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The Allocation of Effort under Uncertainty: The Case of Risk-averse Behavior

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This paper analyzes the labor supply decision of a single economic agent within the expected utility framework. Two formulations of the problem are considered: pure income uncertainty and wage rate uncertainty. In each case, the effects on the labor supply decision of changes in both expected returns and the dispersion of returns (about a constant mean) are investigated. Arguments concerning the “disincentive effects” of uncertainty are shown not to be unambiguously supported by theory.

The pioneering work of Arrow (1965) in the theory of portfolio choice and the recent contributions of Leland (1968, 1971), Mossin (1968*a*, 1968*b*), Sandmo (1969, 1970, 1971), Stiglitz (1969), and others clearly testify to the power of the expected-utility hypothesis in analyzing various problems of choice under uncertainty. But to our knowledge, a general analysis of the effects of uncertainty on the labor supply decision has yet to be done.¹ In this paper we examine the labor supply decisions of a single economic agent in the expected utility framework.

We consider two models and in each analyze the response of the labor supply decision to shifts in the subjective probability distribution of wages and/or income. In model I the individual's income contains a stochastic

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¹ While some aspects of labor market uncertainty have been investigated in the job search literature (e.g., Mortensen 1970) and in Block and Heineke (1972*a*, 1972*b*), the traditional labor supply problem under uncertainty has not been analyzed.

component although the wage rate is deterministic; in model II the wage rate itself is stochastic. Attention is focused on the effects on labor supplied of changes in expected returns and the dispersion of returns about a constant mean. All results are distribution free.

The following definitions will be used: L = the labor allocation, $L \geq 0$; w = the wage rate, $w > 0$, where w is a parameter in model I and a random variable in model II; Y = total income in the period of analysis; Y^0 = autonomous income, that is, income independent of labor force participation; and $U(L, Y)$ = the individual's von Neumann–Morgenstern utility function.

In the work that follows, we assume that (i) income is a commodity ($U_Y > 0$) and labor is a discommodity ($U_L < 0$), and (ii) the individual is risk averse ($U_{YY} < 0$).² As is well known, a general analysis of the allocation decision under uncertainty requires additional information about the agent's preferences. Elements of this situation are similar to those confronting the investigator in most household decision problems. In the labor supply problem under conditions of certainty, assumptions (i) and (ii) are sufficient to sign only direct-substitution terms generated by changes in the wage rate; income effects remain unrestricted in the absence of further assumptions.³ Precisely the same statement applies to the stochastic analogue of this problem, that is, the substitution and income terms generated by a change in the expected wage rate. In both the deterministic and stochastic case, signing income effects requires additional explicit second partial derivative information.

In stochastic models, considerable attention is directed toward determining the effects of changes in the "amount of uncertainty" in the system. Such changes generate Slutsky-like expressions. Just as signing income terms (produced by changes in the wage rate or expected wage rate) requires more detailed information about preference orderings than does signing substitution effects, so does signing "income uncertainty effects" (produced by changes in the "amount of uncertainty") require more information than does signing the "uncertainty substitution effect." The sign of the latter is determined merely by the agent's simple behavior toward risk; that is, whether he has a preference for risk is risk neutral or risk averse, while the former require some knowledge of third derivatives.

An appealing method of providing this information is to postulate plausible hypotheses regarding the agent's behavior toward risk as various arguments of the utility function change. For example, if utility is a function of income, a risk-averse individual may become less risk averse as his

² As usual, subscripts denote partial derivatives; e.g., $U_Y = \partial U / \partial Y$, $U_L = \partial U / \partial L$, and $U_{LY} = \partial^2 U / \partial L \partial Y$.

³ Obviously, signing direct substitution terms requires only assumption (i). Assumption (ii) is neither necessary for signing substitution terms nor sufficient for signing income terms.

income increases. This hypothesis, Arrow-Pratt decreasing absolute risk aversion, has been widely utilized and has yielded interesting results. However, as the recent work of Sandmo (1970, 1971) and Leland (1971) indicates, this hypothesis alone does not provide sufficient information in the multicommodity case.

Therefore, we begin by specifying a coefficient of absolute risk aversion in the multicommodity case, $R(Y, L) = -U_{YY}/U_Y$, and assume this term is invariant to the agent's labor allocation and decreasing in income (the Arrow-Pratt hypothesis). Decreasing absolute risk aversion with respect to income implies that the individual becomes increasingly willing to accept a wager of a given size as his income increases, in the sense that odds demanded diminish. Arrow (1965) argues that both intuition and fact support this hypothesis. Intuitively, constant absolute risk aversion with respect to labor implies that the odds demanded by the individual for taking a risky action are not affected by how much he "works." Formally, (iii) $\partial R/\partial Y < 0$ and (iv) $\partial R/\partial L = 0$.

Assumptions (i)-(iv) and the traditional assumption (v) that in a certain world labor is an inferior commodity,

$$\partial L/\partial Y^0|_{Y=EY} < 0 \quad (\text{v})$$

are sufficient for a distribution-free analysis of the labor allocation problem presented in models I and II.⁴

Model I

In this model we take the rate of return in the labor market to be certain but assume that income generated in the period contains a random component which is distributed independently of the individual's labor supply decision. If A is an asset stock and $x \geq -1$ is the random rate of return on the asset stock, then the case of an individual who "works" and has an uncertain property income of Ax per period closely approximates this situation. Under these conditions, the individual's income opportunities are

$$Y = wL + e, \quad (1)$$

where $e = Ax$ ($=Y^0$) and w is the known rate of return to labor.

The Labor Supply Decision: Income Uncertainty

The individual's labor supply decision is given by

$$\max_L \int_{-A}^{\infty} U(L, wL + e) f(e) de, \quad (2)$$

⁴ For convenience, we denote "certainty" as the case where all realizations of the random variable Y are equal to EY .

⁵ This problem is formally equivalent to that explored by Sandmo (1970) in the section of his paper entitled "Income Risk."

subject to the condition $0 \leq L \leq \bar{L}$ (\bar{L} is total time available) and solving for L .⁶ In (2), $f(e)$ is the individual's subjective probability density and reflects the individual's beliefs as to the intervals in which e , property income, is likely to lie. The necessary condition for a relative maximum is

$$EU_L + wEU_Y \leq 0, \quad (3)$$

with the strict inequality holding in the case where the labor supply decision is $L = 0$.

Only in very special cases will (3) reduce to the certainty decision rule. Obviously, if the individual's utility function is linear in income, expression (3) reduces to the traditional $MRS_{YL} \geq w$. One would expect that if an individual is risk neutral in income ($U_{YY} = 0$), income uncertainty alone would not affect his decisions.⁷ Moreover, since the stochastic component of income is independent of L , (3) will also reduce to the certainty rule if the individual's utility function is quadratic. The labor decision rule above would then be an example of Theil's (1964) "certainty equivalence."

The restrictive nature of both of these utility functions is well known. Risk neutrality clearly contradicts observed behavior, and as Arrow (1965) has emphasized, the assumption of a quadratic utility function, while accounting for risk aversion, implies a serious contradiction of observed portfolio behavior. Therefore, if the present analysis is to account for the income implications of modern portfolio theory, (3) will not be immediately reducible to the certainty decision rule and the effects of uncertainty on the optimum allocation of labor will require explicit consideration.

*The Comparative Statics of Income Uncertainty*⁸

The second-order condition for a relative maximum is

$$H \equiv EU_{LL} + 2wEU_{YL} + w^2EU_{YY} < 0. \quad (4)$$

The effect on labor supplied of a change in the expected value of e when all other moments about the mean are fixed may be investigated by replacing e in (3) with $e + \theta_1$, where θ_1 is a shift parameter, differentiating with respect to θ_1 and evaluating the result at $\theta_1 = 0$. Thence,

$$\partial L / \partial \theta_1 = -E(U_{LY} + wU_{YY}) / H. \quad (5)$$

Since all central moments of the distribution of e remain constant, a change in θ_1 is the stochastic counterpart of a neoclassical lump-sum income transfer; and (5) is clearly negative by assumption (v). That is,

⁶ For simplicity we assume $L < \bar{L}$, i.e., the "upper bound" constraint is nonactive.

⁷ Note that behavior toward risk, while traditionally defined in terms of income, might also be defined in terms of the discommodity labor. Surely the individual can gamble with his labor input as well as income.

⁸ In this section and in the comparative-static analysis that follows, we consider only internal solutions to the maximization problem.

$$\partial L/\partial \theta_1 < 0. \quad (5')$$

Obviously, the inferiority of labor implies that increases in the expected return from "property" lead to decreases in labor supplied (or increases in leisure consumed).

While mean-value shifts have received considerable attention in traditional analysis, the effect on the labor allocation of increased "uncertainty" in nonlabor income have not been adequately explored. The connection between "stability" and work incentives is often implied, but a formal framework appears to be lacking. As an initial step in this direction, we consider the effect of a change in the "amount of uncertainty" in nonlabor returns on the labor supply decision.

Formally, changes in the "amount of uncertainty" may be interpreted as shifts in higher central moments of the distribution of e . Since, for arbitrary probability distributions, the variance of e may be an unsatisfactory measure of dispersion, we follow the suggestion of Arrow (1965), since utilized by Sandmo (1969, 1970, 1971) and Leland (1971), and analyze the effects of a pure increase in dispersion by means of a multiplicative parameter shift followed by additive shift that leaves the mean unchanged.⁹ To proceed, replace e in (3) with $\gamma_1 e + \theta_2$, where γ_1 and θ_2 are shift parameters. Since we desire $E(e)$ to remain unchanged, $dE(\gamma_1 e + \theta_2) = 0$ and $d\theta_2/d\gamma_1 = -E(e) = -\delta$, where $E(e) \equiv \delta$. Differentiating (3) with respect to γ_1 and evaluating the result at $\gamma_1 = 1$ and $\theta_2 = 0$, we have

$$\partial L/\partial \gamma_1 = -\text{cov}(e, U_{LY} + wU_{YY})/H.^{10} \quad (6)$$

Expression (6) is the "income uncertainty effect" on labor supply of a pure dispersion change.

In the discussion following equation (3) above, it is noted that if the individual's utility function is quadratic in income, the stochastic decision rule reduces to its "certainty equivalent" and hence uncertainty has no allocative effects. Equation (6) is a direct assessment of the allocative effects of uncertainty and yields in the case of a quadratic utility function the expected result that $\partial L/\partial \gamma_1 = 0$.

For the more general case, assumptions (iii)-(v) imply that $U_{LY} + wU_{YY}$ is increasing in Y and thus $\text{cov}(e, U_{LY} + wU_{YY})$ is positive.¹¹ Hence,

⁹ For a discussion of this method of analyzing increases in uncertainty, see Leland (1970) and Sandmo (1970, 1971). An excellent discussion of alternative interpretations of increasing risk is given in Rothschild and Stiglitz (1970).

¹⁰ We have written $\partial L/\partial \gamma_1$ as a covariance rather than as $-E(U_{LY} + wU_{YY})(e - \delta)/H$, since it is possible to directly sign the covariance expression.

¹¹ To obtain this result, totally differentiate R with respect to L , noting $\partial R/\partial L \equiv \partial(-U_{LY}/U_Y)/\partial Y$ for continuous functions. Since dR/dL is negative by (iii) and (iv), (v) implies $\partial(U_{LY} + wU_{YY})/\partial Y > 0$.

$$\partial L / \partial \gamma_1 > 0. \quad (6')$$

Inequality (6') is a particularly intriguing result and indicates that the individual uses the labor market as a "hedge" against uncertainty. As the dispersion of nonlabor income increases, the individual compensates by increasing his expected income. While this appears to be a type of "self insurance," it is not a simple analogue of direct insurance. The inequality $\partial L / \partial \gamma_1 > 0$ is an implication of the inferiority of labor and of a particular type of *change* in behavior toward risk as income and labor change, not an implication of behavior toward risk itself. In other words, direct insurance behavior is implied by risk aversion, but, as indicated above, risk aversion alone does not imply $\partial L / \partial \gamma_1 > 0$.

Model II

As indicated at the outset, there has been a very limited concern with the explicit analysis of labor allocation under uncertainty. Institutional restrictions on the supply decision may account for some of this neglect, but it is an unconvincing argument in the light of the attention given to "incentive" or supply effects of tax schemes, etc., in a world of certainty.¹² Perhaps it has been the intuitive feeling that in a fully employed economy the returns to labor are the most certain of factor returns. Nonetheless, as the results above indicate, even if the return to labor were certain, as long as some stochastic element is present in income, labor allocation decisions will depend on risk-taking behavior. Beyond this, we know that some occupations do have uncertain wage payments and that, more significantly, the degree of certainty involved in all wage payments is to some extent institutionally determined. Random income taxes, random inflation, or the absence of property-rights enforcement would drastically alter the dispersion of returns to labor.¹³ A theory of labor allocation should be able to predict the effects of changes in institutional arrangements affecting the certainty of returns in the labor market.

As an extension of the formal analysis of uncertainty and labor "incentives," in this model we take the rate of return in the labor market to be a random variable. Income in the period is given by

¹² For example, see Cooper (1959) and Musgrave (1959); or for evidence of more recent concern, see Cassidy (1970).

¹³ Traditional investigations (nonstochastic) of the effect of taxation on labor allocations, such as Cooper (1959) and Cassidy (1970), emphasize what are essentially mean-value shifts in our model. As the discussion above indicates, there is an additional set of incentive questions associated with dispersion shifts. Clearly, there is an effect on labor allocation due to changes in the degree of certainty surrounding incidence. However, even if the tax is known with certainty, to the extent that the wage rate is stochastic, tax policy will have dispersion effects. In fact, changes in tax rates under these conditions will have both mean and dispersion effects. An intriguing area of further research is the extension of Mossin's (1968) and Leland's (1971) taxation analysis to the labor allocation problem.

$$Y = Y^\circ + wL. \quad (7)$$

Formally, the individual's labor supply decision is determined by

$$\max_{\{L\}} \int_0^\infty U(L, Y^\circ + wL) f(w) dw, \quad (8)$$

subject to $0 \leq L \leq \bar{L}$.¹⁴ The function $f(w)$ is the individual's subjective probability density on w . The first-order condition for a relative maximum is

$$E(U_L) + E(wU_Y) \leq 0. \quad (9)$$

Again, if utility were linear in income, (9) would reduce to the certainty decision rule: $MRS_{YL} \geq E(w)$. However, unlike model I, in this case quadratic utility will not yield "certainty equivalence" and has, as the discussion below indicates, rather special implications for the effects of wage uncertainty on labor allocation.

The Comparative Statics of Wage-Rate Uncertainty

Given an internal solution to (9),

$$G \equiv EU_{LL} + 2E(wU_{YL}) + E(w^2U_{YY}) < 0 \quad (10)$$

is sufficient for a relative maximum. The effect on labor supplied of changes in the expected wage rate may be ascertained in the same manner as above. If we let $w = w + \theta_3$, then

$$\partial L / \partial \theta_3 = -EU_Y / G - LE(U_{LY} + wU_{YY}) / G. \quad (11)$$

The first term is the substitution effect of a change in the expected wage rate and is obviously positive. The second term is the income effect of a change in the expected wage rate and, like equation (5) above, is negative by assumption (v). Consequently, if we accept the often employed assumption that the labor supply function bends back on itself, (11) would generate the stochastic analogue of the neoclassical backward-bending labor supply function.¹⁵

One interesting question remains: Can anything be said about the individual's labor supply decision when the wage rate becomes more uncertain? We proceed as before and investigate a change in wage-rate dispersion about a constant mean. Substituting $w = \gamma_2 w + \theta_4$ into (9), differentiating with respect to γ_2 [with $d\theta_4/d\gamma_2 = -E(w)$ in this case], and evaluating the derivative at $\gamma_2 = 1$, and $\theta_4 = 0$, we have

¹⁴ As before, we assume for simplicity $L < \bar{L}$.

¹⁵ For a recent study that uses this assumption, see Cassidy (1970). Formally, whether the supply function is backward bending or not is a question of whether or not there exists an $L^* < \bar{L}$ such that (9) is maximum at L^* (i.e., whether or not there exists an $L^* < \bar{L}$ such that substitution effects dominate for $L < L^*$ and income effects dominate for $L > L^*$).

$$\partial L / \partial \gamma_2 = -\text{cov}(U_Y, w) / G - L \text{cov}(w, U_{LY} + w U_{YY}) / G. \quad (12)$$

Since equation (12) is a Slutsky-like equation, we call the first term the “uncertainty substitution effect” and the second the “income uncertainty effect.” This substitution effect measures the response of the individual’s labor supply decision due solely to changes in wage-rate uncertainty. As is obvious from (12), and as would be expected, risk-averse behavior implies that this term is negative. The income uncertainty effect in equation (12) measures the response of the labor supply decision attributable solely to the increased income uncertainty implied by the increase in wage uncertainty. This term is analogous to equation (6) and is positive.¹⁶

As indicated above, the income uncertainty result depends on the inferiority of labor and a particular set of assumptions concerning behavior toward risk as Y and L change. For example, if the utility function is quadratic, the income uncertainty effect will be zero. However, in the more general and empirically relevant case, increases in the dispersion of the wage rate induce the individual to supply more effort in an attempt to compensate for the additional income uncertainty.¹⁷

Combining income and substitution effects, we see that changes in wage-rate dispersion lack unambiguous incentive effects. Only through empirically dubious restrictions on the utility function does the ambiguity in (12) disappear.

As (11) and (12) indicate, an increase in wage-rate dispersion has precisely the same qualitative effect on the labor supply decision as a decrease in the expected wage rate. The intuitive explanation is that risk-averse individuals view increased wage uncertainty and decreases in mean wages as “costs” in the sense that each of these parameter shifts has the effect of lowering the chances that a realized wage rate will fall within some specified “acceptable” range. Hence, each shift leads to decreases in labor supplied in substitution terms and increases in labor supplied via income effects.

Summary and Some Implications

An intriguing implication of our analysis is that, contrary to often professed beliefs, increased *income* uncertainty, under very reasonable assumptions, unequivocally brings about increases in labor force participation. However, when the increase in income uncertainty is generated by a change in the dispersion of a wage rate, more than income uncertainty must be

¹⁶ The proof is entirely similar to that of (6) above.

¹⁷ Unlike its implication in the previous model, a quadratic utility function now has allocative significance because of the nonzero substitution term; and an increase in wage dispersion will always decrease the supply of effort.

considered. Since an individual can reduce uncertainty by substituting away from the activity, there is a substitution effect that moves in the opposite direction. It is the substitution effect that provides the theoretical underpinnings for statements about the "disincentive" effects of uncertainty.

It is now clear that arguments concerning the "disincentive" effects of uncertainty are not unambiguously supported by theory. In fact, increases in the dispersion of factor returns may, paradoxically, have strong incentive effects. As we have shown and as Sandmo (1970) had indicated in a different context, to the extent that uncertainty is not directly tied to the allocative decision risk-averse individuals will increase their productive efforts as uncertainty increases. Even in those cases where increases in dispersion are directly related to the allocative decision (wage-rate uncertainty), the "disincentive" effects of increased uncertainty are not derivable from theoretical considerations alone.¹⁸ While the "uncertainty substitution effects" suggest a substitution away from the affected activity, one must have knowledge of relative magnitudes, since the "income uncertainty effect" moves in the opposite direction and may either dominate or be dominated by the substitution effect. Therefore, under very plausible behavioral assumptions, the "conventional wisdom" of the importance of "stability" (the minimization of uncertainty) in factor returns is either a testable hypothesis about relative magnitudes or a normative statement.

In a world of risk-averse individuals, it is true that costless decreases in the dispersion of returns is a Pareto-relevant action. But a fascinating implication, in terms of positive analysis, of the particular set of behavioral assumptions employed in this paper is that public policy directed at reducing uncertainty may have "disincentive" effects. And an appropriately designed increase in income uncertainty may be used as a policy instrument for increasing the supply of labor, but it could not of course be defended with Pareto considerations. Risk-averse individuals lose as uncertainty increases, and their factor reallocation is only a reaction to a less hospitable world.

Our analysis of the labor decision and Sandmo's (1970) analysis of the savings decision both treat one aspect of the household decision in isolation. An interesting avenue of further research is the integration of both decisions into a single model of household decision making in which the supply of labor and savings are jointly determined and both factor returns are stochastic.¹⁹

¹⁸ That is, "distinctive" effects of increased uncertainty do not follow from risk aversion.

¹⁹ For an extension of Sandmo (1970) to the joint labor-savings decision, see Block and Heineke (1972*a*). This paper utilizes relatively strong assumptions, some of which are relaxed in Block and Heineke (1972*b*).

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