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Pricing Software Upgrades: The Role of Product Improvement & User Costs

Ram Bala
Santa Clara University, rbala@scu.edu

Scott Carr

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Ram Bala[1] Scott Carr[2]

Indian School of Business LECG

[1]ISB Campus, Gachibowli, Hyderabad - 500032, India. Ph: 91 40 2318 7139, Fax: 91 40 2300 7038, e-mail: ram_bala@isb.edu

[2]1725 Eye Street NW, Ste 800, Washington, DC, 20006. Ph: 202 466 4422, Fax: 202 466 4487, e-mail: scarr@lecg.com

Abstract

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The computer software industry is an extreme example of rapid new product introduction. However, many consumers are sophisticated enough to anticipate the availability of upgrades in the future. This creates the possibility that consumers might either postpone purchase or buy early on and never upgrade. In response, many software producers offer special upgrade pricing to old customers in order to mitigate the effects of strategic consumer behavior. We analyze the optimality of upgrade pricing by characterizing the relationship between magnitude of product improvement and the equilibrium pricing structure, particularly in the context of user upgrade costs. This upgrade cost (such as the cost of upgrading complementary hardware or drivers) is incurred by the user when she buys the new version but is not captured by the upgrade price for the software.

Our approach is to formulate a game theoretic model where consumers can look ahead and anticipate prices and product qualities while the firm can offer special upgrade pricing. We classify upgrades as minor, moderate or large based on the primitive parameters. We find that at sufficiently large user costs, upgrade pricing is an effective tool for minor and large upgrades but not moderate upgrades. Thus, upgrade pricing is suboptimal for the firm for a middle range of product improvement. User upgrade costs have both direct and indirect effects on the pricing decision. The indirect effect arises because the upgrade cost is a critical factor in determining whether all old consumers would upgrade to a new product or not and this further alters the product improvement threshold at which special upgrade pricing becomes optimal. Finally, we also analyze the impact of upgrade pricing on the total coverage of the market.

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1 Introduction

Upgrades are endemic to many industries that exhibit rapid product innovation. The computer software industry is no exception. By now, this practice of the software industry is known to most consumers and they can usually predict the arrival of newer versions in the future. In fact, today's consumers are tech-savvy enough to have a reasonable estimate of the state of technological evolution. In the software industry, this information can be obtained from various third party sources: magazines such as *PC World*, technology websites such as *Zdnet.com*, and various other internet forums. Very often, firms themselves provide strong hints to consumers about the possible features of future versions, including the projected time of release. For example, top executives at Microsoft (including Bill Gates) have been promoting the complete device inter-operability of Windows Longhorn (projected release in 2006, and now renamed as Windows Vista) since 2003(*USA Today*, 2003). They even provide specific examples of how such inter-operability would help consumers improve their day-to-day computer related tasks. Irrespective of how the consumer obtains knowledge of such innovation, the fact remains that consumers are very likely to project into the future and make a longer term decision. This decision can take two unfavorable outcomes for the producer of software: consumers can choose to buy now and not upgrade at all when the new version comes along. They can also choose to postpone their purchase until the new version arrives.

Firms have adopted different strategies to mitigate such strategic consumer behavior. The most traditional (and yet the most under-explored in academic research) strategy is to offer older consumers an upgrade price: for instance, a new version of Windows Vista Home costs about $215 while users of old versions of Windows can upgrade to this version at only $95 (www.amazon.com, October 2007). This phenomenon is not restricted to operating systems alone. A similar practice is observed with Corel WordPerfect Office X3 Professional Edition: the upgrade sells for $245 while the new version sells for $335. Many other strategies have also emerged in recent times: firms may offer upgrades in modular form and price accordingly; Microsoft has begun to offer a form of

No.	Company	Product	Upgrade Pricing (Y/N)	Full Price	Upgrade price	% Discount
1	Microsoft	Office Home & Student 2007	No	$124.99	-	-
2	Microsoft	Office Professional 2007	Yes	$312.79	$263.95	16%
3	Adobe	Acrobat 8.0 Professional	Yes	$383.99	$156.49	59%
4	Adobe	Photoshop CS3	Yes	$629.49	$195.99	69%
5	IMSI	Turbocad Deluxe V14	No	$71.28	-	-
6	Autocad	LT 2008	Yes	$799.99	$314.99	61%
7	Borland	C++ Builder 6 Personal	No	$79.99	-	-
8	Mathworks	Matlab Individual	No	$1,900	-	-
9	MacKichan	Scientific Workplace	Yes	$845	$295	65%
10	Frontline Systems	Solver Platform SDK	No	$1,495	-	-
11	Corel	Graphics Suite X3	Yes	$337.99	$160.99	52%
12	Macromedia	Studio 8	No	$999	-	-
13	Apple	Final Cut Studio 2	Yes	$1,299	$449.99	65%
14	Apple	iWork '08	No	$74.99	-	-
15	Intuit	Quickbooks Pro 2008	No	$179.99	-	-

Figure 1: Pricing strategies of different firms (Source: www.amazon.com, October 2007)

upgrade insurance where consumers pay upfront fees to cover any potential upgrades in the future; the rise of high speed networks allows firms to deploy software as a service and then deploy either pay-as-you-go and / or subscription pricing schemes which guarantee automatic upgrades. However, the conventional method of offering an upgrade price is still widely used and merits attention.

Figure 1 lists the prices of software products offered by several firms as obtained from amazon.com. Each of the product examples represents a newer version of an already existing product. These examples are in addition to the two examples (Windows Vista and Corel WordPerfect) highlighted before. Upgrade pricing is frequently used though not always. Further, even when upgrade pricing is used, the magnitude of the discount varies widely from 16% of full price to 69%. Also, the deployment of upgrade pricing varies within the product line of the same firm. This raises an important question. When is offering an upgrade price to old customers the best strategy for a firm? What factors does this decision depend on? How does the exact magnitude of the upgrade price vary with these factors?

What factors would consumers consider in making a decision to buy an upgrade? Clearly, the level of product improvement in the upgrade is critical. The software industry is known to be particularly notorious in this regard since many product upgrades are only minor to moderate

improvements over older versions. For instance, many users on amazon.com consider the "plug and play" abilities of Windows XP to be the only improvement over Windows 2000. The upgrade price would certainly be a factor. In addition, users of a software product face an upgrade cost that is not captured by the price of the upgrade. For example, consider a user who upgrades to Windows Vista. Such a user may need to upgrade her hardware configuration to meet the non-negotiable demands of Windows Vista. She may also have to undertake the hassle of reinstalling Windows Vista compatible applications software. This fact is voiced in several popular software forums(*PC World*, 2007). In addition, even Windows experts claim that the user interface for Windows Vista is difficult to get used to (*winsupersite.com*, 2006). Thus, the user upgrade cost may also include the cost of learning. In fact, these costs can be quantified. *PCWorld* comes up with a number for this upgrade cost: $180 in order to upgrade to 2GB RAM and $150 on a new graphics card. Without these additions, experts conclude that it is difficult to effectively utilize the features of Vista. This significantly alters the decision-making process for the user. Of further importance is the fact that these costs could vary widely across products. For instance, in the experience of the authors, upgrading to the latest version of Matlab typically does not come with a hidden upgrade cost. Of course, this could change with a new generation of Matlab which is a significant leap in terms of features over previous versions. This fact also implies that upgrade costs have a tendency to increase as more attributes and features are incorporated in the upgrade.

What motivates a firm to offer upgrade prices? The most important reason would be the ability to implement third degree price discrimination. The product improvement observed by consumers who possess the older version is lower than new consumers. In addition, they also experience different upgrade costs. This might result in lower incremental utility for an old consumer as compared to a new consumer. Hence the firm has to provide a lower price to these consumers to get them to upgrade to the new product. However, older consumers are typically early adopters and hence likely to be higher willingness-to-pay consumers. Consequently, the incremental utility for an old consumer could be higher than a new consumer. This negates the need to offer upgrade pricing. We explore this trade-off by modeling all the above factors simultaneously. Our objective is to answer the question: when is offering an upgrade price to old customers the optimal strategy?

Prior literature on durable goods innovation primarily addresses the issue of "credible commitment" of prices in the presence of strategic customers. The objective of this line of work was to

discover mechanisms through which credible commitment might be possible. One of the insights from past literature is that such credible commitment is never possible for large upgrades when upgrade discounts are offered. We show that this no longer holds once the user upgrade cost is incorporated into the customer's utility function. In particular, credible commitment with upgrade pricing for large upgrades is indeed possible for high enough upgrade costs. Further, even with minor to moderate upgrades, the user upgrade cost in conjunction with the product improvement level has a significant impact on the upgrade pricing decision. The fact that all old customers upgrade is a surprisingly sticky result in prior work which does not quite match reality. Incorporating the user upgrade cost helps us to show that this result is not always true. In particular, high user costs ensure that not all old customers upgrade to the new version. This also changes the product improvement threshold at which upgrade pricing is optimal. Putting together the results for the entire range of product improvement, the following result emerges: for high user costs, upgrade pricing is suboptimal only for an intermediate range of product improvement. This provides guidance to managers on when they should expect to use upgrade pricing. Further, we also analyze the impact of upgrade pricing on profitability and market coverage and find that: upgrade pricing can substantially improve the firm's profitability provided it is viable (for a given set of parameters) but the overall market coverage is lower when compared to a situation when upgrade pricing is disallowed. Thus, this paper bridges the gap between the current state of knowledge in durable goods pricing and in sequential innovation by incorporating a detailed understanding of the operational costs incurred by the consumer when a durable product upgrade is purchased.

The paper proceeds as follows: Section 2 reviews related literature. Section 3 describes the model and states its basic assumptions. Section 4 describes the equilibrium solution structure with a uniform distribution on consumer willingness to pay. Section 5 generalizes some of the results to a broader class of distributions. The proofs for the main propositions are provided in the appendix. Section 6 concludes the discussion and proposes future research directions.

2 Related Literature

Our problem closely relates to sequential innovation and pricing, a topic that has been studied in different streams of literature: new product development, marketing & economics. The earliest

paper in this context is by Dhebar(1994) that looks at the effect of product improvement on the ability of the firm to credibly commit to future period prices. The primary result is that there is an upper limit on the rate of product improvement such that credible commitment is not possible when this upper limit is exceeded. Upgrade pricing is also considered in the same context, but no prescription is provided regarding its optimality. Consumers are given only one option to buy the product (either now or later) and upgrading the product is not possible unless an upgrade price is offered. Kornish(2001) generalizes the utility function used in Dhebar(1994), disallows upgrade pricing but allows old customers to upgrade even without such pricing. She shows that when upgrade pricing is not considered, one can show the existence of an equilibrium where a firm can credibly commit to prices even when the product is improving rapidly. While the utility function used in Kornish(2001) admits the possibility of an upgrade cost, the impact of such a possibility on the final outcome is not studied. We combine aspects of both papers, in that we allow the firm to set an upgrade price, yet consumers can upgrade even when such upgrade pricing is not offered. In addition, we analyze the impact of a user upgrade cost in such a setting. While Dhebar(1994) and Kornish(2001) address the issue of large upgrades, Fudenberg & Tirole(1998) look at moderate upgrades. They study the upgrades and trade-ins issue by considering different information structures that the monopolist has about individual consumers. Closest to the current paper is their 'semi-anonymous' case in which the latter period prices are bound by an arbitrage constraint. The arbitrage constraint, which we also employ, states that any special upgrade pricing that is offered cannot exceed the price offered to new consumers. They conclude that there is no "leapfrogging" (leapfrogging is defined as the situation where all old customers do not upgrade but some new customers buy) when production is costless (as in the case of software). However, this does not match reality since leapfrogging is common in software product markets. We explain how leapfrogging might occur and show that their results will form a special case of our model. Bhattacharya et al(2003) considers optimal product sequencing and briefly visits the upgrade pricing issue using an optimization program; we instead employ the subgame perfection equilibrium concept. Additionally, in their paper, consumers of the initial version of the product cannot purchase the improved version unless an upgrade price is offered just as in Dhebar(1994). As stated earlier, we relax this assumption.

To summarize our contribution, we extend Dhebar(1994) & Bhattacharya et al(2003) by allowing old customers to purchase the improved version even when upgrade pricing is disallowed. We generalize

Kornish(2001) by considering upgrade pricing explicitly. We generalize Fudenberg & Tirole(1998) by studying large upgrades and equilibrium consumer behavior in the context of upgrade costs. To the best of our knowledge, ours is the first paper to address all these issues simultaneously.

There is further work on upgrades that our paper is not that close to. Ellison and Fudenberg(2000) study the effects of upgrades on social welfare, particularly in the context of network externality. Padmanabhan et al (1997) investigate the positioning decision across periods given network externality effects for a fixed set of homogeneous consumers. Other recent work in this area includes Krishnan & Ramachandran(2008) that studies the impact of product design choices such as modular upgradability on consumer upgrade behavior, and Mehra & Seidmann(2008) that looks at upgrade introduction in the context of product lifecycle management. Sankaranarayanan(2007) looks at effectiveness of the upgrade insurance policy that firms such as Microsoft have introduced.

Our modelling methodology is also related to the durable goods monopolist problem in the economics literature. The earliest work in this area is by Coase(1972) who postulates that a monopolist selling a durable good to rational consumers cannot capture monopoly profits. Consumers will look ahead, anticipate a decreasing price trajectory and hence postpone their purchase till price equals cost. Stokey(1981) models this process over an infinite horizon for a non-depreciating durable good and confirms the results. Bulow(1982) uses a two-period model with a second hand market for a durable good and shows that while a monopolist renter can capture monopoly profits, a monopolist seller cannot do the same. Software renting (also known as Software-as-a-Service) is a new business model practised by *application service providers* (ASPs) and may provide greater economic benefits as compared to selling. However, while the benefits of renting in the software services context are well accepted, its implementation suffers because of several issues such as data security, customizability and scalability (*BusinessWeek*, 2008). This fact ensures that while the renting of software will continue to grow, the software product model is unlikely to be displaced anytime soon. The economics of software and technology is also receiving increasing attention within the operations management literature. Druehl & Schmidt(2008) and Liu et al(2007) are recent examples of such work.

Our paper is also related to the literature on versioning of durable goods. Within our upgrade problem, there are two embedded versioning problems. The first one is easy to observe: in a later period, the firm offers a product of lower value (effectively) to the old customers who have already

bought an earlier version. In this case, the two versions are directed towards consumers who differ in their membership (whether they bought earlier) and do not get a choice to pick the alternative product. The second versioning problem is as follows: in a later period, observe the consumers who have bought the earlier period product. Within these consumers, some consumers upgrade to the new version and hence have made use of a product of higher quality in a later period while the consumers who do not upgrade have made use of a product of lower quality. Thus, these two sets of consumers can be considered to have used two different versions of the product. Hence, an understanding of the versioning literature adds value to our work. The academic literature in this area begins with Mussa & Rosen(1978). With the advent of the information goods industry (which includes software) in the late 90s, the topic of versioning has received renewed interest. For instance, popular business literature such as Shapiro & Varian(1999) advocates versioning through quality degradation as an important strategy for firms. However, academic research in this context has provided mixed results. Bhargava & Choudhary(2001) show that for information goods with zero marginal costs of production (as in software), versioning is not optimal for firms with a standard vertical differentiation model. Krishnan & Zhu(2006) show that in the case of development intensive products with negligible marginal costs, versioning may occur as a result of differentiation along multiple dimensions and not quality degradation along one dimension. In recent times, the lack of versioning at optimality when quality is unidimensional has been attributed to the structure of the utility function. Bhargava & Choudhary(2008) and Anderson & Dana(2008) both show that versioning is observed only when the following condition holds (necessary but not sufficient): higher willingness to pay (WTP) consumers value the upgrade strictly higher relative to the older version than lower WTP consumers. Anderson & Dana(2008) refer to this condition as the *increasing percentage differences condition.* As we will see later, the existence of an upgrade cost is critical to satisfying the *increasing percentage differences condition* and this ensures that some consumers do not upgrade to the new version. This results in two intertemporal versions of the product coexisting in the market even though the firm does not offer those two versions in the same period.
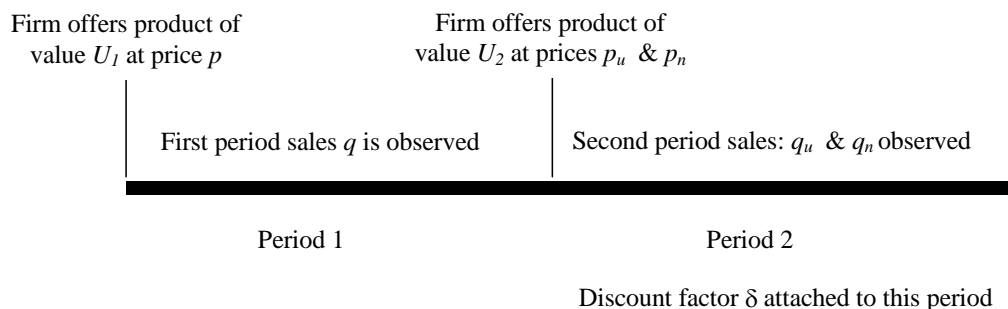
Firm offers product of value U_1 at price p	Firm offers product of value U_2 at prices p_u & p_n
First period sales q is observed	Second period sales: q_u & q_n observed

| Period 1 | Period 2 |

Discount factor δ attached to this period

Figure 2: Sequence of events

3   The Model

First, a brief description of the two-period model; Figure 2 illustrates. In the first period, the firm selects a price p at which it offers the initial version of its software. In the second period, the firm offers an improved version at price p_n; however, consumers who purchased the initial version are allowed to upgrade to the improved version by instead paying a price p_u which is lower than or equal to p_n (but does not exceed p_n). Consumers who decide to purchase the initial version enjoy the software for both periods, and, if they choose to upgrade, they use the improved version in the second period. Consumers who only purchase in the second period enjoy only one period of use, but it is of the improved version. Of course, some (potential) consumers may choose not to buy in either period. Note that the firm offers only one version of the product in either period. This assumption may not be a perfect match with reality but ensures that we are able to focus on the issues under consideration: the relationship between product improvement across time and the pricing decision. Consumers in the second period do not have the option of purchasing from a second hand market. This assumption stems from the software industry context which is characterized by the near absence of secondary markets where used goods can be sold. In part, this is due to the strict licensing agreements of software. But this also results from low utility for consumers from old versions of the product in the face of rapid sequential innovation. At this point, it is important to recognize that the incorporation of secondary markets in the analysis could change the results significantly, but is not that relevant in the software industry context.

Consumers: Consumers are heterogenous with respect to the value they ascribe to the software; each is of a particular "type" that is denoted by θ, and the set of all consumers is represented as an atomless spread of θ values that are distributed over the interval [0,1]. The cumulative distribution function is given by $F(\theta)$ which we typically assume to be a uniform distribution although some of our results can be extended to the larger class of IGFR distributions.

The software product: The critical modelling issue here is to distinguish between the two versions of the software with respect to the consumers' purchasing decisions. The initial and improved version are represented by scalars U_1 and U_2 respectively (the subscripts denote the period in which they are available) with $U_1 < U_2$; a consumer of type θ is assumed to enjoy a baseline utility of $u(\theta, U)$ from using a product of value U. We assume the functional form of $u(\theta, U)$ to be of the following type:

$$u(\theta, U) = \theta U$$

This utility function is in the tradition of Mussa & Rosen(1978). However, unlike earlier models, we assume that users incur a cost when they upgrade. This cost is not captured by special upgrade pricing. This cost is different depending on whether the user upgrades to the new product from an old product or has never purchased an older version before. Greater the jump in features in the new product as compared to the current version that the consumer holds, greater is the upgrade cost for the user. Denoting the upgrade cost by C_u:

$$C_u(U_O, U_N) = \alpha \cdot (U_N - U_O)$$

When a consumer upgrades from product of value U_O (could be zero) to U_N, the user upgrade cost per unit increase in product features is α. This unit upgrade cost is common to all users, irrespective of the willingness to pay (represented by consumer type θ). This fits a setting where all users need to have the same hardware configuration to run the software effectively and hence require a similar upgrade (α) on the hardware setup irrespective of how much they value the software product itself (θ). One question that arises at this stage is whether every user is compelled to incur the upgrade cost whenever a purchase is made. Relaxing this assumption is certainly possible but adds another dimension to the model. Currently, a customer pays an upgrade cost $\alpha \cdot (U_N - U_O)$ in order to enjoy a utility of θU_N as opposed to θU_O. If a customer was given the option to not incur the upgrade cost, the performance of the software may be compromised, thus resulting in a utility of $\theta \beta U_N$

where $\beta < 1$ is a deterioration parameter. This increases the number of potential cases significantly without adding value to some of the key insights we seek to develop. Thus, in all further analysis, we assume that all users will pay the upgrade cost if they wish to purchase the upgrade.

Structure of the game: This is thus a "game" between the firm and its potential consumer base. We assume complete information and employ the subgame perfection equilibrium concept. We model a setting where the firm is committed to offering what is considered "state-of-the-art" at any given point in time. Thus, the firm's selection of U_1 is determined by the state of technology at the beginning of period 1 and its selection of U_2 is constrained by the rate of technological evolution. As explained in the introduction, this rate of evolution is common knowledge to both the firm and the consumers in the software industry. Hence the values of U_1 and U_2 are known to both the firm and the consumers at the beginning of stage 1. Thus, this model has four decision stages: (1) firm selects price p; (2) consumers make first period purchase decisions; (3) firm selects prices p_n and p_u; and (4) consumers make second period purchase/upgrade decisions. As described before, the costs involved in developing the product are fixed costs that occur before the pricing decision in any period. As such, they play no role in the equilibrium pricing structure and hence, for the sake of brevity, we omit them in further analysis. This is similar to Fudenberg and Tirole(1998). As is standard practice, we solve for this game recursively using backward induction.

Arrangement of consumer segments: Although we need to solve recursively using backward induction, an understanding of the arrangement of consumer segments is necessary. So we begin by deriving consumers' two-period buying decisions with all prices (p, p_n, and p_u) presumed known. Four purchasing options are thus available to each consumer; these are listed below together with identifying labels and expressions for u_i, the surplus (utility net of price and upgrade cost) that a consumer derives from selecting the option. We assume a discount factor δ for future time periods and assume this to be common to both utilities and prices. It is also common to both the firm and the consumers. The four purchasing options are:

1. *"decline/decline"* – Buy nothing – surplus $u_1 = 0$

2. *"buy/decline"* – Buy the initial version in period 1 and use that version in both periods (i.e., decline to upgrade) – surplus is $u_2 = \theta(1 + \delta)U_1 - \alpha \cdot U_1 - p$

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3. *"buy/upgrade"* – Buy the initial version and then upgrade in second period – surplus is

$$u_3 = \theta \cdot (U_1 + \delta U_2) - \alpha \delta (U_2 - U_1) - \alpha \cdot U_1 - p - \delta p_u$$

4. *"decline/buy"* – Delay purchase until second period and then buy improved version – surplus is

$$u_4 = \theta \delta U_2 - \alpha \delta U_2 - \delta p_n$$

At equilibrium, consumers select the most fruitful of these options; i.e., each selects the option that provides surplus of

$$\max(u_1, u_2, u_3, u_4). \tag{1}$$

Based on this analysis, we can derive different cases for the arrangement of consumer segments. This is stated in the next lemma.

Lemma 1 *The arrangement of segments along θ for any set of prices is as follows:*

*1) The **decline/decline** segment is always located towards the extreme left consisting of low willingness-to-pay consumers.*

*2) The **buy/upgrade** segment is always located towards the extreme right consisting of high willingness-to-pay consumers.*

*3) The **decline/buy** and **buy/decline** segments are located between the above two segments. Their relative position depends on the relationship between $\frac{U_1}{U_2}$ and δ (Refer figures 3 and 4):*

*a) When $\frac{\delta}{1+\delta} \leq \frac{U_1}{U_2} < 1$ (minor to moderate upgrades) the **decline/buy** segment is to the **left** of the **buy/decline** segment.*

*b) When $0 < \frac{U_1}{U_2} < \frac{\delta}{1+\delta}$ (large upgrades) – the **decline/buy** segment is to the **right** of the **buy/decline** segment.*

The above lemma clarifies the relative positions of the segments for any given prices. Of course, any one of the segments defined above could collapse to zero. Further, arranging the segments in
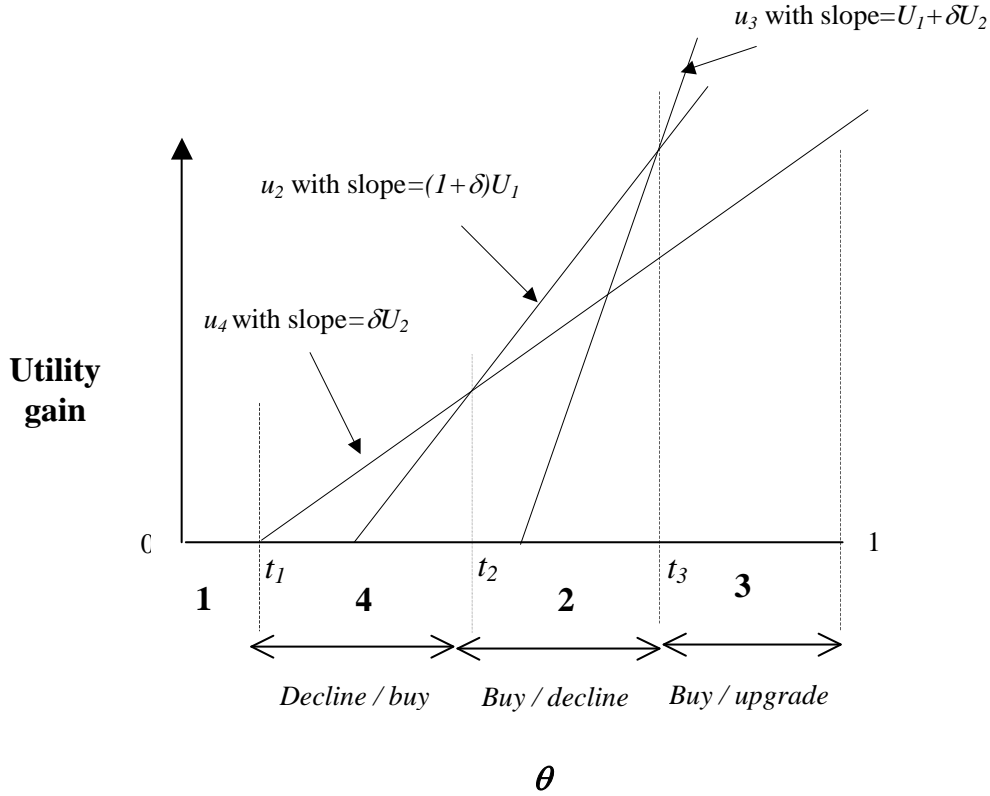
Figure 3: Ordering of customer segments (moderate upgrades)

a particular order enables the computation of the demand function for any given prices and thus enables the calculation of equilibrium prices for a given set of primitive parameters. Label t_1, t_2 and t_3 as the cut-off points that separate the different segments and are arranged by definition in increasing order with $t_1 < t_2 < t_3$. However, since the ordering of the segments is different across the minor/moderate and large upgrade cases, the expressions for the cut-off points are different in the two cases. These cutoffs are used to demarcate the segments in figures 3 and 4. An interesting implication is the ordering of segments depending on whether upgrades are large or minor/moderate. When upgrades are minor/moderate, the decline/buy segment is to the left of the buy/decline segment. This means that the buy/decline segment and the buy/upgrade segment are adjacent to each other ensuring that the first period demand constitutes a contiguous segment in θ space. For large upgrades, the first period demand is no longer contiguous in θ space.

The above lemma and the figures elaborate on the consumer's decision problem. We now describe the firm's decision problem.

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u_3 with slope$=U_1+\delta U_2$

u_4 with slope$=\delta U_2$

u_2 with slope$=(1+\delta)U_1$

**Utility
gain**

O

1

1

t_1

2

t_2

4

t_3

3

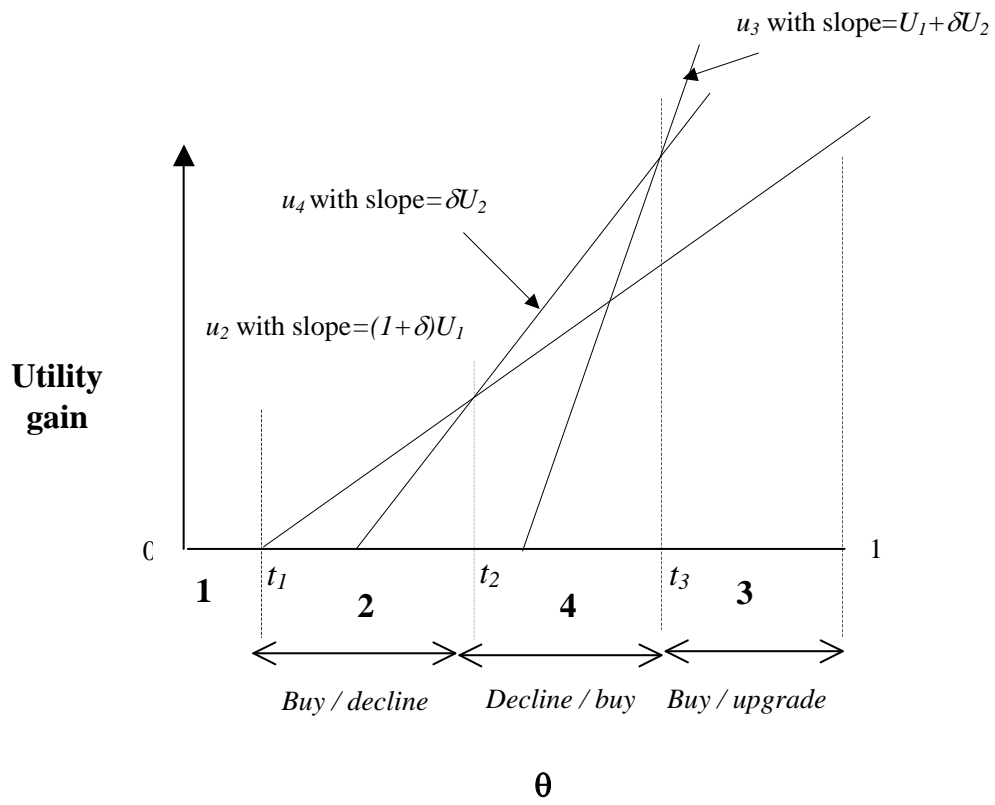*Buy / decline* *Decline / buy* *Buy / upgrade*

θ

Figure 4: Ordering of customer segments (large upgrades)

Producer's profit functions for both periods: Let π_i denote the firm's period-i revenue. The firm's second period revenue is:

$$\pi_2 \triangleq p_u q_u + p_n q_n \tag{2}$$

$$subject\ to\quad 0 \leq q_u \leq q$$

$$0 \leq p_u \leq p_n$$

$$0 \leq q_n \leq 1 - q$$

where q is the first period demand, q_n is the demand for the second period product by consumers who do not own the previous version, and q_u is the number of first period consumers who upgrade to the new version. In addition to non-negativity, the constraints here ensure that only consumers who purchased the initial version will upgrade and that the upgrade price does not exceed the second period purchase price. The second period pricing constraint is reasonable for most cases of off-the-shelf software purchase. It may not hold when consumers buy an operating system pre-installed because in such cases, the price paid for the software by new users as part of the bundle may be smaller than the open market upgrade price that old users pay. We restrict attention to open market software purchases that may require compatible hardware upgrades but do not come pre-installed with the hardware purchase. This still leaves open a large number of examples that would come under the purview of our model.

Finally, an equilibrium is established by a set of prices that maximizes total profit over two periods, π:

$$\pi \triangleq pq + \delta \pi_2^* \tag{3}$$

$$subject\ to\quad 0 \leq q \leq 1$$

$$p \geq 0$$

where π_2^* is equilibrium profit for the firm in the second period subgame.

4 Equilibrium Analysis with $\theta \sim Uniform[0,1]$

Now that we have set up the consumer's decision problem and the producer's objective function, we are ready to analyze the equilibrium consumer behavior and pricing structure. However, to

keep the analysis tractable without losing insight, we restrict the consumer type to be uniformly distributed. This assumption will be relaxed in the next section. We begin with an analysis of large upgrades and then move on to moderate upgrades. Note that all prices and demands at equilibrium are denoted with an asterisk.

4.1 Large Upgrades

While the arrangement of consumer segments was described previously, the analysis of subgame perfection also requires imposition of second period equilibrium conditions. These conditions basically ensure that the firm has no incentive to defect from its first period position (since that would mean that the consumers would anticipate such a defection and change their behavior accordingly). Applying these conditions and solving for subgame perfect equilibria provides us with the final solution. Define α^* by the following expression:

$$\alpha^* = Min \left[Max \left[\frac{8x - 8x^2 - \delta + 9x\delta - 12x^2\delta + 4x^3\delta}{8x - 8x^2 - \delta + 21x\delta - 20x^2\delta + 4x^3\delta}, 0 \right], 1 \right]$$

where $x = \frac{U_1}{U_2}$. We state the equilibrium results using α^* as a threshold.

Proposition 1 *When the firm offers a large upgrade $\left(0 < \frac{U_1}{U_2} < \frac{\delta}{1+\delta} \right)$, there exists a unique subgame perfect equilibrium in pure strategies with the following conditional outcomes:*

a) If $0 \leq \alpha < \alpha^*$, then the firm does not offer upgrade pricing but all old customers upgrade. There are some new consumers.

$$p_u^* = \ p_n^* \quad \& \quad q_u^* = q^* \quad \& \quad q_n^* > 0$$

b) If $\alpha^* \leq \alpha < 1$, then the firm is indifferent towards an upgrade pricing strategy since no new consumers buy the product. Not all old customers upgrade to the new product.

$$q_n^* = 0 \quad \& \quad q_u^* < q^*$$

Part $a)$ of the proposition is essentially a replay of the result from Kornish(2001). Thus, for large upgrades, when upgrade costs are low, we observe an equilibrium where the firm can credibly commit to future prices only when no upgrade pricing is offered. As the upgrade cost increases

beyond a threshold (which is a function of both product improvement and discount factor), it allows credible commitment even with upgrade pricing (although the firm is indifferent towards whether upgrade pricing should indeed be offered, there may be behavioral reasons to do so). This occurs because of the following: high upgrade cost acts as a truncation in the potential market for the product; this enables the firm to credibly convey its lack of incentive to pursue new but lower "willingness to pay" consumers in the second period, thus enabling the use of an upgrade price. From an empirical validation perspective, this provides us with the following testable hypothesis: if upgrade pricing is used by a firm even for large upgrades, it potentially indicates the presence of high upgrade costs. However, a point to note here is that the threshold α^* hits zero for very low $\frac{U_1}{U_2}$ (typically < 0.1). At such high levels of product improvement, the only possible equilibrium for the entire range of non-zero α is the one where upgrade pricing can be utilized. On the other hand, as δ approaches zero, α^* hits 1 and hence only the equilibrium without upgrade pricing is optimal for all values of the upgrade cost. In fact, α^* is decreasing with δ for any given $\frac{U_1}{U_2}$ as can be seen from the sign of the derivative (set $\frac{U_1}{U_2} = x$):

$$\frac{d\alpha^*}{d\delta} = -\frac{32x^2 (3 - 2x)(1 - x)}{(\delta + x(-8 - 21\delta + 4x(2 + \delta (5 - x))))^2} < 0$$

since $x = \frac{U_1}{U_2} < 1$. An increase in the discount factor causes a decrease in α^* and also increases the range over which an upgrade is categorized as large (since $\frac{\delta}{1+\delta}$ increases). This enables the firm to offer upgrade pricing over a larger region in $\left(\frac{U_1}{U_2}, \alpha\right)$ space. The reasoning behind this is as follows: as the emphasis on the second period increases (indicated by an increase in discount factor), the equilibrium which extracts greater revenue in the second period through upgrade customers becomes more favorable. Overall, we conclude that upgrade costs play a strong role in determining the nature of the equilibrium for large upgrades.

The next section addresses a similar analysis for minor/moderate upgrades.

4.2 Minor to Moderate Upgrades

We analyze the equilibrium outcomes when the firm offers a minor to moderate upgrade.

Proposition 2 *When the firm offers a minor/moderate upgrade $\left(\frac{\delta}{1+\delta} \leq \frac{U_1}{U_2} < 1\right)$,*
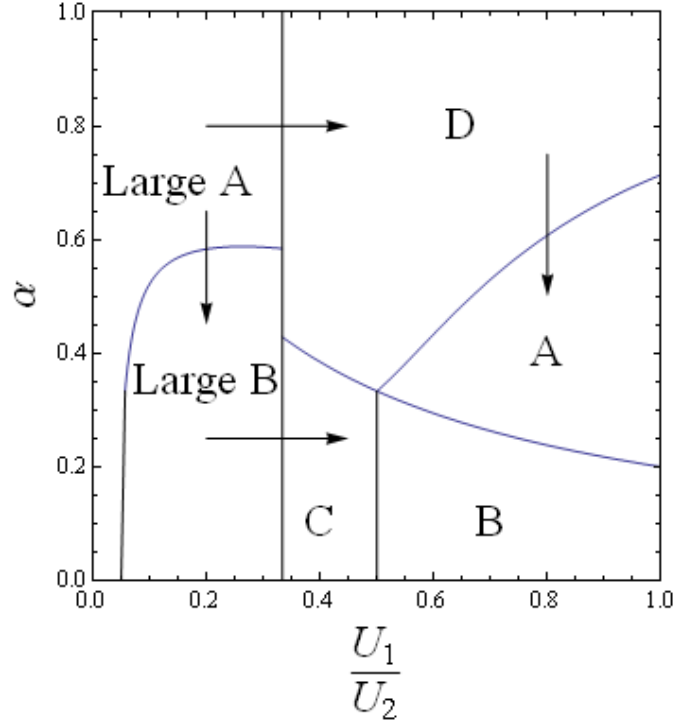
Figure 5: Equilibrium zones in $\left(\frac{U_1}{U_2}, \alpha\right)$ space for $\delta = 0.5$

1) *There is a unique subgame perfect equilibrium in pure strategies.*

2) *There exist 4 regions in $\left(\frac{U_1}{U_2}, \alpha\right)$ space such that the following constraint possibilities are observed:*

$$High \; \frac{U_1}{U_2}, \; High \; \alpha \; = \; Zone \; A = \{p_u^* < p_n^*, \; q_u^* < q^*\}$$

$$High \; \frac{U_1}{U_2}, \; Low \; \alpha \; = \; Zone \; B = \{p_u^* < p_n^*, \; q_u^* = q^*\}$$

$$Low \; \frac{U_1}{U_2}, \; Low \; \alpha \; = \; Zone \; C = \{p_u^* = p_n^*, \; q_u^* = q^*\}$$

$$Low \; \frac{U_1}{U_2}, \; High \; \alpha \; = \; Zone \; D = \{p_u^* = p_n^*, \; q_u^* < q^*\}$$

The different zones and the thresholds that define them are described in figure 5 for the specific case of $\delta = 0.5$.

Each of the labels A, B, C & D is described in proposition 2. The label "Large A" corresponds to the equilibrium outcome in the large upgrade case when $\alpha > \alpha^*$ while the label "Large B" corresponds to the equilibrium outcome when $\alpha \leq \alpha^*$. When the upgrade cost is low (including

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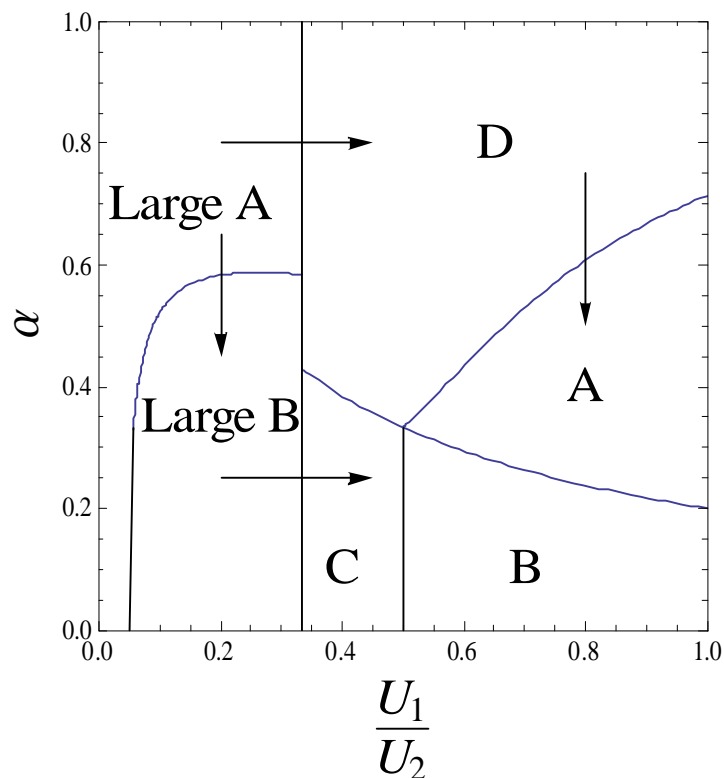
the case $\alpha = 0$), the firm is able to price at equilibrium such that all old customers upgrade and hence "Large A" applies. Thus, upgrade cost is a critical factor in determining whether all old customers upgrade. Within minor/moderate upgrades, whether upgrade pricing will be offered or not depends on the level of product improvement. If product improvement is minor, the firm has to offer an upgrade discount in order to get older (but higher type) consumers to shift to the new product. But such a strategy is not completely effective when upgrade costs are high (Zone A). If product improvement is not minor but moderate, the optimal unconstrained upgrade price exceeds the unconstrained new price offered to newer (but low type) consumers. Consequently, the arbitrage constraint is violated and the firm has to offer the same price to all consumers in period 2 (Zones C & D). However, when upgrade costs are low, the firm hits the upper limit on the number of upgrade consumers and this changes the product improvement threshold at which upgrade pricing becomes optimal (Zone B). A general observation based on combining the results for minor, moderate and large upgrades is that at high upgrade costs $(\alpha > \alpha^*)$ an upgrade pricing strategy is not optimal for a middle range of product improvement (zones "Large B", C & D).

The arrows indicate movement of the thresholds with an increase in the discount factor δ. Based on the equilibrium analysis, we can compare how these regions change for minor/moderate improvements with a change in the discount factor δ. A similar analysis for large upgrades has already been described right after proposition 1.

Proposition 3 *When the firm offers a minor/moderate upgrade $\left(\frac{\delta}{1+\delta} \leq \frac{U_1}{U_2} < 1\right)$, the discount factor δ affects the upgrade pricing decision in the following way:*

1) When α is low enough such that all old customers upgrade to the new product, increase in the discount factor has no impact on the upgrade pricing decision (Zone B does not change) but has a negative impact on the decision to not offer upgrade pricing (Zone C shrinks). At $\delta = 1$, Zone C completely disappears.

2) When α is high enough such that not all old customers upgrade to the new product, there exists a region in $\left(\frac{U_1}{U_2}, \alpha\right)$ space such that the firm does not offer upgrade pricing even when $\delta = 1$ (Zone D area > 0).

An increase in the value of the discount factor indicates a greater value attached to future outcomes. When α is low and all old customers can be made to upgrade, the ability to price discriminate between old and new consumers is not impacted by the discount factor. This ability to price discriminate is significantly muted when upgrade cost is high and hence it is possible to see no upgrade pricing even when $\delta = 1$.

Figures 6 and 7 tabulate the closed form expressions for market sizes and prices respectively in the different zones. A quick scan of the above tables reveals the general trends in market sizes and prices. For instance, the number of new consumers is zero only for the large upgrade case with high upgrade costs. In all other cases, it has a negative relationship with respect to first period sales. These expressions can be used to numerically compute the equilibrium profit for the firm with different parameter values. Figure 8 plots the profit of the firm under 3 different scenarios. The three scenarios are evaluated at $\frac{U_1}{U_2} = 0.1$, 0.45 and 0.8 respectively. These scenarios cover the three possible upgrade cases: large, moderate and minor respectively. For a given set of parameters $\frac{U_1}{U_2}$ and α, the profit is an increasing function of U_2. In order to make the three cases comparable, we set $U_2 = 1$. Thus, the maximum possible profit in each of the three cases is equal. However,

Zones	q^*	q_u^*	q_n^*
A	$\dfrac{2[(1+\delta)U_1-\delta U_2]+2\alpha(\delta U_2-U_1)}{4(1+\delta)U_1-3\delta U_2}$	$\dfrac{1-\alpha}{2}$	$\dfrac{1-q^*-\alpha}{2}$
B	$\dfrac{2(1-\alpha(1-\delta))U_1}{4U_1+\delta U_2}$	q^*	$\dfrac{1-q^*-\alpha}{2}$
C	$\dfrac{2\delta U_1^2+(1-3\delta)U_1U_2+\delta U_2^2-\alpha\left(2\delta U_1^2+(1-4\delta)U_1U_2+\delta U_2^2\right)}{2\left(\delta U_1^2+(1-2\delta)U_1U_2+\delta U_2^2\right)}$	q^*	$\dfrac{(1-q^*-\alpha)U_1}{U_2}$
D	$\dfrac{2(2U_2-U_1)[(1+\delta)U_1-\delta U_2+\alpha(\delta U_2-U_1)]}{(8+11\delta)U_1U_2-4(1+\delta)U_1^2-7\delta U_2^2}$	$\dfrac{2(1-\alpha)(U_2-U_1)+q^*\cdot U_2}{2(2U_2-U_1)}$	$\dfrac{2(1-\alpha)U_2-q^*\cdot(3U_2-U_1)}{2(2U_2-U_1)}$
Large($\alpha<\alpha^*$)	$\dfrac{2(1-\alpha)(U_2-U_1)}{3U_2-2U_1}$	q^*	$\dfrac{2(1-\alpha)U_2-q^*\cdot(3U_2-U_1)}{2U_2-U_1}$
Large($\alpha\geq\alpha^*$)	$1-\alpha$	$\dfrac{1-\alpha}{2}$	0

Figure 6: Equilibrium market sizes for different conditions

Zones	p^*	p_u^*	p_n^*
A	$\dfrac{(1-q^*)[2(1+\delta)U_1-\delta U_2]+\alpha(\delta U_2-2U_1)}{2}$	$\dfrac{(1-\alpha)(U_2-U_1)}{2}$	$\dfrac{(1-q^*-\alpha)U_2}{2}$
B	$\dfrac{(1-q^*)[2(1+\delta)U_1-\delta U_2]+\alpha(\delta U_2-2U_1)}{2}$	$(1-q^*-\alpha)(U_2-U_1)$	$\dfrac{(1-q^*-\alpha)U_2}{2}$
C	$(1-q^*)U_1-\alpha(1-\delta)U_1$	$(1-q^*-\alpha)(U_2-U_1)$	$(1-q^*-\alpha)(U_2-U_1)$
D	$\dfrac{2(1-q^*)[(1+\delta)U_1-\delta U_2]}{2}+$ $\dfrac{\delta[2(1-\alpha)-q^*]\cdot[U_2(U_2-U_1)]}{2(2U_2-U_1)}+\dfrac{\alpha(\delta U_2-U_1)}{2}$	$\dfrac{[2(1-\alpha)-q^*]\cdot[U_2(U_2-U_1)]}{2(2U_2-U_1)}$	$\dfrac{[2(1-\alpha)-q^*]\cdot[U_2(U_2-U_1)]}{2(2U_2-U_1)}$
Large($\alpha<\alpha^*$)	$(1-q^*)U_1-\alpha(1-\delta)U_1$	$\dfrac{(1-\alpha)U_2(U_2-U_1)}{3U_2-2U_1}$	$\dfrac{(1-\alpha)U_2(U_2-U_1)}{3U_2-2U_1}$
Large($\alpha\geq\alpha^*$)	$\alpha\delta U_1$	$\dfrac{(1-\alpha)(U_2-U_1)}{2}$	Arbitrarily high $\geq p_u^*$

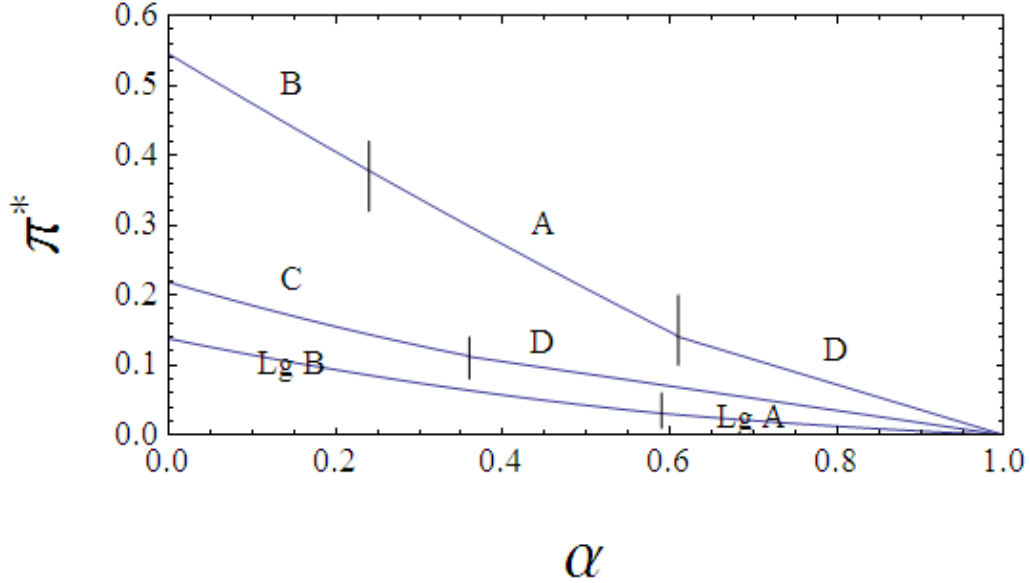Figure 7: Equilibrium prices for different conditions

Figure 8: Firm profit under different conditions

the point to compare would be the extent to which the firm can capture this profit at equilibrium given the strategic behavior of customers. Given a value of U_2 and the ability to modify $\frac{U_1}{U_2}$ and/or α (if possible), what strategy would prove profitable to the firm and what pricing mechanism does the firm use to achieve this profitability? For further ease of comparison, the $\frac{U_1}{U_2}$ values are chosen such that the jump in values across the three cases is equal to 0.35. At the three given values of $\frac{U_1}{U_2}$, the equilibrium traverses through zones (Large B, Large A), (C, D) & (B, A, D) respectively as α increases. This fact can be easily verified from figure 5.

A direct observation from figure 8 is that in each of the three cases, the profit is a decreasing function of α. This should not be a surprise since an increase in α results in a decrease in utility and hence exerts pressure on the ability of the firm to extract consumer surplus through prices. Of more interest is how profit varies depending on the level of $\frac{U_1}{U_2}$. Profit increases as $\frac{U_1}{U_2}$ increases: the highest curve corresponds to $\frac{U_1}{U_2} = 0.8$ while the lowest curve corresponds to $\frac{U_1}{U_2} = 0.1$. Thus, minor upgrades provide higher profit as compared to either moderate or large upgrades, given that the eventual second period product U_2 is the same in all three cases. This indicates that given a target quality level in the second period, offering a significant fraction of the eventual product quality in the first period reduces the impact of strategic customer behavior and allows the firm to charge higher prices. Hence the firm is better off increasing $\frac{U_1}{U_2}$ if possible given a target value of U_2, at all

values of the upgrade cost α.

Although the profit is increasing in $\frac{U_1}{U_2}$, the increase is non-linear. Although $\frac{U_1}{U_2}$ jumps by only 0.35 across the three cases, the jump in profit is substantial as we move from moderate to minor upgrades. Note that the significant difference in pricing strategy between moderate and minor upgrades is the ability to offer upgrade prices. Thus, the ability to offer upgrade prices significantly benefits the firm's profitability. This is true for a large range of α but as α increases further, the firm moves into region D where upgrade pricing is not feasible and this benefit dissipates and eventually disappears as α reaches 1. Combining the above observations reveals that decreasing α and increasing $\frac{U_1}{U_2}$ provide the firm with the ability to offer upgrade prices and hence increase profitability given a target quality level U_2 in the second period.

The above numerical results address the impact of various parameters on the profitability of the firm but do not reveal the overall market coverage impact of upgrade pricing. For instance, a public authority such as the government may be interested in ensuring that a particular software technology is used by a larger percentage of the population (irrespective of whether a specific consumer uses the older or later version). In such cases, it is important to find out the impact of upgrade pricing on the overall market coverage (given by $q^* + q_n^*$). Why might one expect upgrade pricing to have an impact on this decision? When upgrade pricing is disallowed, new consumers are likely to get a lower price (as compared to the situation when upgrade pricing is allowed) and hence more of them may purchase the product. This contributes to an increase in market coverage. However, old customers are likely to get a higher upgrade price (since they no longer get a special discount). Given that consumers have foresight, they may choose not to purchase in the first period if they do not anticipate the appropriate upgrade prices. Consequently, this leads to reduced market coverage. The direction of the ultimate outcome in terms of market coverage is not clear. Thus, while disallowing upgrade pricing will definitely lead to lower profits for the firm (since we are adding a binding constraint to a concave optimization problem), the impact on market coverage needs to be examined. The approach towards evaluating this is to find the market coverage when upgrade pricing is disallowed for those cases where upgrade pricing is optimal for the firm. In order to keep the analysis tractable, we evaluate this at $\delta = 1$. Further, we analyze this only for minor/moderate upgrades since even when $\alpha > \alpha^*$, the firm is indifferent towards upgrade pricing for large upgrades and hence disallowing upgrade pricing has no real impact on

market coverage. The next proposition details the results.

Proposition 4 *When the firm offers a minor to moderate upgrade, disallowing upgrade pricing **increases** the overall market coverage irrespective of the magnitude of upgrade cost parameter α.*

Note the difference in outcomes depending on the level of product improvement. For large upgrades, disallowing upgrade pricing for the case $\alpha > \alpha^*$ does not affect market coverage since the firm can set the two prices equal without any loss in profit (since the firm earns its entire second period revenue from upgrade consumers). This is no longer true for minor to moderate upgrades, where the loss in first period consumers is more than compensated by the acquisition of new consumers due to lower second period price.

An observation discussed earlier is that there exists a threshold on α such that when α is below this threshold, we find that all old customers upgrade to the new product. The next section analyzes this result by generalizing consumer type beyond the uniform distribution.

5 Equilibrium Analysis for Minor to Moderate Upgrades with $\theta \sim IGFR[0,1]$

As stated in the previous section, we extend the results to IGFR distributions with a study of consumer upgrade behavior as the primary motive. The next proposition extends previous results for the minor/moderate upgrade case to IGFR distributions on θ.

Proposition 5 *When the firm offers a minor to moderate upgrade*

1) At any first period price p, there exists a unique pure strategy equilibrium in the second period subgame. At this equilibrium, there exists a threshold on $\frac{U_1}{U_2}$ above which the firm offers an upgrade price strictly lower than the new price:

$$p_u^* < p_n^*$$

2) There exists a subgame perfect equilibrium in pure strategies. At this equilibrium, $\alpha > 0$ is a necessary but not sufficient condition to ensure that not all old customers upgrade to the new

product.

$$q_u^* < q^*$$

Fudenberg and Tirole (1998) have analyzed the above results in the absence of upgrade costs and find that no leapfrogging occurs when production is costless. They define leapfrogging as a situation in which not all old customers upgrade and some lower type consumers buy the new version of the product directly. Their result does not reflect reality in the sense that software products fit the example of costless production and yet frequently display leapfrogging in their consumer markets. Our result provides a viable explanation for the presence of leapfrogging (which requires that not all old customers upgrade) in software markets by showing that a necessary condition for this to occur is the presence of positive user upgrade costs. This can be further verified by borrowing from the versioning literature. Bhargava & Choudhary(2001b) and Anderson & Dana(2006) both show that versioning is observed only when the following condition holds (necessary but not sufficient):

$$\frac{\partial}{\partial \theta} \left(\frac{u(\theta, U_H)}{u(\theta, U_L)} \right) > 0 \qquad (4)$$

where U_L is the lower value version and U_H is the higher value version. This condition basically states that higher type θ customers value the upgrade strictly higher relative to the older version than lower type θ customers. Anderson & Dana(2006) refer to this condition as the *increasing percentage differences condition.* This condition can never hold for a utility function that is separable in θ and U since:

$$\frac{u(\theta, U_H)}{u(\theta, U_L)} = \frac{f(\theta) \cdot g(U_H)}{f(\theta) \cdot g(U_L)} = \frac{g(U_H)}{g(U_L)} \text{ independent of } \theta$$

In the second period, observe customers q who buy the first period product U_1. Within these q customers, the q_u customers who upgrade to U_2 have made use of a product of quality U_2 in period 2 while the $(q - q_u)$ customers who do not upgrade have made use of a product of quality U_1 in period 2. Thus, these two sets of customers can be considered to have used two different versions of the product. This also implies that the condition $q_u < q$ is equivalent to the condition where two different versions are sold to the q customers. When $q_u = q$, offering two versions is not optimal for the firm. Further, these customers have already incurred the upgrade cost in period 1 and do not have to incur any further upgrade cost if they simply use U_1. Checking for the versioning condition specified in inequality (4):

$$\frac{u(\theta, U_2)}{u(\theta, U_1)} = \frac{\theta \cdot U_2 - \alpha(U_2 - U_1)}{\theta U_1} = \frac{U_2}{U_1} - \left(\frac{\alpha}{\theta} \right) \left(\frac{U_2 - U_1}{U_1} \right)$$

$$\frac{\partial}{\partial \theta}\left(\frac{u(\theta, U_2)}{u(\theta, U_1)}\right) = \left(\frac{\alpha}{\theta^2}\right)\left(\frac{U_2 - U_1}{U_1}\right) > 0 \text{ for } \alpha > 0$$

The increasing percentage differences condition (which requires $\alpha > 0$ in our model) is a necessary condition for versioning but is not sufficient. This corresponds exactly to the result shown in Proposition 5.

6   Concluding Remarks

Firms have adopted different strategies in dealing with strategic consumer behavior caused by rapid technological innovation. One such strategy is for firms to price discriminate by offering old customers special upgrade prices. The objective of our research is to understand whether this is effective across all levels of product improvement while incorporating knowledge about the consumers' willingness-to-pay and operational issues such as user upgrade costs. Our results reveal that offering upgrade prices is not always optimal. Upgrade pricing is useful for large upgrades in the context of high user costs by enabling credible commitment to a strategy where no new consumers are acquired. It is also effective when upgrades are minor, since old customers have to be given an incentive to adopt upgrades that are only minor improvements over older versions. However, for moderate upgrades, special upgrade pricing is not effective and the firm is better off offering the same symmetric price to all consumers. Thus, when user upgrade costs are high, upgrade pricing is not effective for a middle range of product improvement. The exact product improvement threshold at which upgrade pricing becomes effective is a function of whether all old customers upgrade to the new product. This upgrade behavior is governed by the user upgrade cost. Hence, user costs also play an important indirect role in the effectiveness of upgrade pricing.

Upgrade pricing also has a significant impact on profitability. If the firm had a fixed quality target to achieve over a longer horizon, the ability to offer upgrade prices ameliorates strategic consumer behavior and increases profits significantly. However, making upgrade pricing viable requires some flexibility in the quality offered in earlier time periods in addition to the ability to reduce the upgrade costs. If an important social objective is to increase the overall market coverage for a particular type of product (this is analogous to decreasing the deadweight loss), evaluating the impact of upgrade pricing on coverage is important. Our analysis reveals that disallowing upgrade pricing does not change market coverage when the firm offers a large upgrade while it increases

coverage when the firm offers a minor to moderate upgrade. This provides a prescription for policy makers who can make the appropriate decisions.

We apply our results to some of the examples that were discussed in the introduction. For example, almost everyone agrees that Vista comes with significant hidden upgrade costs and Microsoft certainly claims that Vista is a large upgrade over XP. Given that they also offer upgrade pricing, it is a reasonable conjecture that "new sales" of Windows Vista are negligible. Essentially, a consumer who postponed purchase of Windows XP is unlikely to buy a new copy of Vista. At the same time, not all XP users are likely to upgrade to Vista (a situation not far from reality). Any new sales for Vista can occur due to an increase in the potential market caused by the arrival of a new cohort of consumers. We do not model such a scenario but this would be a productive agenda for future research. On the other hand, our experience with Matlab reveals low upgrade costs. Yet, a new version of Matlab is offered at the same price to all consumers. Our results indicate that this would happen only when the upgrade is moderate or large and that most old customers are likely to upgrade to the new version.

While the evidence in the preceding paragraph is merely anecdotal, it would be worthwhile to empirically evaluate the validity of the model conclusions. Consumer upgrade costs and the level of product improvement between any two versions could both be determined by polling relevant experts who evaluate such products. For example, the introduction of this paper lists the upgrade cost that a user incurs when purchasing Windows Vista. This upgrade cost was evaluated by *PC World*. Similar figures could be estimated for other products. As for product improvement, websites such as zdnet.com and cnet.com list ratings on a 10-point scale for various products. Recent empirical studies in this area such as Ghose & Sundararajan(2007) use such ratings to evaluate quality differences between versions. In order to test hypotheses concerning these parameters and the equilibrium upgrade pricing / consumer behavior, we need pricing and consumer upgrade information. Pricing information is relatively easy to get by observing prices at retailers, both online and "brick and mortar". Consumer upgrade information is available with the firm (which aggregates this information across retailers). While we make no claims about the ease of getting such data, it is not impossible to conceive of a study which evaluates the implications of our model.

As always, our results are restricted by the reality of our assumptions. Restricting the analysis to the software industry might seem to make the problem domain narrow since upgrades can and

do occur in many other industries and product categories. Some of the results derived can be generalized to other industries but with some caution. The reason for this caution rests on the key differences that software product markets display when compared to that of other products. For software products, the variable cost of producing a single copy of the product is negligible. Most of the costs are embedded in the fixed costs of development and testing. Another important aspect of our model setup is that many of our assumptions about the market stem from observations of individual consumers of software. Clearly, enterprise contexts may deviate from these conditions and may lead to different results. Most of our results can be interpreted clearly in the individual user context. One potential area of research is to generalize the information structure available to the consumer in terms of product improvement and future prices. Consumers may not have complete information about future producer strategies. Future research will have to explore the mechanisms that firms and consumers will use to convey and acquire such private information respectively.

7   Acknowledgement

8   Appendix

Proof of Lemma 1

Each of the functions inside the maximization equation (1) is linear, increasing in θ, and decreasing in the prices. Thus, the function (1) is convex non-decreasing piecewise linear in θ with at most four pieces. This implies:

- The $[0,1]$ segment is partitioned into at most four intervals such that the consumers with θ values within each interval make the same purchase decision.

- The breakpoints between these intervals are determined by the prices together with U_1 and U_2.

- The length of each interval is also a fraction of the potential consumers who select that option. Let M denote the total number of potential consumers. Let $[a,b] \subseteq [0,1]$ with $l = |b-a| \leq 1$ be an interval of length l. This corresponds to demand of $M \cdot (F(b) - F(a))$. For a uniform distribution on θ, this corresponds to $M.l$. For simpler analysis, this M is set identically equal to 1; this is a matter of scaling and is without loss of generality.

- The segments are arranged (from left to right along the θ axis) in increasing order of slope $\frac{\partial u_i}{\partial \theta}$. (as is standard in vertical differentiation models. See Mussa & Rosen(1978) for more details)

- The breakpoints between segments are at the θ values for which consumers would be indifferent between the options that correspond to two adjacent segments.

- These intervals can be defined by t_1, t_2, and t_3 with $0 \leq t_1 \leq t_2 \leq t_3 \leq 1$ where t_1 is the supremum θ of consumers who don't buy anything, t_3 is the infimum θ of consumers who both buy and upgrade, and t_2 is the point of indifference between the other two options. It is of course possible for any of the segments to be empty. This would be represented by one or more of the inequalities just above holding as equalities.

- Under our assumptions, the *decline/decline* interval is leftmost, the *buy/upgrade* interval is right-most, and the ordering of the *buy/decline* and *decline/buy* intervals depends on the ratio of U_1, and U_2.

Proof of Proposition 1

a) Given the consumer segmentation in figure 4, in period 2, the firm sees two discontinuous segments that bought the first period product ($[t_3, 1]$ and $[t_1, t_2]$). Consequently, the firm can optimally set the following prices in the second period:

$$p_u = t_3 \cdot (U_2 - U_1) - \alpha \text{ and } p_n = t_2 \cdot U_2 - \alpha(U_2 - U_1) \tag{5}$$

Next, we layout the conditions for rational consumer behavior in the first period for large upgrades by writing the indifference conditions for the three cut-off points t_1, t_2 and t_3:

$$t_1 = \frac{p}{(1+\delta)U_1} + \frac{\alpha}{1+\delta}, \quad t_2 = \frac{\delta p_n - p}{\delta U_2 - (1+\delta)U_1} + \alpha \left(\frac{\delta U_2 - U_1}{\delta U_2 - (1+\delta)U_1} \right) \tag{6}$$
$$t_3 = \frac{p - \delta(p_n - p_u)}{U_1} + \alpha(1 - \delta)$$

The expressions from (5) and (6) then solve in terms of a given p, to give $t_1 = t_2 = t_3$ which further implies that $q_u^* = q^*$ and $q_n^* = 0$. This violates the starting assumption that each of the four possible segments is non-zero. Hence, an equilibrium can only be evaluated by collapsing either one or both of the *buy/decline* and *decline/buy* segments. In either case, the first period sales q becomes a contiguous segment. The critical issue to be examined is whether the firm has an incentive in the second period to deviate from its earlier position once first period demand is revealed to be contiguous. We write out the second period profit function for the firm (given by equation (2) for any first period contiguous sales q:

$$\pi_2 = p_u \left(1 - \alpha - \frac{p_u}{U_2 - U_1} \right) + p_n \left(1 - q - \alpha - \frac{p_n}{U_2} \right)$$

Solving for optimal prices, we get:

$$p_u^* = (1 - \alpha) \left(\frac{U_2 - U_1}{2} \right)$$

$$p_n^* = (1 - q - \alpha) \left(\frac{U_2}{2} \right) \tag{7}$$

Substituting back into the demand function, we get the market sizes for the unconstrained problem:

$$q_u^* = \frac{1 - \alpha}{2} \tag{8}$$

$$q_n^* = \frac{1 - q - \alpha}{2} \tag{9}$$

An examination of these expressions reveals that we cannot have $q_u^* = q$ and $q_n^* = 0$ simultaneously for $\alpha < 1$. So only one of them will be satisfied.

Case 1: $q_u^* = q$

We set the expression for q_u given in equation (8) equal to q. This means that the firm sets first period price p such that given optimal pricing (as anticipated by consumers) in period 2, a demand of $q = \frac{1-\alpha}{2}$ is observed. Substituting for q in the price equation (7) for p_n^*:

$$p_n^* = \frac{1 - \alpha}{4} \cdot U_2 < (1 - \alpha) \left(\frac{U_2 - U_1}{2} \right) = p_u^* \text{ for large upgrades}$$

Hence, we must have a symmetric price across both segments. Setting $p_u^* = p_n^* = p_s^*$ in the second period profit function (given by equation (2)):

$$\pi_2 = p_s \left(2(1 - \alpha) - q - \frac{p_s}{U_2 - U_1} - \frac{p_s}{U_2} \right)$$

The optimal second period price is:

$$p_s^* = [2(1 - \alpha) - q] \left(\frac{U_2 (U_2 - U_1)}{2 (2U_2 - U_1)} \right)$$

The number of upgrade consumers is given by the demand function:

$$q_u^* = 1 - \alpha - \frac{p_s^*}{U_2 - U_1}$$

Substituting for p_s^* and solving for q by setting $q_u^* = q$ provides us with the equilibrium value for q:

$$q^* = \frac{2(1 - \alpha)(U_2 - U_1)}{3U_2 - 2U_1}$$

Since all old customers upgrade, we have the following: $t_1 = t_2$ and $t_3 = 1 - q$. Writing the first period indifference condition for the marginal consumer t_3:

$$(1 - q)\delta U_2 - \alpha\delta U_2 - \delta p_s^* = (1 - q)(U_1 + \delta U_2) - \alpha\delta(U_2 - U_1) - \alpha U_1 - p - \delta p_s^*$$

Solving for first period price p:

$$p^* = (1 - q^*)U_1 - \alpha(1 - \delta)U_1$$

All prices and quantities can now be computed. The overall profit function is:

$$\pi_{case1}^* = p^* q^* + \delta p_s^* (q^* + q_n^*) \tag{10}$$

Case 2: $q_n^* = 0$

We set the expression for q_n given in equation (9) equal to 0. This means that the firm sets first period price p such that given optimal pricing (as anticipated by consumers) in period 2, a demand of $q = 1 - \alpha$ is observed. Since $q_u = \frac{1-\alpha}{2}$, the constraint $q_u^* < q^*$ is satisfied. Also, no new consumers are acquired for any non-zero price p_n^*. Thus, raising price p_n^* such that the arbitrage constraint is satisfied does not affect firm profit. Hence p_n^* can be any value equal to or above $p_u^* = (1 - \alpha)\left(\frac{U_2 - U_1}{2}\right)$. Since no new consumers buy the product, we have the following: $t_2 = t_3$ and $t_1 = 1 - q$. Writing the first period indifference condition for the marginal consumer t_1:

$$0 = (1 - q)(1 + \delta)U_1 - \alpha U_1 - p$$

Simplifying, we get:

$$p^* = \alpha\delta U_1$$

All prices and quantities can now be computed. The overall profit function is:

$$\pi_{case2}^* = p^* q^* + \delta p_u^* q_u^* \tag{11}$$

Given that the firm sets the first period price p before consumers make purchase decisions, the firm can pick the equilibrium that provides higher profit by using first period price as a signal. Only a comparison of the profits across the two potential equilibria can reveal the optimal strategy for the firm. First, observe the following:

π_{case1}^* is a convex function of α

π^*_{case2} is either a convex or a concave function of α.

Setting $\pi^*_{case1} = \pi^*_{case2}$ and solving for α provides the following roots:

$$\alpha = \frac{8x - 8x^2 - \delta + 9x\delta - 12x^2\delta + 4x^3\delta}{8x - 8x^2 - \delta + 21x\delta - 20x^2\delta + 4x^3\delta}$$

where $x = \frac{U_1}{U_2}$ and

$$\alpha = 1$$

At $\alpha = 0$, $\pi^*_{case1} > \pi^*_{case2}$.

Define α^* such that:

$$\alpha^* = Min\left[Max\left[\frac{8x - 8x^2 - \delta + 9x\delta - 12x^2\delta + 4x^3\delta}{8x - 8x^2 - \delta + 21x\delta - 20x^2\delta + 4x^3\delta}, 0\right], 1\right]$$

Putting all these facts together, we get:

$$\pi^*_{case1} \geq \pi^*_{case2} \text{ for } \alpha \leq \alpha^*$$

$$\pi^*_{case1} < \pi^*_{case2} \text{ for } \alpha > \alpha^*$$

This gives us the required result.

Proof of Proposition 2: 1) We first solve this problem without regard for the constraints and then check to see that they are not violated. The profit function for second period given by equation (2):

$$\pi_2 = p_u\left(1 - \alpha - \frac{p_u}{U_2 - U_1}\right) + p_n\left(1 - q - \alpha - \frac{p_n}{U_2}\right)$$

which is concave as shown by:

$$\frac{\partial^2 \pi_2}{\partial p_n^2} = \frac{-2}{U_2} < 0 \text{ and } \frac{\partial^2 \pi_2}{\partial p_u^2} = \frac{-2}{U_2 - U_1} < 0$$

By inspection, π_2 is also separable in p_u and p_n, so solving it is a matter of setting the respective partials to zero.

Setting $\frac{\partial \pi_2}{\partial p_n} = 0$ gives p_n^*, the optimal[1] value for p_n and hence q_n^*, the optimal value for q_n:

$$p_n^* = (1 - q - \alpha) \frac{U_2}{2} \ ; \ q_n^* = (1 - q - \alpha) \frac{1}{2} \tag{12}$$

Setting $\frac{\partial \pi_2}{\partial p_u} = 0$ gives p_u^*, the optimal value for p_u and hence q_u^*, the optimal value for q_u:

$$p_u^* = (1 - \alpha) \left(\frac{U_2 - U_1}{2} \right) \text{ and } q_u^* = \frac{1 - \alpha}{2} \tag{13}$$

The constraint $q_u \leq q$ has thus far been ignored, so the last expression is only valid when $q \geq \frac{1}{2}$. It then follows from the concavity of π_2 that if $q \leq \frac{1}{2}$ then q_u^* will exactly equal q. That is:

$$q_u^* = Min \left(\frac{1 - \alpha}{2}, q \right). \tag{14}$$

Also, by inspection, p_u^* and p_n^* are always ≥ 0 as required. The same holds true for q_u^* and q_n^*. Writing out the equilibrium second period profit as function of first period parameters:

$$\pi_2^* = (1 - \alpha)^2 \left(\frac{U_2 - U_1}{4} \right) + (1 - q - \alpha)^2 \frac{U_2}{4}$$

Differentiating this twice with respect to q:

$$\frac{\partial \pi_2^*}{\partial q} = - (1 - q - \alpha) \frac{U_2}{2}$$

$$\frac{\partial^2 \pi_2^*}{\partial q^2} = \frac{U_2}{2}$$

To solve for the producer's problem of selecting the first period price, we derive the demand curve for the first period given optimal pricing in the second period. Substituting the price p_n^* from equation (12) into the indifference equation for $t_2 \ (= 1 - q)$:

$$(1 - q)\delta U_2 - \alpha \delta U_2 - \delta (1 - q - \alpha) \frac{U_2}{2} = (1 - q)(1 + \delta)U_1 - \alpha U_1 - p$$

Solving for p:

$$p = (1 - q) \left[(1 + \delta)U_1 - \frac{\delta U_2}{2} \right] + \frac{\alpha \delta U_2}{2} - \alpha U_1$$

Substituting this into π using equation (3):

$$\pi = (1 - q)q \left[(1 + \delta)U_1 - \frac{\delta U_2}{2} \right] + \alpha \left(\frac{\delta U_2 - 2U_1}{2} \right) q + \delta \pi_2^*.$$

---

[1]Throughout these proofs, the values derived are optimal given that customers' purchasing decisions are as implied by the thresholds t_1, t_2, and t_3 as described in the main text. Since these thresholds specify consumers' optimal purchasing behavior in response to posted prices, these optimal prices are also the equilibrium prices.

Taking the partial twice with respect to q :

$$\frac{\partial \pi}{\partial q} = (1 - 2q)\left[(1+\delta)U_1 - \frac{\delta U_2}{2}\right] + \alpha\left(\frac{\delta U_2 - 2U_1}{2}\right) + \delta\frac{\partial \pi_2^*}{\partial q} \tag{15}$$

$$\begin{aligned}
\frac{\partial^2 \pi}{\partial q^2} &= -(2(1+\delta)U_1 - \delta U_2) + \delta\frac{\partial^2 \pi_2^*}{\partial q^2} \\
&= -2(1+\delta)U_1 + \delta U_2 + \delta\frac{U_2}{2} \\
&= -\frac{1}{2}\left(4(1+\delta)U_1 - 3\delta U_2\right) < 0
\end{aligned}$$

Concavity of the overall objective function in q along with linear constraints ensures a unique equilibrium solution in the first period. For every q, there is exactly one equilibrium solution in the second period. Consequently, we have a unique subgame perfect equilibrium.

2) Continuing from part 1) of the proposition, concavity ensures that setting the first derivative to zero in equation (15) maximizes profits:

$$q^* = \frac{2\left[(1+\delta)U_1 - \delta U_2\right] + 2\alpha(\delta U_2 - U_1)}{4(1+\delta)U_1 - 3\delta U_2}$$

Using this expression in the second period constraints given by equation (2) gives us lower and upper bounds on α :

$$\alpha' = \frac{1}{1 + 4\left(\frac{U_1}{U_2}\right)} \leq \alpha \leq \frac{2\delta + 4(1+\delta)\cdot\left(\frac{U_1}{U_2}\right)^2 - (2+5\delta)\cdot\left(\frac{U_1}{U_2}\right)}{2\delta + 4(1+\delta)\cdot\left(\frac{U_1}{U_2}\right) - (2+3\delta)\cdot\left(\frac{U_1}{U_2}\right)} = \alpha''$$

When α obeys the bounds strictly, we get non-binding second period constraints (Zone A).

When α violates both bounds, we must have $\alpha'' \leq \alpha'$ and since $\alpha' < \alpha < \alpha''$ for non-binding constraints, the second period constraints must be binding when $\alpha'' \leq \alpha \leq \alpha'$ (Part of Zone C)

Setting $\alpha' = \alpha''$ and solving for $\frac{U_1}{U_2}$ gives us three points of intersection:

$$\frac{U_1}{U_2} = 0, \frac{3}{4}\left(\frac{\delta}{1+\delta}\right) \text{ and } \frac{1}{2}$$

Of these, only $\frac{U_1}{U_2} = \frac{1}{2}$ is over $\frac{\delta}{1+\delta}$ and we need to consider only this comparison because $\frac{U_1}{U_2} \in [\frac{\delta}{1+\delta}, 1]$ for minor to moderate upgrades. This gives us a clean bifurcation in terms of $\frac{U_1}{U_2}$ for the binding and non-binding constraint regions.

When we have $\alpha \leq Min(\alpha', \alpha'')$, we must have $\alpha \leq \alpha'$. When this is true, we know that $q_u^* = q^*$.

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Setting $q_u = q$, we can derive the optimal upgrade price in terms of q:

$$p_u^* = (1 - q - \alpha)(U_2 - U_1) \tag{16}$$

Using this value of p_u^*, we can rewrite the second and first stage profit at equilibrium second period pricing for $\alpha \leq \alpha'$ as:

$$\pi_2^* = (1 - q)q(U_2 - U_1) - \alpha q(U_2 - U_1) + (1 - q - \alpha)^2 \frac{U_2}{4} \tag{17}$$

$$\pi = (1 - q)q \frac{(2(1 + \delta)U_1 - \delta U_2)}{2} + \alpha \left(\frac{\delta U_2 - 2U_1}{2} \right) q + \delta \pi_2^*$$

Taking the derivatives with respect to q and simplifying:

$$\frac{\partial^2 \pi}{\partial q^2} = - \left(2U_1 + \delta \frac{U_2}{2} \right) < 0$$

Concavity ensures that setting the first derivative equal to zero gives us a profit maximizing solution:

$$q^* = \frac{(1 - q^*)\left[2(1 + \delta) U_1 - \delta U_2 \right] + \alpha (\delta U_2 - 2U_1)}{2} \tag{18}$$

Applying the second period pricing constraint $p_u < p_n$:

$$(1 - q - \alpha)(U_2 - U_1) < (1 - q - \alpha) \frac{U_2}{2}$$

Simplifying:

$$\frac{U_1}{U_2} \geq \frac{1}{2} \tag{19}$$

This marks the remaining part of Zone C and entire Zone B.

Finally, we have the case where $\alpha \geq Min(\alpha', \alpha'')$. For this case, we must have $\alpha \geq \alpha''$. When this is true, we know that $p_u^* = p_n^*$. We set $p_u = p_n = p_s$ in the second stage of the model and solve the unconstrained problem using equation (2):

$$\pi_2 = p_s \left(1 - \alpha - \frac{p_s}{U_2 - U_1} \right) + p_s \left((1 - q) - \alpha - \frac{p_s}{U_2} \right)$$

π_2 is concave in p_s as shown by $\frac{\partial^2 \pi_2}{\partial p_s^2} = -2 \left(\frac{1}{U_2 - U_1} + \frac{1}{U_2} \right) < 0$

Setting $\frac{\partial \pi_2}{\partial p_s} = 0$ gives p_s^*, the optimal value for p_s:

$$p_s^* = (2(1 - \alpha) - q) \left(\frac{U_2(U_2 - U_1)}{2(2U_2 - U_1)} \right) \tag{20}$$

This now enables us to calculate the expressions for q_u and q_n as functions of q.

The analysis has heretofore ignored the constraint $q_u \leq q$. Applying this constraint, we find that:

$$q \geq \frac{2(1-\alpha)(U_2 - U_1)}{3U_2 - 2U_1}$$

It follows from the concavity of π_2 that if $q \leq \frac{2(1-\alpha)(U_2-U_1)}{3U_2-2U_1}$, then q_u^* will exactly equal q, and generally:

$$q_u^* = Min\left(\frac{2(1-\alpha)(U_2-U_1)}{3U_2 - 2U_1}, q\right) \tag{21}$$

Using the indifference equation for t_2 and the expression for p_s, we can derive the first period demand curve and the overall profit function. Differentiating this profit function with respect to q, we get:

$$\frac{\partial^2 \pi}{\partial q^2} = -2U_1 + \frac{(U_2 - U_1)(7U_2 - 4U_1)}{2(2U_2 - U_1)}\delta$$

$\frac{\partial^2 \pi}{\partial q^2} \leq 0$ for $\frac{U_1}{U_2}$ in the range $\left[\frac{8+11\delta-\sqrt{64+64\delta+9\delta^2}}{8+8\delta}, \frac{8+11\delta+\sqrt{64+64\delta+9\delta^2}}{8+8\delta}\right]$ of which $\left[\frac{\delta}{1+\delta}, 1\right]$ is a subset. Setting $\frac{\partial \pi}{\partial q} = 0$ provides the optimal solution:

$$q^* = \frac{2(2U_2 - U_1)\left[(1+\delta)U_1 - \delta U_2 + \alpha(\delta U_2 - U_1)\right]}{(8+11\delta)U_1U_2 - 4(1+\delta)U_1^2 - 7\delta U_2^2}$$

Applying the constraint on upgrade consumers using equation (21), we get the following condition on α:

$$\alpha \geq Min\left[Max\left[\frac{\delta - \left(\frac{U_1}{U_2}\right)\left(2 + 5\delta - \left(\frac{U_1}{U_2}\right)\left(5 + 6\delta - 2(1+\delta)\left(\frac{U_1}{U_2}\right)\right)\right)}{\delta - \left(\frac{U_1}{U_2}\right)\left(2 + 11\delta - \left(\frac{U_1}{U_2}\right)\left(5 + 13\delta - 2(1+2\delta)\left(\frac{U_1}{U_2}\right)\right)\right)}, 0\right], 1\right]$$

Label the expression on the right as α'''. Analysis of this expression with respect to α' and α'' is analytically intractable for any general δ. Hence, we perform a grid search where we vary δ from 0 to 1 in steps of 0.05 to observe the following:

$$\alpha''' < \alpha' \text{ for } \delta > 0.21$$

Thus, for reasonably large δ, this threshold does not affect the thresholds developed so far and the entire area given by $\alpha > Max(\alpha', \alpha'')$ is classified as Zone D. If δ is smaller, the threshold between Zones C & D is given by the following function:

$$Max\left[\frac{1}{1 + 4\left(\frac{U_1}{U_2}\right)}, \frac{\delta - \left(\frac{U_1}{U_2}\right)\left(2 + 5\delta - \left(\frac{U_1}{U_2}\right)\left(5 + 6\delta - 2(1+\delta)\left(\frac{U_1}{U_2}\right)\right)\right)}{\delta - \left(\frac{U_1}{U_2}\right)\left(2 + 11\delta - \left(\frac{U_1}{U_2}\right)\left(5 + 13\delta - 2(1+2\delta)\left(\frac{U_1}{U_2}\right)\right)\right)}\right]$$

Proof of Proposition 3

The boundary between Zone A and Zone B is the curve in $\left(\frac{U_1}{U_2}, \alpha\right)$ space given by:

$$\alpha = \frac{1}{1 + 4\left(\frac{U_1}{U_2}\right)} \tag{22}$$

The boundary between Zone B and Zone C is given by the line:

$$\frac{U_1}{U_2} = \frac{1}{2}$$

The boundary between Zone C and Zone D is given by:

$$\alpha = Max\left[\frac{1}{1 + 4\left(\frac{U_1}{U_2}\right)}, \frac{\delta - \left(\frac{U_1}{U_2}\right)\left(2 + 5\delta - \left(\frac{U_1}{U_2}\right)\left(5 + 6\delta - 2\left(1 + \delta\right)\left(\frac{U_1}{U_2}\right)\right)\right)}{\delta - \left(\frac{U_1}{U_2}\right)\left(2 + 11\delta - \left(\frac{U_1}{U_2}\right)\left(5 + 13\delta - 2\left(1 + 2\delta\right)\left(\frac{U_1}{U_2}\right)\right)\right)}\right]$$

The boundary between Zones D and A is specified by the curve:

$$\alpha = \frac{2\delta + 4(1 + \delta)\cdot\left(\frac{U_1}{U_2}\right)^2 - (2 + 5\delta)\cdot\left(\frac{U_1}{U_2}\right)}{2\delta + 4(1 + \delta)\cdot\left(\frac{U_1}{U_2}\right)^2 - (2 + 3\delta)\cdot\left(\frac{U_1}{U_2}\right)}$$

The boundary between the Zone "large" and the Zones C & D is given by the vertical line:

$$\frac{U_1}{U_2} = \frac{\delta}{1 + \delta}$$

1) Zone C is bounded by the following curves:

$\alpha = 0$ (bottom), $\frac{U_1}{U_2} = \frac{\delta}{1+\delta}$(left), $\alpha = Max\left[\alpha', \alpha'''\right]$(top), and $\frac{U_1}{U_2} = \frac{1}{2}$(right)

The bound on the left is an increasing function of δ. It can be shown that α''' is a decreasing function of δ by observing the sign of the second derivative:

$$\frac{d\alpha'''}{d\delta} = -\frac{(2 - x)^2(2x - 1)(2x - 3)}{(\delta - x\left(2 + 11\delta - x\left(5 + 13\delta - 2x\left(1 + 2\delta\right)\right)\right))^2}$$

where $x = \frac{U_1}{U_2}$. $\frac{d\alpha'''}{d\delta} < 0$ for $x < \frac{1}{2}$(moderate upgrades: since this corresponds to Zone C). Since α' is independent of δ, the function $Max\left[\alpha', \alpha'''\right]$ is decreasing in δ. As δ increases, this bound on the

left moves towards the right (increases) and the bound on the top moves downwards (decreases). Consequently, the size of Zone C decreases. At $\delta = 1$, the bounds on the left and right coincide, thus Zone C disappears.

2) At $\delta = 1$, Zone D is bounded by the following curves:

$\alpha = \dfrac{2\delta + 4(1+\delta)\cdot\left(\frac{U_1}{U_2}\right)^2 - (2+5\delta)\cdot\left(\frac{U_1}{U_2}\right)}{2\delta + 4(1+\delta)\cdot\left(\frac{U_1}{U_2}\right)^2 - (2+3\delta)\cdot\left(\frac{U_1}{U_2}\right)}$ (bottom), $\frac{U_1}{U_2} = \frac{1}{2}$ (left), $\alpha = 1$ (top), and $\frac{U_1}{U_2} = 1$ (right)

Using the fact that $\delta = 1$, the bound from the bottom simplifies to:

$$\alpha = \frac{2 + 8x^2 - 7x}{2 + 8x^2 - 5x}$$

where $x = \frac{U_1}{U_2}$.

The area of Zone D at $\delta = 1$ is given by the following expression:

$$Area(D) = \frac{1}{2} - \int_{\frac{1}{2}}^{1} \left(\frac{2 + 8x^2 - 7x}{2 + 8x^2 - 5x}\right) dx$$

$$= 0.272 > 0$$

Proof of Proposition 4

To analyze the market coverage when upgrade pricing is disallowed, we go back to the profit function in the second period but with a symmetric second price. However, we need to do this separately for Zones A & B since the parameter space for each case is different and this has an implication on whether all old customers upgrade.

Case 1: Zone A $\left(\alpha' < \alpha < \alpha'' \ \& \ \frac{U_1}{U_2} > \frac{1}{2}\right)$

The second period profit function can be written from equation (2):

$$\pi_2 = p_s\left(1 - \alpha - \frac{p_s}{U_2 - U_1}\right) + p_s\left((1-q) - \alpha - \frac{p_s}{U_2}\right)$$

The analysis from here onwards is similar to that in Zone D. Thus, the outcome $q_u^* = q^*$ occurs only when $\alpha \leq \alpha'''$. But we can show that for $\delta = 1$, $\alpha''' < \alpha'$, given $\delta = 1$ and $\frac{U_1}{U_2} > \frac{1}{2}$ (proof

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available with the authors). Since $\alpha > \alpha'$, we can take $q_u^* < q^*$ at equilibrium and the expressions for prices and market sizes in Zone D hold. Thus, the market coverage can be obtained from the tables in the main text:

$$
\begin{aligned}
coverage(disallowed) &= q^* + \frac{2(1-\alpha)U_2 - q^* \cdot (3U_2 - U_1)}{2U_2 - U_1} \\
&= \frac{2(1-\alpha)U_2}{2U_2 - U_1} - \frac{q^*U_2}{2U_2 - U_1} \\
&= \frac{2(1-\alpha)U_2}{2U_2 - U_1} - \frac{2U_2\left[2U_1 - U_2 + \alpha\left(U_2 - U_1\right)\right]}{19U_1U_2 - 8U_1^2 - 7U_2^2}
\end{aligned}
$$

$$
coverage(allowed) = \frac{1-\alpha}{2} + \frac{\left[(1+\delta)\,U_1 - \delta U_2\right] + \alpha(\delta U_2 - U_1)}{4\,(1+\delta)\,U_1 - 3\delta U_2}
$$

Applying the condition $coverage(disallowed) > coverage(allowed)$, we get the following condition on α:

$$
\alpha < 1 - \frac{8x\,(2 - x\,(5 - 2x))}{16 - x\,(77 - x\,(169 - 64x\,(3 - x)))} \tag{23}
$$

where $x = \frac{U_1}{U_2}$. Denote the expression on the right as α_u. We can show that $\alpha'' < \alpha_u$ for $x > \frac{1}{2}$ (proof available with the authors). Since we have $\alpha' < \alpha < \alpha''$ in Zone A, this ensures that inequality (23) is satisfied and the market coverage when upgrade pricing is disallowed is higher.

Case 2: Zone B $\left(\alpha \le \alpha' \ \& \ \frac{U_1}{U_2} > \frac{1}{2}\right)$

The second period profit function can be written from equation (2):

$$
\pi_2 = p_s\left(1 - \alpha - \frac{p_s}{U_2 - U_1}\right) + p_s\left((1 - q) - \alpha - \frac{p_s}{U_2}\right)
$$

The analysis from here onwards is similar to that in Zone D. Thus, the outcome $q = q^*$ occurs only when $\alpha \le \alpha'''$. Earlier in this proposition, we showed that for $\frac{U_1}{U_2} > \frac{1}{2}$ and $\delta = 1$, we have $\alpha''' < \alpha'$ Thus, we get two subcases. When $0 \le \alpha \le \alpha'''$ we take $q_u^* = q^*$ at equilibrium and the expressions for Zone C apply. When $\alpha''' \le \alpha \le \alpha'$, we take $q_u^* < q^*$ at equilibrium and the expressions for prices and market sizes in Zone D hold. First, we state the expression for market coverage when upgrade pricing is allowed.

$$
coverage(allowed) = \frac{1-\alpha}{2} + \frac{U_1}{4U_1 + U_2}
$$

Subcase 1: $0 \le \alpha \le \alpha'''$ and $\frac{U_1}{U_2} > \frac{1}{2}$

We use the market size expressions corresponding to Zone C from the tables in the main text.

$$coverage(disallowed) = (1 - \alpha)\frac{U_1}{U_2} + q^* \left(\frac{U_2 - U_1}{U_2}\right)$$

$$= (1 - \alpha)\frac{U_1}{U_2} + \left(\frac{2U_1^2 - 2U_1U_2 + U_2^2 - \alpha\left(2U_1^2 - 3U_1U_2 + U_2^2\right)}{2\left(U_1^2 - U_1U_2 + U_2^2\right)}\right)\left(\frac{U_2 - U_1}{U_2}\right)$$

Applying the condition $coverage(disallowed) > coverage(allowed)$, we get the following condition on α:

$$\alpha < \frac{2 + x}{1 + 4x}$$

where $x = \frac{U_1}{U_2}$. This is true since for Zone B, we have $\alpha \leq \alpha' < \frac{2+x}{1+4x}$.

Subcase 2: $\alpha''' \leq \alpha \leq \alpha'$ and $\frac{U_1}{U_2} > \frac{1}{2}$

We use the market size expressions corresponding to Zone D from the tables in the main text.

$$coverage(disallowed) = \frac{2(1 - \alpha)U_2}{2U_2 - U_1} - \frac{2U_2\left[2U_1 - U_2 + \alpha\left(U_2 - U_1\right)\right]}{19U_1U_2 - 8U_1^2 - 7U_2^2}$$

Applying the condition $coverage(disallowed) > coverage(allowed)$, we get the following condition on α:

$$\alpha < \frac{4 + x\left(-19 + x\left(57 - 86x + 32x^2\right)\right)}{(1 + 4x)\left(4 + x\left(-3 + x\left(-11 + 6x\right)\right)\right)}$$

where $x = \frac{U_1}{U_2}$. Label the expression on the right as α_u. We can show that for $x > \frac{1}{2}$ and $\delta = 1$, we have $\alpha' < \alpha_u$ (proof available with the authors).

Since $\alpha < \alpha'$ in Zone B, we have the required result.

Proof of Proposition 5: 1) The second period profit function can be written out from equation (2) as follows:

$$\pi_2 = p_u\left(1 - F\left(\alpha + \frac{p_u}{U_2 - U_1}\right)\right) + p_n\left(1 - q - F\left(\alpha + \frac{p_n}{U_2}\right)\right)$$

Maximizing π_2 over p_u and p_n without regard for the constraints provides unique solutions for p_u and p_n (denote them by $s_u = \frac{p_u}{U_2 - U_1}$ and $s_n = \frac{p_n}{U_2}$) and are obtained implicitly as solutions to:

$$s_u \cdot h\left(\alpha + s_u\right) = 1 \tag{24}$$

and

$$(1 - F(\alpha + s_n)) \cdot (1 - s_n \cdot h(\alpha + s_n)) = q \tag{25}$$

where h is the hazard rate function. This result follows from Lariviere(2006). Thus we have a unique pure strategy equilibrium in the second period subgame.

Set γ to be the lowest type consumer who purchases the product in period 1. Then, based on the arrangement of segments, we have:

$$q = 1 - F(\gamma) \tag{26}$$

The second period constraints can then be derived in terms of first period decision γ.

Pricing constraint $(p_u \leq p_n)$:

$$(U_2 - U_1) \cdot s_u \leq U_2 \cdot s_n$$

Rearranging this equation, we get:

$$\frac{U_1}{U_2} \geq 1 - \frac{s_n}{s_u} \tag{27}$$

The above equation is valid at second period subgame equilibrium for any first period price / demand. The rhs of equation (27) provides the required threshold.

2) Applying the upgrade sales constraint $(q_u \leq q)$ using the definition of s_u and equation (26):

$$1 - F(\alpha + s_u) \leq 1 - F(\gamma)$$

or equivalently:

$$\gamma \leq \alpha + s_u \tag{28}$$

The equilibrium second period profit is:

$$\pi_2^* = (U_2 - U_1) \cdot s_u \cdot (1 - F(\alpha + s_u)) + U_2 \cdot s_n \cdot (1 - (1 - F(\alpha + s_n)) \cdot (1 - s_n \cdot h(\alpha + s_n)) - F(\alpha + s_n))$$

$$= (U_2 - U_1) \cdot s_u \cdot (1 - F(\alpha + s_u)) + U_2 \cdot s_n \cdot (1 - F(\alpha + s_n)) - U_2 \cdot q \cdot s_n \text{ (from equation (25)}$$

Note here that s_u is a known number from equation (24) while s_n is a function of q and hence a function of γ.

The first period demand curve is derived by writing the indifference condition for t_2:

$$p = \delta U_2 \cdot s_n + \gamma \cdot ((1 + \delta)U_1 - \delta U_2) + \alpha \cdot (\delta U_2 - U_1) \tag{29}$$

The overall profit function including the first period is given by equation (3):

$$\pi = pq + \delta\pi_2^* \tag{30}$$

$$= ((1+\delta)U_1 - \delta U_2) \cdot \gamma \cdot (1 - F(\gamma)) + \alpha \cdot (\delta U_2 - U_1)(1 - F(\gamma))$$

$$+ \delta(U_2 - U_1) \cdot s_u \cdot (1 - F(\alpha + s_u)) + \delta U_2 \cdot s_n \cdot (1 - F(\alpha + s_n))$$

The above profit function is continuous function over a compact set $[0, 1]$. Hence, it must have an optimal solution in γ. This is a pure strategy equilibrium for the game between the firm and consumers. For every such equilibrium, there exists a unique pure strategy equilibrium in the second period subgame as shown in part 1). Hence, there always exists a subgame perfect equilibrium in pure strategies.

Taking a derivative with respect to γ and observing that s_u is independent of γ while s_n is a function of γ :

$$\frac{\partial \pi}{\partial \gamma} = ((1+\delta)U_1 - \delta U_2) \cdot (1 - F(\gamma)) \cdot (1 - \gamma h(\gamma)) - \alpha \cdot (\delta U_2 - U_1) f(\gamma)$$

$$+ \delta U_2 \cdot (1 - F(\alpha + s_n)) \cdot (1 - s_n \cdot h(\alpha + s_n)) \cdot \frac{\partial s_n}{\partial \gamma}$$

Observe that $\frac{\partial s_n}{\partial \gamma} > 0$ using equations (25) and (26). Hence, the last term in the profit expression is strictly increasing. First, we show that an optimum cannot occur at the boundaries of the domain $[0, 1]$.

If $\gamma = 1$, then from equation (28) we must have $s_u = 1 - \alpha$, which is impossible since this violates equation (24). If $\gamma = 0$, then equation (25) is violated when $\alpha > 0$. Also, $s_n = 0$ when $\alpha = 0$ from equation (25) and the overall profit turns out to be:

$$\pi^* = \delta(U_2 - U_1) \cdot s_u \cdot (1 - F(s_u))$$

This cannot be the equilibrium since one can always find a strategy that earns higher profit that this. For instance, consider the strategy $p = \infty$ (high enough to cause $q = 0$), and optimize only for the second period for over p_n. This would give overall profit corresponding to:

$$\pi^* = \delta U_2 \cdot \beta \cdot (1 - F(\beta)) > \delta(U_2 - U_1) \cdot s_u \cdot (1 - F(s_u))$$

where β is the solution to equation (24) and hence $\beta = s_u$. Hence γ must occur in the interior of $[0,1]$. For an optimum γ to occur in the interior of the range $[0,1]$, the first derivative must

necessarily be zero. For this, the first two terms of the profit expression in equation (30) must together be non-increasing at the optimal γ or in terms of the derivative:

$$((1 + \delta)U_1 - \delta U_2) \cdot (1 - F(\gamma)) \cdot (1 - \gamma h(\gamma)) - \alpha \cdot (\delta U_2 - U_1) \cdot f(\gamma) \leq 0$$

This simplifies to:

$$(\gamma - \alpha) h(\gamma) + \alpha \cdot \left(\frac{\delta U_1}{(1 + \delta)U_1 - \delta U_2} \right) \cdot h(\gamma) \geq 1 \tag{31}$$

Notice the structure of this first term in the above expression as a function of the variable γ. It is similar to equation (24) but with an extra term. Given that $h(\gamma)$ is an increasing function of γ, the inequality (31) is solved for a value of γ lower than in equation (24). The solution in terms of γ is:

$$\gamma^* \geq s_u - k(\alpha)$$

where $k(\alpha)$ is a positive value and $k(0) = 0$. This when combined with equation (28) ensures that:

$$\alpha + s_u \geq \gamma^* \geq s_u - k(\alpha) \tag{32}$$

This collapses to $\gamma = s_u$ when $\alpha = 0$. Hence, $\alpha > 0$ is a necessary condition. It is not sufficient as shown by the counter example in Proposition 2 where $F(\theta)$ is uniformly distributed and there exists a strictly positive α below α' such that not all customers upgrade.

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