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Until quite recently, scholars have been nearly unanimous in accepting as correct Plato's characterization of Zeno of Elea as a faithful disciple of Parmenides. They have also adopted a reconstruction of his first argument against plurality made by Fränkel[2]. Both this view of Zeno and the accepted interpretation of the reconstructed argument, however, have lately been subjected to incisive criticism by Friedrich Solmsen[3]. Solmsen has claimed that Zeno was no Eleatic, but rather a dialectician without positive philosophical commitments; and he has used the argument reconstructed by Fränkel to support his interpretation.

Solmsen's case rests on three claims. First, he argues that there is no reason to believe, and good reason not to believe, that Plato, the oldest source of the orthodox interpretation, was aiming at historical accuray in the portrait of Zeno he offers in the *Parmenides*. Second, he asserts that the later sources, particularly Simplicius, cannot provide independent confirmation of Plato's account because of their reliance on it. Finally, he argues that the argument reconstructed by Fränkel is really directed not against plurality alone, but against the Eleatic One also.

These criticisms are powerful; however, they do not establish the correctness of Solmsen's view of Zeno. He has shown that modern scholars have been somewhat credulous in accepting Plato's account as historically accurate, but he has not shown that Plato did not correctly interpret the philosophical purpose of Zeno's work[4]. He has shown that Simplicius relied on Plato's account and followed

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[2] Hermann Fränkel, "Zeno of Elea's Attacks on Plurality", *American Journal of Philology* 53 (1942), pp. 14—18.

[3] Friedrich Solmsen, "The Tradition about Zeno of Elea Re-examined", *Phronesis* 16 (1971), pp. 116—141.

[4] A detailed defense of the historical accuracy of Plato's account, prompted by Solmsen's article, is to be found in Gregory Vlastos, "Plato's testimony concerning Zeno of Elea", *Journal of Hellenic Studies* 95 (1975), pp. 136—162.

Plato's interpretation, but not that his interpretive bias "is reflected in the selection, the presentation, the paraphrases and the interpretation of the passages he quotes"[5]. The objectivity of Simplicius's presentation of the evidence, if not his interpretation, is attested to by the fact that Solmsen can draw most of the textual support for his own, anti-Platonic account, from Simplicius.

Most important, Solmsen has established that the Fränkel reconstruction of the first argument concerning plurality does not provide an adequate refutation of the pluralist thesis; but he has not succeeded in showing that his own interpretation of the argument is itself satisfactory.

In this paper, therefore, I shall attempt to point out the shortcomings of Solmsen's interpretation. I shall also offer an alternative argument, drawn from textual material in the Fränkel reconstruction and from additional material that is in all likelihood Zeno's, that is free from the deficiencies of the Fränkel reconstruction, in both the orthodox interpretation and Solmsen's. I cannot assert with confidence that this alternative argument was originally a single argument in Zeno's work, though I think it likely that it represents fairly Zeno's reasoning. At the very least, I hope to show that an argument can be constructed from these Zenonian texts that does justice to Zeno's reputation as a dialectician and to the historical influence of his work. The argument I offer will also be seen to be fully compatible with the traditional, Platonic interpretation of Zeno's work, and incompatible with Solmsen's; that is, it will be an argument directed solely against plurality[6].

I

Fränkel offered his reconstruction to complete an argument ([b—d] below) that is obviously incomplete as it stands. The reconstructed argument goes as follows[7]:

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[5] Solmsen, pp. 127—128.

[6] I take no stand on the question wheter the argument was directed at a particular group, such as the Pythagoreans. The argument works against all who accept the hypotheses of infinite divisibility and of elementary particles, as I indicate below.

[7] Simplicius, *In Aristotelis Physicorum Libros Quattour Priores Commentaria*; in Hermann Diels, ed., *Commentaria in Aristotelem Graeca* (Berlin, 1882), vol. 9, p. 139, 18—19 and 10—15, and p. 141, 2—8. The translation of (a) is mine, that of (b—d) is that of H. D. P. Lee, *Zeno of Elea* (Amsterdam, 1967), pp. 19, 21. All future reference to Simplicius appear in the text, enclosed by parentheses.

a) Nothing has size, because each of the many is the same as itself and one.
b) What has neither magnitude nor thickness nor mass does not exist at all. For, ... if it were added to something else, it would not increase its size; for a null magnitude is incapable, when added, of yielding an increase in magnitude. And thus it follows that what was added was nothing. But if, when it is subtracted from another thing, that thing is no less; and again, if, when it is added to another thing, that thing does not increase, it is evident that both what was added and what was subtracted were nothing. If what is had no magnitude, it would not exist at all.
c) But, if it is, then each one must necessarily have some magnitude and thickness and must be at a certain distance from another. And the same reasoning holds good of the one beyond: for it will also have magnitude and there will be a successor to it. It is the same to say this once and to say it always: for no such part will be the last nor out of relation to another.
d) So, if there is a plurality, they must be both small and large. So small as to have no magnitude, so large as to be infinite.

I refer the reader to Solmsen's article for the details of his critique of this argument. In essence, he shows that (a—c) do not establish (d), and that (a) is peculiarly out of place in an argument for (d). Instead, he claims that we should take (a—b) as an attack on the Eleatic One, and (c—d) as an attack on plurality.

Let us investigate first the claim that (a—b) constitute an attack on the One. Part of the claim is Solmsen's contention that "from a) we know that 'no *megethos*' is true of the *hen*"[8]. Do we know this from (a)? Perhaps; for (a) tells us that nothing has size; and, since the Eleatic One is at least a putative something, presumably it would not have size. But to claim that the argument applies only or even chiefly to the One would be to understate critically its scope; for the initial 'nothing' must also range over 'each of the many' referred to in the latter part of (a).

Solmsen notes that (a) is a Simplician paraphrase of a Zenonian text in which 'each of the many' may not even have appeared, and confidently asserts of the many "that they did not figure as the subject of *ouden echei megethos*"[9]. Solmsen's use of 'subject' here is peculiar, but surely the most natural way to take (a) as it stands is to have the 'nothing' of the first part of the sentence take each of the many as values. The only way to attack this reading is to attack the text, as Solmsen does; but what then becomes of the textual basis of his interpretation?

[8] Solmsen, p. 135.
[9] Ibid., p. 134.

If we keep (a) as it is, then, (a—b) applies clearly to the entities of the pluralist's world. The problematic aspect of the argument is the claim of (a) that unity entails sizelessness, but this is just another formulation of the Zenonian principle that size entails multiplicity (see below, p. 251). If we accept this principle, then (a—b) should persuade us that not merely the One, but nothing of any kind exists. Zeno would have required no other argument to refute both monism and pluralism, so Solmsen's claim that it is directed against the One would be considerably understated.

There is a qualification that must be added. Although it is clear that (a) must include at least the many, it is possible that the force of 'each of the many' was to limit the scope of the argument solely to the many, excluding the One. In this case, of course, Solmsen's interpretation would be contradicted, rather than supported, by the text. This interpretation was that of Simplicius; but, as Solmsen has stated, he was under the influence of the Platonic account.

On behalf of the Simplician interpretation of (a), though, one must note that the argument of (b) is hardly apt for a refutation of monism. For (b) shows that sizeless entities are nothing because they do not increase or decrease entities to which they are added or from which they are subtracted; but this argument could at best be applied to the One hypothetically, since the monist rules out, at least as a matter of fact, other entities to which the One could be added.

In short, these features of (a—b) make it certain that it was not directed, as Solmsen thinks, only against the One, and at least plausible that it was directed solely at the many, with the One excluded from its scope.

As to the second argument in Solmsen's interpretation, (c—d), the major fault with it is that it is invalid. By Solmsen's own account, (a—c) cannot establish (d); *a fortiori*, (c) alone cannot. Solmsen admits this, but sees in (c—d) certain "parallel illogicalities"[10] that make him think the argument would have satisfied Zeno. This gives Zeno too little credit; fairness to his historical stature and to the power evident in his other preserved arguments requires that we at least attempt to find a way to strengthen this argument.

Further, Solmsen argues that the combination in (a—d) of arguments against unity and against plurality is a good example of Zeno's strategy of *sunagein ta enantia*[11]. Yet Zeno's technique

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[10] Ibid., p. 136.
[11] Ibid., p. 137.

was not to refute two incompatible hypotheses, but, as Simplicius
shows (139. 5—9), as the arguments of the *Parmenides* make clear,
and as Solmsen himself acknowledges as the "obligatory" format
of his arguments[12], to deduce contradictory conclusions from one
hypothesis or attribute contradictory properties to one kind of
entity. On Solmsen's interpretation, only (c—d) does this, not
(a—b) or (a—d) as a whole.

For the above reasons, Solmsen's interpretation of the argument
must be rejected. Solmsen's critique of the orthodox interpretation,
on the other hand, has cast grave doubts on it. I suggest, therefore,
that we discard Fränkel's reconstruction by removing (a), and
attempt to find alternative premisses from which (d) can be deduced
in conjunction with (b) and (c).

II

There is material for the missing premisses in Simplicius. We
may take it as certain that Zeno did attempt to prove (d), given
the snippets of argument for it we find in Simplicius (139. 5—15,
141. 1—8), its conformity with Zeno's general format of argument,
and Simplicius's own testimony that (d) was one of the theses to
be established (139. 5—9). Obviously, Zeno did not attempt to
prove (d) because he thought it true, but because its proof would
provide a *reductio* of the pluralist position.

Three premisses provide the background for the argument:

1. All bodies are infinitely divisible.
2. Any divisible body is a plurality and not a unity.
3. Any true unity is indivisible. (138. 5—6, 139. 20—21).

From these we can infer that no unity is a body; and, hence, that
any materialist must be a pluralist, whereas any monist must be an
immaterialist[13]. Thus an argument that relied on the divisibility of
an entity could not, on Zeno's principles, affect the One[14].

The key to the argument for (d) occurs in a passage attributed by
Porphyry to Parmenides. Simplicius, who quotes Prophyry, thinks

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12 Ibid., p. 130.

13 Of course, an immaterialist need not be a monist, and a position such as Berkeley's
which postulates a plurality of immaterial substances, would be unaffected by
this argument. Such a position does not seem to have concerned Zeno, in any case;
clearly, it is pluralistic materialism that he is interested in refuting.

14 This is further evidence for excluding the One from the scope of (a); cf. above,
pp. 249 f.

the argument is Zeno's; and in this he has been followed by most scholars[15]. At least the argument is unlike anything extant from Parmenides' stylus, and it is in keeping with the nature of Zeno's known work. In the unlikely event that Parmenides *did* invent the argument, Zeno would certainly have been familiar with it and would have known how to adapt it to his own purposes.

Here is Porphyry's testimony, as quoted by Simplicius:

> Parmenides had another argument which was thought to prove by means of dichotomy that what is, is one only; and accordingly without parts and indivisible. For, he argues, if it were divisible, then suppose the process of dichotomy to have taken place: then either there will be left certain ultimate magnitudes, which are minima and indivisible, but infinite in number, and so the whole will be made up of minima but of an infinite number of them; or else it will vanish and be divided away into nothing, and so be made up of parts which are nothing. Both of which conclusions are absurd. It cannot therefore be divided, but remains one.
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> (139. 27—32; Lee, transl.).

I believe that (b) and (c) serve to elucidate the absurdity of both conclusions. The parts of the divided entity cannot be divided into nothing because (b) shows that parts of no size could never generate a whole which has size. The parts cannot be infinite in number and of any given size, however small, because (c) shows that the construction of an entity from such parts would produce an entity of infinite size.

The argument quoted by Porphyry is not an argument for (d), but for the indivisibility of the One. On order to adapt it to the proof of (d), we should have to reformulate it something like this:

A) Let there be a body, a part of the observable physical universe, of a given finite size. (Everyone who believed in a plurality of extended, corporeal things would have to maintain that such bodies exist). Call this body 'M'.

B) M is divisible into an infinite number of parts (from 1, above).

C) These parts are either of some definite, minimal size, or of no size[16].

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[15] Vlastos states in his notes that the language of part of the passage is un-Zenonian, so that only a portion of the text in question can be authentic. Yet it is certainly possible that Porphyry offered his readers a modern paraphrase of an authentic text from Zeno.

[16] This premise, as it stands, is false. Zeno omits a third possibility: that the parts may be infinite in number but ever-diminishing in size, so that there is no minimal size for them. Many have seen in this omission a fatal flaw in the argument; but I argue below (pp. 254f.) that Zeno might have found this possibility, had it occurred to him, irrelevant to his purposes.

D) In either case it will prove impossible to construct M from its parts (bacause of b and c).

E) But this makes it impossible that a body such as M exists; so we must abandon (A), and thus pluralism.

F) If pluralism is to be saved, we must assert that M is constructed of an infinite number of parts which are at the same time of some (minimal) size and without size.

G) (F) entails that M will be both infinitely large and no size at all (again because of b and c); in other words, (d)[17].

Of course, (d) and (D) are equally absurd; this is the dilemma of the pluralist. The argument is no mere sophism. It does not show that pluralism is incoherent; but it does show that infinite divisibility and the existence of indivisible elements of things are incompatible hypotheses. The argument has bite for the ordinary person, because he accepts the infinite divisibility of objects in space as a fact, not as a fiction of mathematics, and at the same time thinks that by physically dividing things he can eventually reach their ultimate components.

It is interesting to note that the three major responses to Zeno in classical times were the physical systems of Plato, Aristotle, and

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[17] Doubts about this argument as a reconstruction of Zeno may arise for several reasons, among them that the wording of (b) and (c) in the original version of the first argument against plurality seems to suggest that they were employed for some other purpose than proving my (D) and (G). It would have been neater, for my purposes, had (b) stated explicitly that "a whole of finite size cannot be generated from parts of no size", instead of its explicit conclusion that what has no magnitude is nothing. It would also have been neater if (c) had not left implicit the desired conclusion that "a whole made up of an infinite number of parts, each part having some definite size, is itself infinite in size".

In defense of the reconstructed argument, let me note three things. First, there is a strong parallel between the language of (b) and the statement in the Porphyry passage that, if the division of the entity had not left an infinite number of minima, "it will vanish and be divided away into *nothing*, and so be made up of parts which are *nothing*". In both cases the wording is stronger than the wording of (d), (D), and (G); but the stated conclusion would entail the weaker one.

Second, although (c) alone does not give us the explicit conclusion we desire, it gives us all the information that we need to draw that conclusion: that each of the infinite number of parts of the entity divided must have some definite size. We cannot expect from ancient philosophers the degree of explicitness and rigor we expect from ourselves, so it is not surprising to find some parts of the argument left only implicit. I have no doubt, for instance, that my paraphrase of the reconstructed argument on pp. 252f. is far fuller than the Zenonian original I hypothesize.

Finally, the problem of the wording of (b) and (c) would seem to affect equally *any* argument that concludes (d) from them, not just mine.

the Atomists. Plato came to a conclusion similar to Zeno's, that phenomenal objects "partake of opposites," and postulated the Forms as entities free from this condition[18]. Aristotle gave up the idea of an actual infinity of parts of objects and the entire program of constructing the universe from elementary particles (his 'elements' are only qualitatively simple). The Atomists gave up the mathematical claim that every extended entity is infinitely divisible by postulating atoms, and accepted the conclusion that an infinity of atoms produce an entity of infinite size. Each bowed in some way to Zeno's argument, though none accepted monism.

It is often said that the consequences of Zeno's argument can be avoided by appeal to the concept of an infinite series of ever smaller quantities, approaching the limit zero. Such an infinite series *would* constitute an entity of finite size. It is likely that the existence of such a series did not occur to Zeno (although its possibility is suggested by others of his arguments, notably the Achilles and the Dichotomy); but this is more than an unhappy historical accident.

Even if the existence of such a series had occurred to Zeno, he might justifiably have questioned the relevance of its existence to the dissolution of *his* paradox. For the pluralist, in the historical

[18] If, at any rate, we can take at face value Socrates' words at *Parmenides* 128E – 130A. Earlier, Socrates had stated that the point of Zeno's argument was that if there were many things, they must be both like and unlike, which is impossible; and Zeno had agreed with this statement (127E). Now, Socrates accepts Zeno's conclusion, as it applies to things of the physical world, and states that there is no impossibility in their participation in opposite properties; but he denies that the opposites themselves, that is the Forms, partake of each other. This view is similar to the view of the Forms expounded in the *Phaedo*.

As the opposite properties Socrates attributes to the things of this world are in general 'incomplete' or relative terms, both of which may be applied to an object if the *relatum* of each is different, there is no real incompatability of the kind found in Zeno's arguments. Yet Plato may well not have recognized, at the time of the *Phaedo* at least, that incomplete terms form a separate class from terms of the kind Zeno deals with. He may not have known, in other words, that he could accept participation in opposites without accepting Zeno's conclusion. At any rate, he makes no attempt to refute Zeno's arguments in the *Parmenides*, but tries only to limit their scope to entities of the physical world.

As the arguments of the *Parmenides* purport to show that even the Forms, to some extent, must also "partake of opposites", and as the *Sophist explicitly* adopts that position, the view that only phenomena participate in opposite properties cannot be Plato's ultimate view on the matter. Still, the testimony of the *Phaedo* and *Parmenides* indicates that a major motivation for Plato's postulation of the Forms was his desire to have entities that did not suffer from the "contradictory" nature of phenomena.

context of the argument, is committed to the view that division of entities yields elements from which the entities can be constructed; and the ever-diminishing quantities in the series that approaches zero offer nothing that could serve as such an element. So adoption of such a series as an answer to Zeno implies the abandonment of indivisible magnitudes and the constructivist program.

It has also been stated that the paradoxical result achieved by the argument is of only academic interest, because a division of entities into an infinite number of particles could not be accomplished in a finite time. Two remarks are in order here. First, the argument does not assert that such a dichotomy be a physical possibility, but only that this assumption that it is performed would entail certain absurd results. Second, one who maintains that it is impossible to complete an infinite series of actions in a finite time is then faced with the awkward consequence that Achilles can never overtake the tortoise (as G. E. L. Owen has noted, Zeno's arguments work in groups[19]).

It has often been noted that Zeno's arguments produced problems resembling those that arose in the early days of the calculus. The resemblance between the pluralist's elementary particles, which must be at once of some size and of no size, and the mathematician's infinitesimals, is clear from the following:

> The idea of the infinitesimal . . . was the idea of a fractional quantity infinitely close to zero, yet different from zero. It seemed to be needed in the study of rates, which was the business of the differential calculus . . .
>
> We are used to there being no end of smaller and smaller numbers, $1/8$ and $1/16$ and so on, nearer and nearer zero. But these are not infinitesimals. An infinitesimal is supposed to go into 1 not just sixteen times, or a thousand times, but infinitely many times.
>
> The idea was seen as absurd. 1 divided by infinity is simply 0 and not infinitesimally more[20].

Like the problems raised by Zeno, the problems of the calculus could not be solved by pointing out a fallacy in the argument; they had to be solved by a fundamental rethinking of the notion of infinite divisibility.

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[19] G. E. L. Owen, "Zeno and the Mathematicians", *Proceedings of the Aristotelian Society*, N. S. 58 (1957—1958), pp. 199—222.

[20] W. V. O. Quine, "Foundations of Mathematics", in *The Ways of Paradox* (New York, 1966), pp. 24—25.

III

It should be clear that the argument outlined above attacks the pluralist hypothesis in the service of monism. If this argument, or something like it, can be attributed to Zeno, then the traditional account of his views is correct. Though the argument is drawn from material attributable to Zeno, and material compatible with what we know of his interests and method, and though the argument enables us to see why the philosophers who succeeded him responded to Zeno as they did, still it would be speculative to claim that the argument outlined above reconstructs with certainty Zeno's actual argument against plurality[21]. At the very least, however, it represents what we should *want* Zeno to have said, given his temporal position in the history of philosophy.

A final note: Solmsen argues that Zeno was a dialectician, not an Eleatic. I have argued that Zeno's adherence to the philosophy of Parmenides is much more likely than his independence from it. Yet it cannot be denied that Zeno's philosophical importance is due not to his Eleaticism, but to his dialectical skill. He was apparently quite unsuccessful in winning converts to monism, which did not require philosophical refutation to seem unacceptable. He was, on the other hand, enormously successful in shaking the conceptual foundations of our ordinary view of the world. The tremors are still being felt today, as the voluminous literature on the paradoxes attests. Perhaps this is all Solmsen needs to bring to our attention; the purposes of the historical Zeno may well be irrelevant for an evaluation of his place in the history of philosophy.

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[21] The possibilities remain that Zeno thought (b—d) adequate as it stood, in spite of the incompleteness obvious to us; or that he augmented it with premisses lost to us. Due to the fragmentary nature of the material with which we have to deal, certainty on this point is impossible.