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Rigid pricing policies and profit maximization

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RIGID PRICING POLICIES AND PROFIT MAXIMIZATION

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In this paper we present a model of a profit maximizing firm in which one price is set for the entire multiperiod planning horizon. One of the consequences of such a pricing policy, if one employs widely used assumptions about demand and cost functions, is a decision rule for choosing the optimal price which may be interpreted as the "full-cost" pricing equation of much recent controversy. The significance of this result lies in the fact that full-cost pricing and profit maximization have often been held to be inconsistent.[1] In addition, this model integrates the pricing decision with the other decisions taken by the firm.

Introductory Remarks

Our model takes the firm to be confronting a T period planning horizon and acting to maximize the discounted stream of profits over the horizon. The rationale for a rigid pricing policy is of course, the existence of costs associated with changing price. These costs may be charges affiliated with publishing new price listings, brochures and cataloges, as well as the additional sales effort needed to "explain" the price changes to established buyers. In oligopoly they may take the form of an added dimension of uncertainty which arises when price is changed. Empirical investigations regarding the pricing policies of apparently effectively colluding oligopolists have frequently indicated that product price changes in response to short-run fluctuations in demand and cost are avoided, with price remaining constant often for extended periods of time.[2] A number of reasons are presented for rigid pricing policies in such industries, with major emphasis placed on uncertainty regarding reactions of rivals to price changes. If a price increase by a single oligopolist is not followed, the loss in market share and goodwill can indeed be significant. Further, while a competitive firm cannot influence present or future demand, the oligopolist has more discretion and is often depicted as viewing price in a longer term perspective.

Variants of the rigid pricing policy have been employed by a number of authors in an attempt to "explain" the pricing policies of imperfectly competitive firms.[3] An often used assumption is that firms price by applying a standard markup to material and labor costs. With this "full cost" pricing scheme, price will respond to changes in the cost of

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[1] For a good review of the debate see, R. B. Heflebower, "Full Costs, Cost Changes, and Prices," in *Business Concentration and Price Policy*, a report of the National Bureau of Economic Research, Princeton University Press, 1955, p. 361–96.

[2] A classic study is that of R. L. Hall and C. J. Hitch, "Price Theory and Business Behavior," *Oxford Economic Papers*, No. 2, 1939. Also see A. D. N. Kaplan, J. B. Dirlan and R. F. Lanzilotti, *Pricing in Big Business – A Case Approach*, Brookings Institution, 1958; and R. F. Lanzilotti, 'Pricing Objectives of Large Companies," *American Economic Review*, December 1958.

[3] A limited selection of sources include: *Hearings, Before the Subcommittee on Antitrust and Monopoly of the Judiciary, U.S. Senate, 85th Congress, 1st Session, Part 1: Opening Phase – Economists' Views*, Government Printing Office, 1957; Gardner Ackely, "A Third Approach to the Analysis and Control of Inflation," in *The Relationship of Prices to Economic Stability and Growth; Compendium of Papers Submitted by Panelists*, pp. 619–36. C. L. Schultze, *Prices, Costs and Output in the Postwar Decade*, Committee for Economic Development, 1960; C. L. Schultze, "Uses of Capacity Measures for Short Run Economic Analysis," *American Economic Review*, Papers and Proceedings, May 1963, pp. 295–96; Otto Eckstein, "A Theory of Wage-Price Process in Modern Industry," *Review of Economic Studies*, October 1964, pp. 267–86; and Otto Eckstein and Gary Fromm, "The Price Equation," American Economic Review, December 1968, pp. 1164–66.

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producing "standard" or "normal" output, either because of changes in input prices or technology, but will not be adjusted for short term fluctuations in demand or costs. The markup is usually considered dependent in some unspecified fashion on industry structural factors such as barriers to entry, concentration, product differentiation, etc.

There are a number of apparent deficiencies in pricing rules which derive from a "full cost" formulation or its variant, the target rate of return on investment criterion. First, such pricing techniques fail to take explicit account of demand elasticities or the size of marginal, rather than average, costs. Second, the "full cost" criteria are often revealed as *ad hoc* relations derived from questionnaires posed to businessmen familiar with pricing procedures – a method which has been repeatedly challenged.[4] Third, many investigations of oligopolistic pricing fail to integrate the price decision with related decisions involving inventory policy and backlogging of orders, even though rigid prices over time imply a transference of at least part of intertemporal rationing to nonprice variables.[5] For example, in the literature of management science and operations research, price is frequently taken as exogenously determined with the rate of inventory accumulation (production) selected on the basis of a given time path of expected sales.[6] This approach, while suitable for particular decision problems, is unacceptable if price and inventory are viewed as interdependent decisions. Granted that many econometric studies of oligopolistic pricing add inventory and backlogged order variables to a specific price equation, little work has been done towards specification and estimation of a model in which price, inventory and backlogged orders are simultaneously determined decision variables of an imperfectly competitive firm.

The model presented in this paper eliminates each of these deficiencies. The decision rules of the model instruct the entrepreneur to set one price for the entire horizon, that price being equal to the marginal cost of "standard" or "normal" output, plus a markup which varies inversely with the elasticity of demand. In addition, the decisions of the firm are completely integrated as price, inventory holdings and order backloggs are simultaneously determined from the derived decision rules.

The Model

The following definitions will be used:

S_t = sales in period t, $0 < S_t$

c_t = production in period t, $0 < q_t$

x_t = net inventory holdings in period t, $-\infty < x_t < \infty$

P = the price chosen for the T period horizon, $0 < P$

δ = the discount rate, $0 \leq \delta \leq 1$

π = the discounted stream of profits

$C(q_t)$ = the cost associated with q_t units of production

$g(q_t - q_{t-1})$ = the cost associated with changing the production rate from q_{t-1} to q_t.

$\phi(x_t)$ = the cost associated with holding x_t units of inventory

$$= \begin{cases} I(x_t)\,, & x_t > 0 \\ \psi(x_t)\,, & x_t < 0 \end{cases}$$

[4]In short, the opinion of the businessman is not considered to carry much weight in the evaluation of objective economic functions, since businessmen are unlikely to think in the language of an economist. Often evidence is summarized from accounting data having little to do with economic concepts. Friedman, e.g., argues that "... the answers by businessmen to questions about the factors affecting their decisions ... is about on a par with testing theories of longevity by asking octogenarians how they account for their long life. ..," *Essays in Positive Economics,* University of Chicago Press, 1966, p. 31. For a review of the criticisms relating to the questionnaire approach see, Fritz Machlup, 'Marginal Analysis and Empirical Research," *American Economic Review,* Sept. 1966, pp. 135–48, esp. Sec. 2.

[5]In a somewhat different context, the work of Edwin Mills attempts such an integration. See, *Price, Output and Inventory Policy,* John Wiley & Sons, 1962. Also, A. J. Nevins, "Some Effects of Uncertainty: Simulation of a Model of Price," *Quarterly Journal of Economics,* Feb. 1966, pp. 73–87.

[6]See, K. Arrow, S. Karlin, and H. Scarf, *Studies in the Mathematical Theory of Inventory and Production,* Stanford University Press, 1958, or C. Holt, F. Modigliani, J. Muth and H. Simon, *Planning, Production, Inventory and Work Force,* Prentice-Hall, 1960.

$\phi(x_t)$ is then the function which assigns the cost to intertemporal transportation of production. $I(x_t)$ is inventory storage cost; $\psi(x_t)$ is the cost of backlogging orders.

As noted above, the model to be presented presupposes the existence of costs which make it desirable to set one price for the entire planning horizon. In such a case,

$$(1) \qquad \pi = \sum_1^T \delta^{t-1}[P\cdot S_t - C(q_t) - \phi(x_t) - g(q_t - q_{t-1}) + \lambda_t(q_t - S_t + x_{t-1} - x_t) + \mu_t(P - f_t(S_t))]$$

where $f_t(S_t)$ denotes the demand function in period t and μ_t and λ_t are undetermined Lagrangean multipliers in period t. Necessary conditions for a maximum are:[7]

$$(2) \qquad P - \lambda_t - \mu_t f_t'(S_t) = 0, \qquad t=1,2,\ldots,T,$$

$$(3) \qquad -C'(q_t) - g'(q_t - q_{t-1}) + \delta g'(q_{t+1} - q_t) + \lambda_t = 0, \qquad t=1,2,\ldots,T,$$

$$(4) \qquad -\phi'(x_t) - \lambda_t + \delta\lambda_{t+1} = 0 \qquad t=1,2,\ldots,T-1$$

$$(5) \qquad \sum_1^T \delta^{t-1}(S_t + \mu_t) = 0,$$

$$(6) \qquad q_t - S_t + x_{t-1} - x_t = 0, \qquad t=1,2,\ldots,T, \text{ and}$$

$$(7) \qquad P - f_t(S_t) = 0, \qquad t=1,2,\ldots,T.$$

Substitution of (3) into (2) and (4), and then (2) into (5) yields

$$(8) \qquad \sum_1^T \delta^{t-1}\ [S_t \cdot f_t'(S_t) + P - C'(q_t) - g'(q_t - q_{t-1}) + \delta g'(q_{t+1} - q_t)]/f_t'(S_t) = 0, \text{ and}$$

$$(9) \qquad -\phi'(x_t) - C'(q_t) + \delta C'(q_{t+1}) - g'(q_t - q_{t-1}) + 2\delta g'(q_{t+1} - q_t) - \delta^2 g'(q_{t+2} - q_{t+1}) = 0,$$
$$t=1,2,\ldots,T-1.$$

Equations (6), (7), (8) and (9) are 3T equations in 3T+1 variables.[8] Given "well behaved" functions, specification of x_T allows solution. According to equation (8) the price selected for the entire T periods is chosen such that the discounted sum of revenue changes, due to a price change, is equal to the discounted sum of changes in costs, due to a price change, with the summation taken over the entire T period planning horizon. Equations (9) indicate the optimal amount of inventory (backlogged orders) in period t is that quantity such that marginal inventory cost equals the discounted change in marginal production costs between periods t and t+1 plus the discounted changed in marginal costs associated with production rate changes between periods t and t+1.

As we saw in the preceeding paragraph, equations (8) and (9) have straightforward interpretations as they stand. Nevertheless, it is of some interest to assume the functions in these equations may be adequately approximated with quadratic functions and then take another look at the decision rules. We choose to approximate these functions because in practice they *are* approximated – with quadratics being overwhelmingly the most popular approximating function[9] – and it may well be the case that some of the alleged discrepancies between observed pricing policies and the

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[7]We assume π is convex.

[8]Here x_o and q_o are data. The value of production in the first period of the next horizon, q_{T+1}, must be assigned.

[9]For example see, C. Holt et. al., op. cit., especially chapters 3–6. This book is directed to deriving operational optimal decision rules for the firm and quadratic approximations are used throughout.

pricing policies predicted by economic models stem from the fact that in practice the class of convex functions used by economists is much to broad to allow estimation, and that managers approximate these functions with families of curves dependent upon only a small number of parameters.[10] The consequence of such action would be the specialization of the firm's decision rules into a form which may, at first glance, seem inconsistent with the more general rules derived by economists, but which in fact are not. We are not suggesting that all, or even most, managers *explicitly* make quadratic approximations, but rather that many "rule of thumb" decision rules may reflect implicit assumptions about the form of the relevant functions. In what follows, we will focus our attention on equation (8), due to the long standing controversy over the pricing equation.

We now assume revenue and cost functions are quadratic. Specifically:

$$(10) \quad C(q_t) = boq_t + b_1 q_t{}^2 \ , \ b_o > 0$$

$$(11) \quad g(q_t - q_{t-1}) = g \cdot (q_t - q_{t-1})^2 \ , \ g > 0 \ .$$

$$(12) \quad \phi(x_t) = cx_t{}^2 \ , \ c > 0 \ \text{and}$$

$$(13) \quad f_t(S_t) = a_o + a_1 S_t + \alpha_t \ , \ a_o > 0, \ a_1 < 0$$

where a_t is a shift parameter in the two dimensional demand function in period t. Hence $f_t(S_t)$ instead of $f(S_t)$. We also assume $\delta = 1$. Since this model is concerned with the short term decisions of the firm, such an assumption seems relatively innocuous.

Equation (8) is now

$$(8') \quad P + a_1 \bar{S} = b_o + 2b_1 \bar{q} + (2g/T) \left[(q_1 - q_o) - (q_{T+1} - q_T) \right]$$

where $\bar{S}$ and $\bar{q}$ are the average production and sales rates over the horizon. Since $a_1 = f_t'(S_t)$, the left hand side of (8) is the marginal revenue associated with the average sales rate; $b_o + 2b_1\bar{q}$ is the marginal cost of the average production rate.

Letting $\eta = -(d\bar{S}/dP)\,(P/\bar{S})$ equation (8′) becomes

$$(8'') \quad P = b_o + 2b_1 \bar{q} + (b_o + 2b_1 \bar{q})\ \eta\text{-}1 + (2g/T) \left[(q_1 - q_o) - (q_{T+1} - q_T) \right] \eta/\eta\text{-}1$$

The last term of (8″) becomes insignificant for large values of T and may be ignored, Therefore, if a rigid pricing policy is pursued and if managers approximate cost and revenue functions with quadratics, price is set equal to the marginal cost of the average production rate plus a "markup" which varies inversely with the elasticity of demand at the average sales rate. One could easily interpret $\bar{q}$ as the "standard output" rate in which case price is fixed to equal the marginal cost associated with the standard output plus a markup.

In summary, not only is the "full costing" equation a natural consequence of profit maximization in this model, but also since the values of all decision variables are determined simultaneously by equations (6) – (9) the pricing decision is integrated with the other decisions of the firm.

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[10] A small selection of articles addressing this alleged discrepancy follows: R. Hall and C. Hitch, op. cit., R. A. Lester, "Shortcomings of Marginal Analysis for Wage-Employment Problems," *American Economic Review,* March, 1946, p. 63, F. Machlup, op. cit., R. A. Gordon, "Short-Period Price Determination in Theory and Practice," *American Economic Review,* June, 1948, p. 265, and J. S. Early, "Marginal Policies of Excellently Managed Companies," *American Economic Review,* March, 1956, p. 44.