

1967

# Stochastic Reserve Losses

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## Recommended Citation

Birch, Eleanor M., and John Heineke. "Stochastic Reserve Losses." *The American Economist* 11.1 (1967): 20-28.

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## STOCHASTIC RESERVE LOSSES\*

By Eleanor M. Birch and John M. Heineke\*\*

### *Introduction*

In an article in the September, 1961, issue of the *American Economic Review*, Daniel Orr and W. G. Mellon introduced the notion of uncertainty into the well-known comparative static analysis of bank credit expansion.<sup>(1)</sup> This paper discusses their findings, the nature of their assumptions, and some possible extensions of their results.

### *Orr's and Mellon's Stochastic Reserve Losses*

Orr and Mellon began their analysis with the individual bank. They wanted to concentrate on credit expansion rather than portfolio selection with its problems of deciding upon different securities at varying interest rates. Therefore, they made the simplifying assumption that the individual bank's assets are held either as reserves which earn no return or as loans which earn the going interest rate,  $i$ , and that some expense is incurred when the latter are converted to the former. The bank is subject to random changes in its reserves, both positive and negative. Its decision problem is how far to extend credit, given the random nature of its reserve changes and the legal reserve it must hold.

The variables of their model are:

R: The volume of excess reserves at the beginning of the evaluation period.

D: The volume of new deposit liabilities created during the period.

L: The loss of reserves during the period.

$\rho$ : The legal reserve ratio ( $0 < \rho < 1$ ).

Reserves are legally sufficient if  $R - L \geq \rho(D - L)$  at the end of the evaluation period. Clearly, then, the largest reserve loss that can be tolerated without violating the legal reserve requirement is:

$$v = \frac{(R - \rho D)}{(1 - \rho)}$$

The bank's gross revenue is  $iD$  but this may be reduced if its reserves fall below the legal minimum. When that happens, the bank must pay a penalty consisting of a lump sum,  $M$ , and a variable cost,  $r$ , per dollar of reserve deficit. But this event is uncertain; its expected value is determined by the probability distribution of  $L$ . Thus the bank's profit function, which it seeks to maximize is:

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\*Paper delivered at the Graduate Student Session of the American Economic Association Meetings in San Francisco, December 27, 1966.

\*\*Mrs. Birch is a National Science Foundation Graduate Fellow and Instructor; Mr. Heineke is an Instructor. Both are at the University of Iowa.

$$P = iD - M \int_0^{\infty} \phi(l) dl - r \int_0^{\infty} l \phi(l) dl. ^2$$

The authors made three assumptions about the distribution of L:

1.  $\Phi(l)$  is normal
2.  $E(L)$  is linearly dependent on  $D$ .
3.  $\text{Var}(L)$  is independent of  $D$ ; it is a given constant.

These can be summed up by saying that  $L$  is  $N: (kD, \sigma L)$ . The assumptions imply that cash flows (or reserve losses) in different periods are random and independent, and their variation is not affected by changes in deposit liabilities.

The authors then differentiated the profit function with respect to  $D$ , set it equal to zero, and solved for the optimal value of  $D$ , given various values of the parameters,  $M$ ,  $r$ ,  $i$ ,  $\rho$ , and  $k$ , and the independent variable,  $R$ . (These results are reproduced in Table 1 below for reference.) They found that, under conditions of uncertainty, the ratio of marginal credit expansion to excess reserves is less than that of total credit expansion to total reserves. They also showed that, given a constant variability, the presence of uncertainty leads a decentralized banking system to expand credit less than a centralized one.

### Critique

In the December, 1962, issue of the *American Economic Review*, H. Laurence Miller, Jr., took issue with the nature of the profit function used by Orr and Mellon.<sup>(3)</sup> He pointed out that whenever  $i > r$ , it would pay the bank to expand loans indefinitely. He suggested that this condition is often met in the real world (since the Federal Reserve does not customarily set its discount rate at a penalty level), yet the predicted behavior seems not to be empirically verified. Orr and Mellon in their reply took Professor Miller to task for attempting "to substitute a simple out-of-pocket cost for the penalty in our profit function."<sup>(4)</sup> In their defense, they pointed to a statement in their original article where they said a bank that would set  $r < i$  "takes into account only the money gained, and ignores such non-quantified factors as reputation for soundness."<sup>(5)</sup> This defense, while valid, becomes superfluous once the behavior premises of their model are fully explicated. The Orr-Mellon model specifically assumed that a bank holds its assets in only two forms, reserves or loans. When a bank finds itself in a reserve-deficit position, what can it do? There is some confusion on this point. Let us suppose that once a loan is made, a bank can do nothing about it. If it gets into reserve difficulties, it has only one escape: it can borrow from the Federal Reserve. Now if the problem were presented in this form, and if we continued to assume the banker is most reluctant to borrow from the Fed., then the problem would be one of avoiding ruin (at least figuratively), and might be approached via ruin theory.

But, alternatively, let us suppose that borrowing from the Fed. is not the only way out; assume there is some way to cancel loans. There is a lump-sum cost associated with any such transaction and a variable cost, e.g., like a brokerage cost, associated with each dollar the bank is short. So far, this sounds just like the Orr-Mellon parameters,  $M$  and  $r$ . But note that the  $r$  here *must* exceed  $i$ , because in addition to what we have called the brokerage cost, there is the gain foregone when the loans are cancelled. Since this opportunity cost always equals  $i$ , the brokerage cost plus the opportunity cost clearly must be greater than  $i$ . So this formulation of the problem rescues Orr and Mellon from Miller's criticism.

There were some other difficulties in the article connected with their assumptions about  $L$ . They assumed that  $L$  is normally distributed.<sup>(6)</sup> They made no justification of this, unless the flat statement that "the normal distribution has a great deal of a priori appeal" can be construed as such. As we shall see later, a case might have been made here for the normal distribution but the authors did not make it. They also assumed that the expected value of  $L$  was linearly dependent on  $D$ . This leads the reader to think that  $E(L)$  is of the form  $a+kD$ , but Orr and Mellon later revealed that  $E(L) = kD$ . Thus, their assumption was stronger than mere linearity; they actually assumed strict proportionality. The variance of  $L$  was assumed to be a given constant, independent of  $D$ . To simplify their computations, the authors assumed this constant to be unity, which had some interesting effects on their results, as we shall see later.

The profit function, reproduced above, also seems inappropriate since the penalty  $r$  is paid on every dollar of reserve loss,  $L$ , even though it should be charged only to those losses in excess of the critical value,  $v$ . Thus, the final term of their profit function,

$$-r \int_v^{\infty} l \phi(l) dl \text{ should read } -r \int_v^{\infty} (l-v) \phi(l) dl.$$

In the process of taking the derivative of their own profit function, Orr and Mellon also made some errors, as Professor Tsiang later pointed out.<sup>(7)</sup> Two  $k$ 's were omitted from the final equation (the one on which Orr and Mellon based the results shown in Table 1). On procedural rather than substantive grounds, we might note, too, that another term was omitted from their derivative, viz.,  $-r \int_v^{\infty} \phi(l) \frac{\partial l}{\partial D} dl$ .<sup>8</sup> Apparently, Orr and Mellon assumed that

$\partial l / \partial D = 0$  which makes the whole term equal to zero. This seems like a justifiable assumption, but it should have been identified as such rather than left for the reader to deduce. Thus, the derivative of the profit equation should be:

$$\frac{\partial P}{\partial D} = i + \phi(v)M\left(\frac{dv}{dD} - k\right) + \left(\frac{dv}{dD} - k\right)r[1 - \Phi(v)].$$

An interesting question is why these errors did not affect their calculations by any notable amount (Table 2). The answer is that their assumption of a unit variance for  $L$  made their table highly insensitive to such changes as well as to changes in the parameters. For example, on page 618 of their article, they mentioned that credit expansion under uncertainty is sensitive to  $r$ , among other things. Yet, in their table, a 50-fold change in  $r$ , from  $r=.01$  (Case 1) to  $r=.5$  (Case 5), *ceteris paribus*, changes credit expansion not a whit. Similarly, when the correct profit equation is used, and the derivative properly obtained, the results (shown in Table 2) do not differ much from those shown in Table 1.

The assumption that  $E(L)=kD$  also leads to some interesting questions. First, we may ask: How does it happen that  $E(L)=kD$ ? There are two alternative interpretations. First, if the bank were assumed to lose reserves to the full amount of  $D$  to other banks, then  $k$  would be the proportion lost in the "one period." Thus, there would be some remaining loss,  $(1-k)D$ , in future periods. But if this were the case, in any single period  $E(L)$  would not be  $kD$  but rather  $a+kD$  where  $a$  is the amount of reserve loss associated with previously issued loans.

According to the second interpretation, if all deposit liabilities were assumed to clear during the "one period", then  $k$  would be the proportion of reserves lost to other banks in the system. In a footnote, Orr and Mellon make it clear that this latter explanation is their interpretation of  $k$ . But if this is so, then there cannot be any deposits outstanding at the beginning of the period, because all the deposit liabilities created in earlier periods have presumably cleared during their own "periods." Yet Orr and Mellon stated that, under uncertainty, the ratio of marginal credit expansion to excess reserves is less than that

of total credit expansion to total reserves. Nowhere in their model, however, is there any specific consideration of total credit expansion or total reserves. Their analysis is solely in terms of  $D$  which, at first sight, seems to be the marginal credit expansion. But their assumption that  $E(L) = kD$ , as we saw above, rules out any initial outstanding deposit liabilities. Thus, initial credit expansion must be zero. So it becomes doubtful whether  $D$  should be interpreted as marginal or total credit expansion.<sup>(9)</sup>

It is clear though that Orr and Mellon considered  $D$  to be the marginal credit expansion. If we follow their lead and assume some arbitrary initial level of deposit liabilities, it follows that, under uncertainty, the ratio of marginal credit expansion to excess reserves is less than that of total credit expansion to total reserves, as they stated. What they did not state is that this is also generally true under certainty, except in the case of a monopoly bank (Case 7 of Table 1), where  $\rho D^* = R$ .<sup>(10)</sup> As long as  $\rho D^* < R$ , which always holds unless  $k=0$ , then the ratio of the marginals is lower than that of the totals. What Orr and Mellon probably intended to suggest is that the gap between the two ratios is larger under uncertainty; this follows from their result that  $D$  is generally less than  $D^*$  (except for their Case 6, admittedly an atypical case).

### *Choice of Distribution*

Let us examine more closely the nature of the events that give rise to the random variable, net reserve losses,  $L$ . Changes in reserves arise chiefly through check clearings. If a check drawn on Bank A is deposited in another account in Bank A, bank A's reserves and deposits do not change, so  $L=0$ . If a check drawn on Bank B is deposited in an account in Bank A, bank A's reserves and deposit liabilities both increase, so  $L < 0$ . If a check drawn on bank A is deposited in an account in bank B, bank A's reserves and deposit liabilities both decrease, so  $L > 0$ . The basic event that gives rise to our random variable,  $L$ , is a check's clearing. This can be viewed as an event occurring at random along the time continuum, i.e., as a Poisson process. The Poisson distribution is quite convenient to work with because this time continuum can be subdivided or sliced into segments small enough to obtain a good fit for the experimental data. Therefore, if we assume that these check clearings are independent of each other, then the number of checks cleared (i.e., the number of reserve losses, or, more generally, reserve changes) during a time period,  $t$ , has a Poisson distribution. The time period, as indicated above, can be adjusted to make the parameter,  $\lambda t$ , equal to  $t$ ;  $t$  is then the expected number of checks to be cleared (or expected number of reserve changes) in a period of length  $t$ .

But this still doesn't give us the random variable we want. Each check that clears carries along with it, in a kind of piggy-back fashion, the random variable we're interested in, viz., the face amount, which corresponds exactly to a reserve change. Let  $Z$  be the face value of a check; then  $F(z)$ , the distribution function of  $Z$ , gives the probability that, if a reserve change occurs, it will be less than or equal to  $z$ , i.e.,  $\Pr(Z \leq z \mid \text{a reserve change occurs})$ . Let  $F^n(x)$  be the distribution function of  $X$ , the total reserve change (which, if the

bank is small, we may assume will be losses), where  $X = \sum_{i=1}^n z_i$ . Let  $X(t)$

be the total reserve change during a period of length  $t$  and  $G(x, t)$  be its distribution function. The function  $G(x, t)$  is equal to the product of: 1.  $F^n(x)$ , given that the bank experiences  $n$  reserve changes; and 2. the probability that  $n$  changes are experienced, summed over all  $n$ . Since the number of reserve changes is a Poisson variable, we have:

$$G(x, t) = \sum_{n=0}^{\infty} \frac{e^{-t} t^n}{n!} \cdot F^n(x)$$

where  $G(x, t)$  is the distribution function of total reserve changes.

Let  $q_1$  denote the expected value of  $Z$  and  $q_2$  the expected value of  $Z^2$ ; then it follows that the mean and variance of  $X(t)$  are  $q_1t$  and  $q_2t$ , respectively. In the problem at hand, it is very important to be able to compute  $G(x,t)$  for large values of  $X$ , i.e., for values of the loss function above the Orr-Mellon critical value,  $v$ . The function  $F(z)$  can be observed and the moments,  $q_1$  and  $q_2$ , estimated. An approximate solution can be obtained by standardizing  $X(t)$  and appealing to the Central Limit Theorem. But for large values of  $X$ , and these are precisely the ones of interest to us, the normal approximation can be rather poor. The Esscher approximation has been developed to deal with this problem.

Note that the appeal to the Central Limit Theorem above and the properties of asymptotic normality are justified from the nature of the basic event and are not the same as the Orr-Mellon assumption of a normally distributed loss function. No assumption concerning the distribution function,  $F(z)$ , was made in the exposition above.

The danger of blithely using the normal approximation has been pointed out in risk theory. Collective risk theory was first developed by European, especially Scandinavian, actuaries and statisticians. Cramér has dealt with this as part of the modern theory of stochastic processes.<sup>(11)</sup> In a recent article on reinsurance, P. M. Kahn discussed a problem in calculating stop-loss premiums similar to our problem here:

... Mr. Feay used the normal distribution. In our discussion of his paper we furnished a quotation from Ammeter<sup>(2)</sup> and some examples to show that this approximation is not always satisfactory. We wish here to provide an alternative method for approximating  $F(x,t)$  and for calculating stop-loss premiums and to compare his methods with those produced by collective risk theory, particularly by Esscher's method. In point of fact, a most important stimulus to the development of the distribution branch of collective risk theory was dissatisfaction with the use of the normal distribution as an approximation to  $F(x,t)$ .<sup>(12)</sup>

Mr. Kahn gives numerical examples of reinsurance premiums calculated under the normal approximation and, alternatively, under the Esscher approximation. In all instances, the former method produces a smaller required premium than the latter. Applying similar techniques to the Orr-Mellon problem would confirm their statement that their assumptions tended to understate the uncertainty in the system. The use of the compound Poisson distribution and the Esscher approximation would tend to give a more realistic weight to uncertainty.

But if the above formulation is correct, it does not necessarily follow that the uncertainty in the system has, in fact, been understated by Orr and Mellon. Everything depends on how their estimate of  $oL$  is calculated. They did not consider this question since they assumed the variance of  $L$  is a given constant. This procedure is acceptable for pedagogical purposes. But in the real world, variances do not drop like manna from heaven; they must be estimated. Now there is nothing in the Orr-Mellon paper on this, but the unwary reader would probably think that all one had to do was observe the reserve losses over a number of periods and then estimate  $oL$  from those data. But if reserve losses are the outcome of a compound Poisson process as postulated above, then this estimate of  $oL$  may be quite misleading, as is well-known from the modern theory of stochastic processes:

Let  $N$  = the number of checks cleared in a period

$Z_i$  = the face amount of the  $i^{\text{th}}$  check

$$X = \sum_{i=1}^n z_i \quad z_i = \text{the total face amount of all } N \text{ checks} = \text{total reserve change in a period.}$$

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$\sigma$  = lower case sigma.

It is known that:

$$E(X) = E(N) \cdot E(Z)$$

$$\text{Var}(X) = E(N) \cdot \text{Var}(Z) + \text{Var}(N) \cdot E(Z)^2^{(13)}$$

Now consider a simple numerical example. Assume the following data are available for 6 periods:

Period	N	Z	X
1	4	7 1 3 9	20
2	2	13 7	20
3	1	20	20
4	3	3 10 7	20
5	2	12 8	20
6	5	2 1 6 4 7	20
Totals	17	120	120
Means	2.833	7.059	20
Variances	2.169	23.93	0

$$\begin{aligned} \text{Var}(X) &= E(N) \text{Var}(Z) + \text{Var}(N) \cdot E(Z)^2 \\ &= (2.833)(23.93) + (2.169)(49.829) \\ &= 67.79 + 108.08 \\ &= 175.87 \end{aligned}$$

In this case, if the observer were to examine X, i.e., total reserve losses, alone, he would estimate the variance of X as 0. We would be back in the deterministic framework again where we need not guard against variation in reserve losses. But if we followed this lead, our bank would soon be in dire straits, since the true variance of X, based on N and Z, is estimated at 175.87.

The opposite result can also occur, as we see in the following example:

Period	N	Z	X
1	2	6 4	10
2	1	1	1
3	2	3 7	10
4	1	1	1
5	2	5 5	10
6	1	1	1
Totals	9	33	33
Means	1.5	3.67	5.5
Variances	0.3	5.25	24.3

$$\begin{aligned} \text{Var}(X) &= E(N) \cdot \text{Var}(Z) + \text{Var}(N) \cdot E(Z)^2 \\ &= (1.5)(5.25) + (0.3)(13.44) \\ &= 7.875 + 4.032 \\ &= 11.91 \end{aligned}$$

Here the direct method of estimating  $\sigma L^2$  would yield 24.3 while the estimate based on the information available on N and Z would yield 11.91. If one calculated  $\sigma L^2$  by the direct method and used it in a loss function of the Orr-Mellon type, then the results could no longer be said to understate the uncertainty in the system. The use of the proper estimate of  $\sigma L^2$  would lower the uncertainty still more.

Other modifications to the Orr-Mellon approach could be made. For example, a highly cautious banker might not want to hazard a guess about the nature of the distribution function of his losses. In this event, he could use the Tchebycheff inequality to get an upper bound for the uncertainty in the process. This would tend to restrict his credit expansion to a much greater degree than the Orr-Mellon results indicate. Such a cautious approach might satisfy Professor Miller, whose criticism suggests that banks normally follow restrictive credit policies. Most modifications of this type would have the effect Orr and Mellon predicted, viz., that of increasing uncertainty in the system. But the exposition above on the proper calculation of  $\sigma L$  is more significant because the numerical examples presented there show that the change could go in either direction.

### Conclusion

The Orr-Mellon paper was a valuable contribution in that it showed us how to look at an old question in a new way. Its weaknesses do not diminish its pedagogical value. They merely reflect the inadequacy of a static model for describing behavior that is essentially dynamic. Our introduction of the compound Poisson process is a very small step toward such a dynamic model where time,  $t$ , enters in an important way.

### APPENDIX (1=1)

$$P = iD - M \int_{-\infty}^{\infty} \phi(1) d1 - r \int_{-\infty}^{\infty} (1-v) \phi(1) d1$$

$$\frac{\partial P}{\partial D} = i - M[-\phi(v) \frac{\partial v}{\partial D} + \int_{-\infty}^{\infty} \frac{\partial \phi(1)}{\partial D} d1] - r[\int_{-\infty}^{\infty} (1 \frac{\partial \phi(1)}{\partial D} + \phi(1) \frac{\partial 1}{\partial D} - v \frac{\partial \phi(1)}{\partial D} - \phi(1) \frac{\partial v}{\partial D}) d1]$$

$$\text{where } \phi(1) = \frac{1}{\sqrt{2\pi}} \exp [-(1-kD)^2/2]$$

$$\text{and } \frac{\partial \phi(1)}{\partial D} = \phi(1)[k(1-kD)]$$

$$0 = i + M\phi(v) \frac{\partial v}{\partial D} + Mk[\phi(1) \int_{-\infty}^{\infty} 1] + rk[1\phi(1) \int_{-\infty}^{\infty} 1 - \int_{-\infty}^{\infty} \phi(1) d1] - vkr[\phi(1) \int_{-\infty}^{\infty} 1] + r \frac{\partial v}{\partial D} \int_{-\infty}^{\infty} \phi(1) d1$$

$$= i + M\phi(v) \frac{\partial v}{\partial D} - Mk\phi(v) - rkv\phi(v) - rk[1-\phi(v)] + vkr\phi(v) + r \frac{\partial v}{\partial D} [1-\phi(v)]$$

$$i + M\phi(v) [\frac{\partial v}{\partial D} - k] + [1-\phi(v)][\frac{\partial v}{\partial D} - k]r$$

$$\text{Assuming } \phi(1) \frac{\partial 1}{\partial D} = 0$$

$$\text{and letting } \phi(a) = \int_{-\infty}^a \phi(1) d1$$



Table 1  
Optimal Values of D Under Uncertainty Compared with Traditional  
Values (D\*), as calculated by Orr and Mellon

Case	M	r	i	k	R	D	D*
1	20	.01	.0025	.7	10	8.9	13.2
2	20	.01	.0025	.7	1000	1311	1316
3	0	.01	.0025	.7	10	10.7 <sup>a</sup>	13.2
4	20	.01	.0025	.3	10	15.7	22.8
5	20	.5	.0025	.7	10	8.9	13.2
6	arbitrary	.01	.01+	.7	arbitrary	∞	13.2
7	20	.01	.0025	0	10	35.4	50 <sup>a</sup>

<sup>a</sup>These are corrected values supplied by Orr and Mellon after original table was published.

Source: Orr and Mellon, "Stochastic Reserve Losses," *American Economic Review*, Vol LI, No. 4, Sept., 1961, p. 619.

Table 2  
Revised Values of Table 1 Based on Corrected Equations

Case	M	r	i	k	R	D	D*
1	20	.01	.0025	.7	10	8.9	13.2
2	20	.01	.0025	.7	1000	1311	1316
3	0	.01	.0025	.7	10	12.5	13.2
4	20	.01	.0025	.3	10	15.7	22.8
5	20	.5	.0025	.7	10	8.9	13.2
6	arbitrary	.01	.01+	.7	arbitrary	∞	1.3R
7	20	.01	.0025	0	10	35.4	50

Source: Author's calculations

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1. Daniel Orr and W. G. Mellon, "Stochastic Reserve Losses and Expansion of Bank Credit," *American Economic Review*, Vol. LI, No. 4, Sept., 1961, pp. 614-623.
2. Their profit equation did not distinguish between the random variable,  $L$ , and its realization,  $l$ . We conform here to normal usage.
3. H. Laurence Miller, Jr., "Stochastic Reserve Losses and Expansion of Bank Credit: Comment," *American Economic Review*, Vol. LII, No. 5, Dec., 1962, pp. 1118-1120.
4. Daniel Orr and W. G. Mellon, "Stochastic Reserve Losses and Expansion of Bank Credit: Reply," *American Economic Review*, Vol. LII, No. 5, Dec., 1962, pp. 1120.
5. *Ibid.*, p. 1121.
6. Their notation for this was somewhat odd.  $\phi(1)$  was said to be  $N(kD, oL)$ . It would have been better form to say  $L:N(kD, oL)$  since densities do not have distributions; random variables do.
7. See *American Economic Review*, Vol. LIII, No. 4, Sept., 1963, "Errata," p. 745.
8. See Appendix.
9. Professor Miller alluded to this in his "Comment".

<sup>10</sup>Let  $D^1$  = initial deposit liabilities,  $R^1$  = initial reserves,  
 $D$  = marginal credit expansion, and  $R$  = excess reserves. Then  
 $R^1 = \phi D^1$ . The inequality Orr and Mellon refer to is:  

$$\frac{D}{R} < \frac{D^1 + D}{\phi D^1 + R}$$
. This is equivalent to:  $D\phi D^1 + RD < RD^1 + RD$  or  $\phi D^1 < R$ .

In the certainty case, we merely replace  $D$  by  $D^*$ . Thus, the inequality holds whenever  $\phi D^* < R$ .

11. Harald Cramér, "Model Building with the Aid of Stochastic Processes," *Technometrics*, Vol. 6, No. 2, May, 1964, pp. 133-159.
12. Paul M. Kahn, "An Introduction to Collective Risk Theory and Its Application to Stop-Loss Reinsurance," in *Society of Actuaries Transactions*, Vol. XIV, Part 1, 1962, p. 407. Reference is to H. Ammeter, "The Calculation of Premium Rates for Excess of Loss and Stop-Loss Reinsurance Treaties," in *Non-proportional Insurance*, ed. by S. Vajda, Brussels: Arithbel S. A., 1955.
13. See: Emanuel Parzen, *Stochastic Processes* (Holden-Day, Inc., San Francisco, 1962), p. 56. Also, Hans Buhlmann, "A Distribution Free Method for General Risk Problems," *The Astin Bulletin*, Vol. III, Part II, Aug., 1964, pp. 144-152.

## COMMENT\*

by John T. Boorman\*\*

### Introduction

This critique of the paper presented by Mrs. Birch and Mr. Heineke, "Stochastic Reserve Losses", <sup>(1)</sup> will be divided into two parts. In the first part, we shall have to make reference not only to the original article by Daniel Orr and W. J. Mellon <sup>(2)</sup> but also to various 'comments' on this article which have

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\*I wish to extend my great thanks to Mr. T. V. S. Rama Mohan Rao for his help in deriving the differentiated form of the profit function and for his other very helpful comments. Thanks are also due to Dr. Michael DePrano, Mr. W. Roger Watson and Mr. John Cownie, all of whom read this manuscript in some stage of preparation and made helpful suggestions.

\*\*Mr. Boorman is a graduate student at the University of Southern California.

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