $b_2\lambda_N^2 + b_1\lambda_N + b_0$ is a concave function. Notice λ_{N-} is the larger root. Then we can conclude that $\Pi_M^N - \Pi_M^P < 0$ (Case P $\succ$ Case N) when $\lambda_N > \lambda_{N-} = \bar{\lambda}_N^M$ in the feasible region.

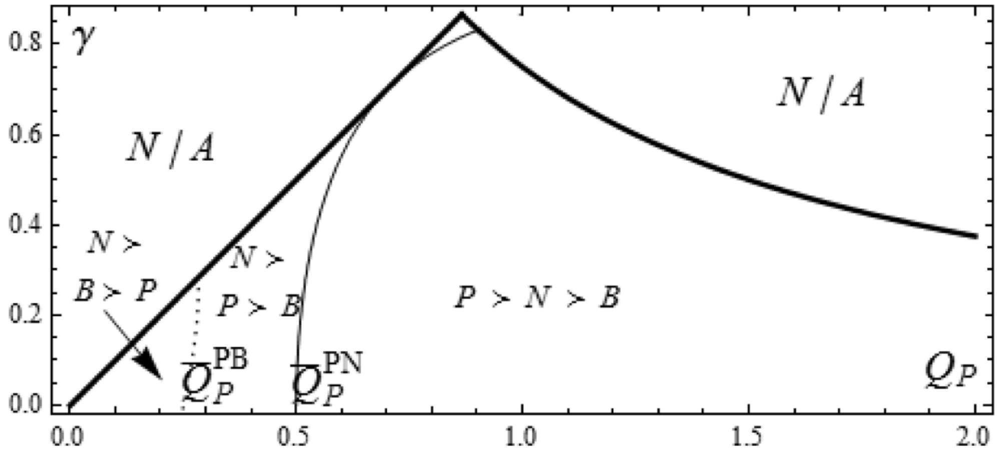
Comparison between the two thresholds

By solving two constrained nonlinear programming problems given $a_2 \leq 0$, we find $-3.73 \leq \bar{\lambda}_N^R - \bar{\lambda}_N^M \leq -1.29$, which means $\bar{\lambda}_N^R < \bar{\lambda}_N^M$ in the feasible region. Q.E.D.



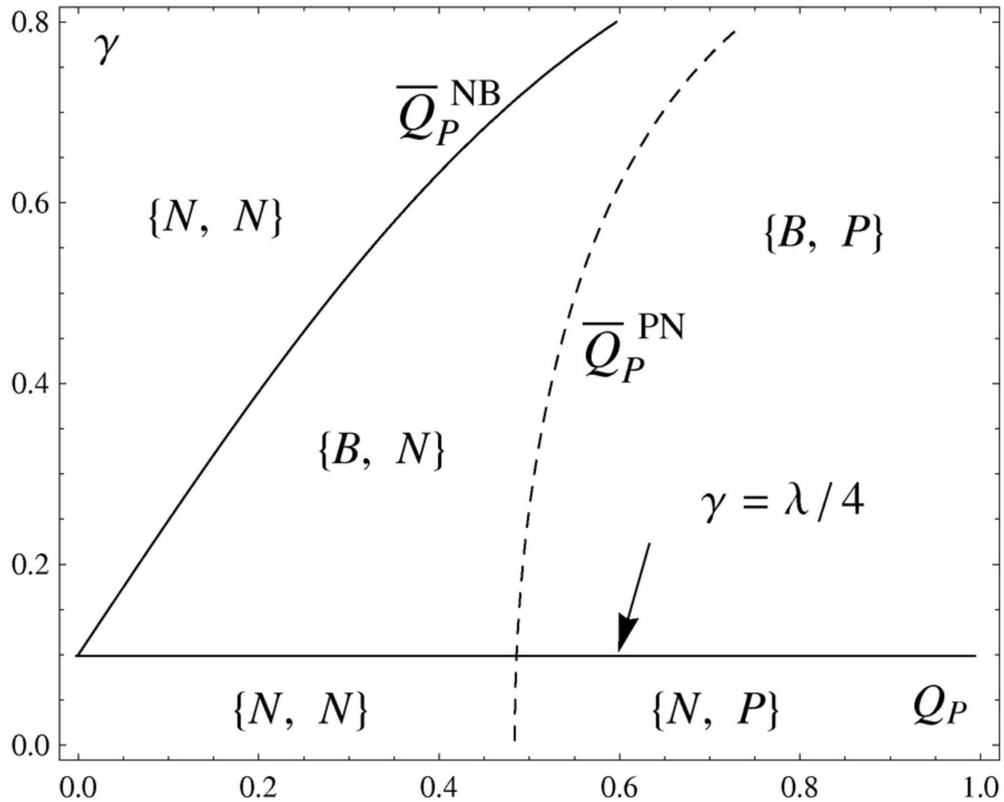
The NB manufacturer's preference with the same spillover/quality.

202x86mm (300 x 300 DPI)



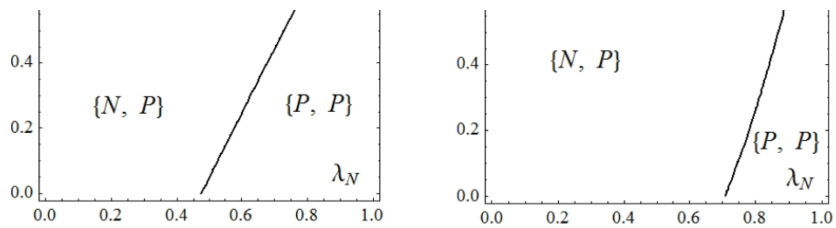
The retailer's preference without spillover.

200x92mm (300 x 300 DPI)



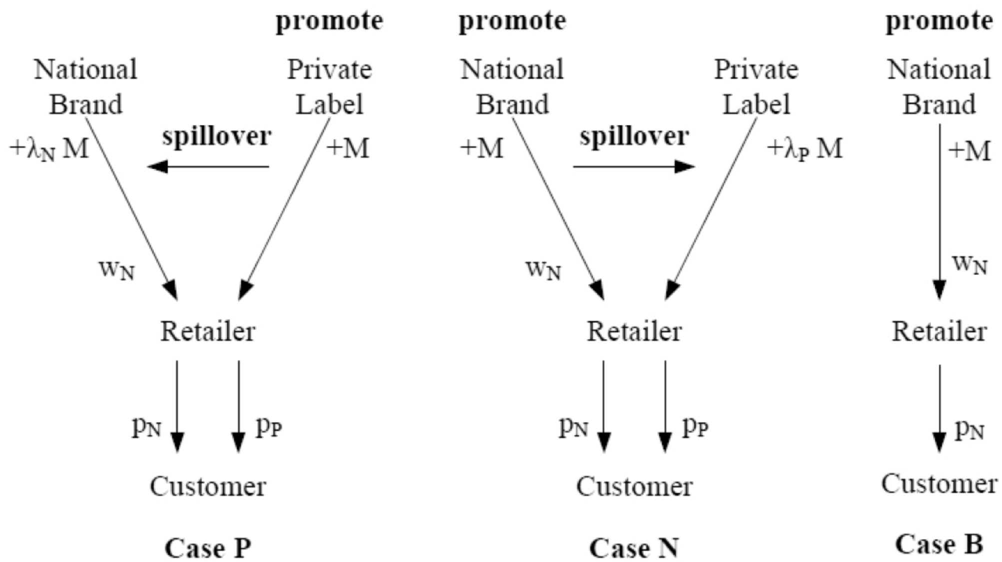
The thresholds of the \{Manufacturer's, Retailer's\}
preference as PL's quality varies with fixed $\lambda=0.4$.

76x62mm (300 x 300 DPI)



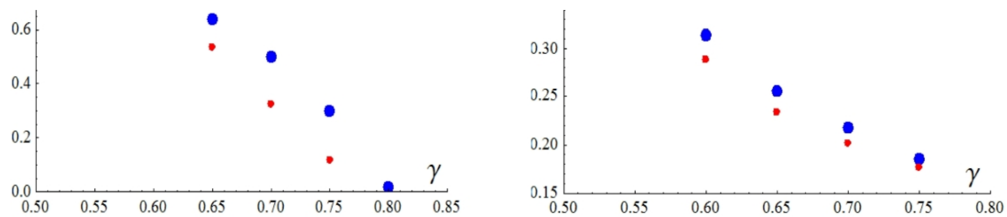
{Manufacturer's, Retailer's\} preferences of Case P or Case N
with asymmetric spillover rates.

288x201mm (300 x 300 DPI)



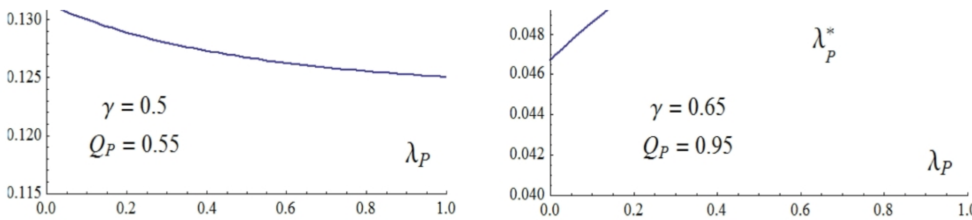
Channel structures

312x173mm (300 x 300 DPI)



Optimal spillover rate (a) and promotional effort (b) when $%
\protect\gamma $ increases

288x201mm (300 x 300 DPI)



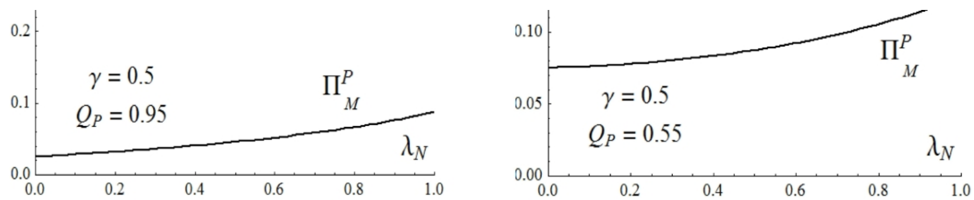
The retailer's profit in Case N

288x201mm (300 x 300 DPI)

		$\gamma = 0.1$			
		Case P (λ_N)		Case N (λ_P)	
		Π_M	Π_R	Π_M	Π_R
$Q_P = 1$	$\lambda = 0.1$	0.102	0.384	0.144	0.322
	$\lambda = 0.5$	0.142	0.405	0.176	0.355
	$\lambda = 0.9$	0.219	0.443	0.229	0.428
	Increase	$\boxed{\mathbf{0.117}}$	**0.058**	**0.085**	$\boxed{\mathbf{0.106}}$
$Q_P = 0.7$	$\lambda = 0.1$	0.109	0.218	0.151	0.198
	$\lambda = 0.5$	0.140	0.233	0.173	0.217
	$\lambda = 0.9$	0.201	0.264	0.211	0.259
		$\boxed{\mathbf{0.091}}$	**0.046**	**0.060**	$\boxed{\mathbf{0.061}}$
$Q_P = 0.4$	$\lambda = 0.1$	0.116	0.112	0.158	0.119
	$\lambda = 0.5$	0.137	0.122	0.170	0.128
	$\lambda = 0.9$	0.183	0.145	0.193	0.147
		$\boxed{\mathbf{0.067}}$	**0.033**	$\boxed{\mathbf{0.035}}$	**0.028**

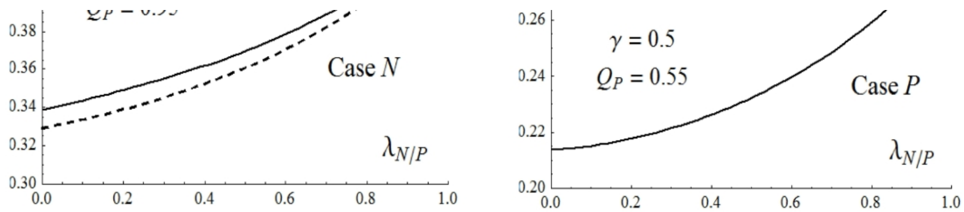
The increase of both firms' profits as $\protect\lambda$ grows.

157x137mm (300 x 300 DPI)



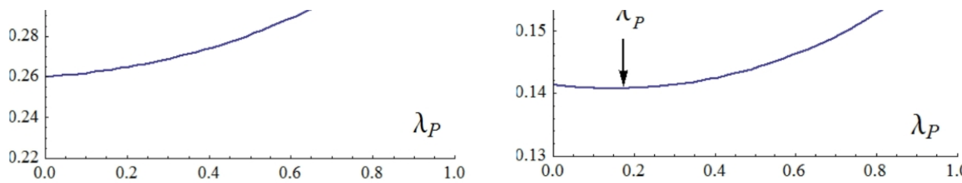
The two firms' profits in Case P

288x201mm (300 x 300 DPI)



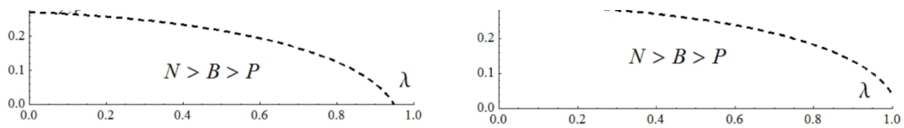
The supply chain's profit in Case P and N

288x201mm (300 x 300 DPI)



The retailer's profit in Case N

288x201mm (300 x 300 DPI)



The trends of $\bar{Q}_{P}^{PN}$ and $\bar{Q}_{P}^{PB}$ for the retailer when $\protect\lambda $ increases.

288x201mm (300 x 300 DPI)

Retailer Manufacturer	γ is low	γ is moderate	γ is high
Q_P is high	High High	High Moderate	High or Low Low
Q_P is low	High High	High or Low Low	Infeasible

The \{retailer's, manufacturer's\} preferences on the spillover rate ($\protect\lambda _{P}^{\ast }$) in Case N.

554x179mm (300 x 300 DPI)